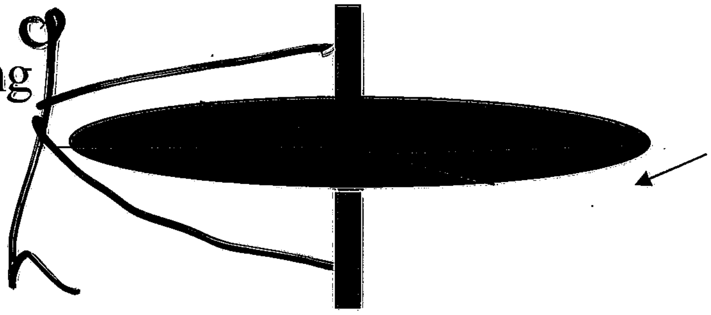
José sits on frictionless ice, holding a spinning bicycle wheel. View from above it is going **clockwise** (CW). Neglect external friction.



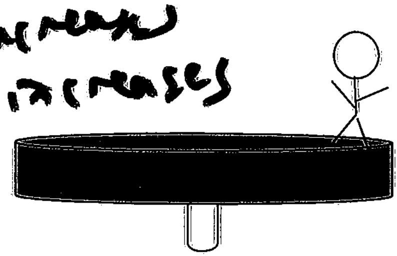
Clicker quiz 1: If he grabs on to the wheel edge firmly and "stops" it he will then be

a. turning CW (viewed from the top)
b. turning CCW
c. not turning

$\Sigma L_{bef} = \Sigma L_{aft}$

Maria is on a spinning merry-go round. What will happen to its rotational speed if she…

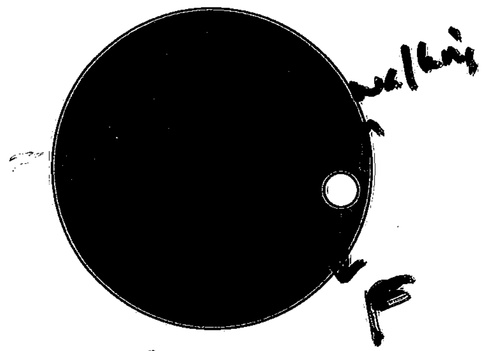
I decreases
ω increases



Clicker quiz 2: Walks towards the center?

a. it slows down
b. it stays same speed
c. it speeds up

$L = I\omega$
$L = r_\perp p$



Clicker quiz 3: Runs opposite to the spinning so she is at rest vs the ground? (same choices) (c)

$L_{1\,bef} + L_{2\,bef} = L_{1\,aft} + 0$ ← she has no L after

Clicker quiz 4: Slips off when she steps on a frictionless icy part? (same choices) (b)

L not conserved (system)
L conserved (just merry go round)

Formulas Review

Definitions and Fundamental Laws

Final exam: you will be expected to know these

Definition of momentum: $\vec{p} = m\vec{v}$

Conservation of momentum: $\sum \vec{p}_{before} = \sum \vec{p}_{after}$ (if …) no net external force

Angular formulas from old formulas:

$x \rightarrow \theta$
$v \rightarrow \omega$ } e.g. definitions/three kinematic equations
$a \rightarrow \alpha$

$\frac{1}{2}mv^2 \rightarrow \frac{1}{2}I\omega^2$ $p = mv \rightarrow L = I\omega$

$m \rightarrow I$ e.g. rotational kinetic energy, angular momentum

$F \rightarrow \tau$ e.g. Newton's 2nd Law for torques $\Sigma F = ma$

$p \rightarrow L$ $\Sigma \tau = I\alpha$

Definition of torque: $\tau = r_{\perp}F = rF_{\perp} = rF\sin\theta$

Conservation of L: $\sum L_{before} = \sum L_{after}$ (if …) → no net external torque

s = distance

New stuff, but not quite as basic

Final exam: I will give you these

arc length: $s = r\theta$

tangential $v = r\omega$

tangential $a = r\alpha$

$a_c = v^2/r$

"Impulse eqn": $\vec{F}\Delta t = \Delta\vec{p}$ → $F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}$

"velocity reversal eqn": $(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$ for elastic collisions

Universal gravity: $F_g = \frac{GMm}{r^2}$, $PE_g = -\frac{GMm}{r}$ ↑ increases

0 ←

negative

Moments of inertia:

$I_{pt\,mass}$ = $I = mR^2$ (point mass going in circle)

I_{sphere} = $I = 2/5\ mR^2$ (sphere rotating about center)

I_{hoop} = $I = mR^2$ (hoop rotating about its axis)

I_{disk} = $I = 1/2\ mR^2$ (disk or cylinder about its main axis)

$I = 1/12\ mL^2$ (rod about its center)

$I = 1/3\ mL^2$ (rod about its end)

Angular momentum, definition 2: $L = r_{\perp} p = r p_{\perp} = rp \sin\theta$

↳ mv = kg m/s

<u>Things which *you* might consider to be formulas</u>

(but I don't really, so I won't give them to you on exam)

Relationship between speed and period: $v = 2\pi r/T$ ← distance/time

Quick derivation of satellite orbital velocity:

$$\Sigma F = ma_c \rightarrow \frac{GMm}{r_{orbit}^2} = \frac{mv_{orbit}^2}{r_{orbit}}$$

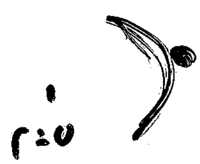
Quick derivation of escape velocity:

$$E_{bef} = E_{aft} \rightarrow -\frac{GMm}{r_{planet}} + \frac{1}{2} m v_{escape}^2 = 0$$

(be clear about which r to use…)

$r = \infty$

$I_{tot} = I_1 + I_2 + \ldots$



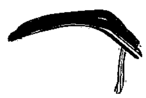
Kepler's 3rd Law

Exam 3 - Review of important concepts

1. Momentum
   a. Definition
   b. Conservation Law
   c. Collision problems $(\Sigma p_x)_{bef} = (\Sigma p_x)_{aft}$
      i. 1D $(\Sigma p_y) = (\Sigma p_y)$
      ii. 2D – think of momentum components in each direction
      Inelastic: use cons. of momentum
   d. Elastic vs. Inelastic Elastic: use cons. of mom. + velocity reversal eqn
   e. Combination problems (e.g. bullet into block of wood)
   f. Center of mass motion
   g. Impulse eqn
2. Rotational motion
   a. Angular quantities: θ, ω, α
      i. How they relate to "regular" quantities
      ii. Radians – 2π radians in a circle
   b. Connection between linear and rotational motion: $v = \omega r$, etc. $a = \alpha r$ → tangential acceleration
   c. Kinematic equations for constant angular acceleration
      i. also constant *tangential* acceleration
   d. Period vs. velocity vs. ω

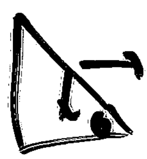
$$v = \frac{2\pi r}{T}$$

3. Centripetal acceleