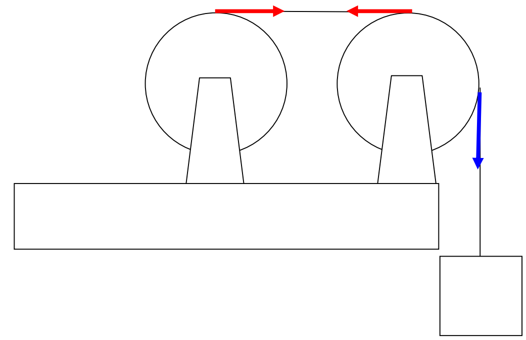
Lecture 17 Announcements

1. Results of class voting
   a. New problem solving session?
      i. Yes! Fri 3-4 pm, room 455 MARB
      ii. (old one still Mon 1-2 pm, room 455 MARB)
   b. More vs. fewer problems worked in class?
      i. Fewer won by a little, but (more + same) combined were about as much
      ii. Probably I won't change too much, perhaps just go a little bit slower through problems
   c. Warmup comments at start of class?
      i. "Fine as is" won the voting
      ii. Probably I won't change too much
   d. An additional (evening) exam review session?
      i. Yes! Needs to be scheduled still if I can find a TA
   e. HW graded as soon as you turn it in
      i. Yes! Now implemented!
   f. Lecture notes with handwritten stuff from class
      i. Yes. Will be posted to website.

(From last time)



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is **no slipping of the rope on either wheel and both wheels accelerate the same**. The tension in the rope is
   a. Largest between the disks
   b. Largest above the mass
   c. The same in both places.

What's different with our old "massless pulleys"?

Angular Correspondences Review

Kinematics

Distance: x	Angle: θ
Velocity: v	Angular velocity: ω
Acceleration: a	Angular accel.: α
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Mass

Mass: m	Moment of inertia: I

Force/Newton's 2nd Law

Force: F	Torque: τ
$\sum \vec{F} = m\vec{a}$	$\sum \tau = I\alpha$

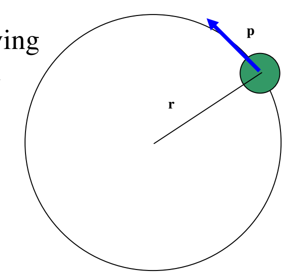
Energy

$KE_{trans} = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$

Momentum…

Momentum: p $(= mv)$	Angular momentum??

Angular momentum



Imagine a mass *m* on a thin rod moving in a circle, with constant speed v. It has linear momentum $\vec{p}$ =_______.
Is $\vec{p}$ constant?_______
Is $|p|$ constant?_______

What do we need to change $|p|$?

<u>Force-momentum relationship</u>
Start with Newton 2:
$\sum \vec{F} = m\vec{a}$

<u>Torque-ang. mom. relationship</u>
$\sum \tau = I\alpha$

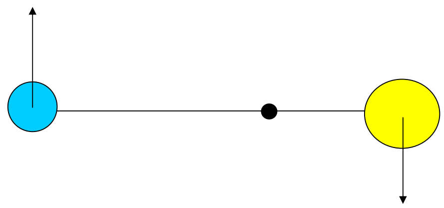
Define $L = I\omega$

If no external forces, no change in momentum

If no external *torques*, no change in *angular* momentum

Conservation of Angular momentum

$\Sigma L_{bef} = \Sigma L_{aft}$ → if and only if *no external torques*



Imagine two space stations connected by a cable. They are rotating about their center of mass.

Someone in the blue station pulls the cable in so they are each closer to the center of rotation. What happens?

Clicker quiz: Is rotational kinetic energy conserved? The total energy afterwards is:

a. more
b. less
c. the same

Hint: is there any non-conservative work done?

Skaters spinning

(frictionless ice)

Clicker quiz: When an ice skater brings her arms close to her body during a spin, her moment of inertia

a. decreases
b. increases
c. stays the same

Clicker quiz: When an ice skater brings her arms close to her body during a spin, her angular momentum

a. decreases
b. increase
c. stays the same

Demo: Hoberman sphere
Demo: spinning chair
Videos: train on circular track, pocket watch

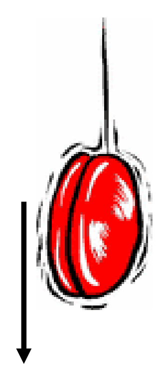
Worked Problem: A skater has an initial ω of 2 rad/s and $I = 30$ kg·m^2. When she brings in her arms, $I = 10$ kg·m^2. What is the final ω?

How much energy did it take to do this?

Answers: 6 rad/s, 120 J

Is L conserved in these cases?

"Teacups": Hands-on-post is connected to the platform floor

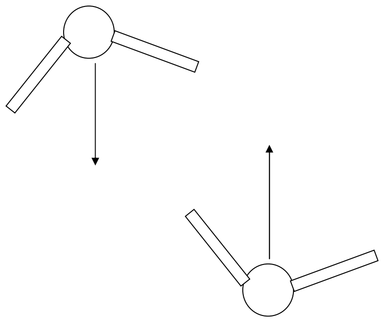


Demo: gyroscopes

Food for thought: two skaters joining hands

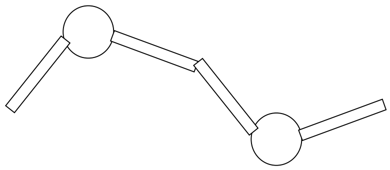
Before

Any L?



After

Any L?



Is angular momentum conserved?
Yes if there are no external torques

Clicker quiz: Was there an external torque here?
a. Yes
b. No

Another expression for L…

Start with
$\tau = r_{\perp} F$

Remember
$F_{net} = \Delta p/\Delta t$
$\tau_{net} = \Delta L/\Delta t$

Result: $\boxed{L = r_{\perp} p}$ $(= r\,p_{\perp} = r\,p\,\sin\theta)$

Worked Problem: Skaters on previous page have 0.7 m arms and are each 62 kg. They come together at 3.5 m/s. How fast (rad/s) are they turning afterwards?

Answer: 5 rad/s

Comment on vectors… (aka L has a direction!)

Does ω have a direction? ___________

Therefore ___________________

Thus with **no external torques**…

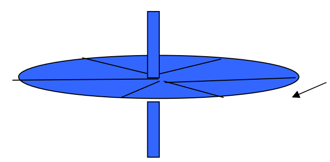
Demos: gyro

With external torque weird things happen!

Demos: briefcase, bicycle wheel (Ralph's question)

José sits on frictionless ice, holding a spinning bicycle wheel. View from above it is going **clockwise** (CW).

Neglect external friction.

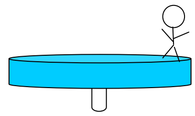


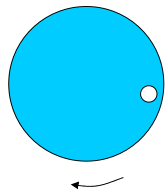
Clicker quiz: If he grabs on to the wheel edge firmly and "stops" it he will then be
a. turning CW (viewed from the top)
b. turning CCW
c. not turning

Clicker quiz: If instead of stopping it he flips the wheel over, so it is going CCW (viewed from the top), he will be
a. turning CW, but slower than in the previous problem
b. turning CCW, slower than in the previous problem
c. turning CW, faster than in the previous problem
d. turning CCW, faster than in the previous problem

Demos: "lazy Susan platform"—bicycle wheel (Ralph's question), combined bike wheels

A girl is on a spinning merry-go round.





What will happen to the rotational speed ω of the merry-go-round if she…

HINT: Sometimes it's easier to think of the **forces (torques)** she put on the merry-go-round to change, rather than conservation of L.

Clicker quiz 1: Walks towards the center?
- a. it slows down
- b. it stays same speed
- c. it speeds up

Clicker quiz 2: Runs opposite to the spinning so she is at rest vs the ground? (same choices)

Clicker quiz 3: Slips off when she steps on a frictionless icy part? (same choices)

Clicker quiz 4: She jumps so that she is moving radially outward versus the ground? (same choices)

Clicker quiz 5: Jumps tangentially off the merry-go-round so that she goes faster relative to the ground than she was before she jumped? (same choices)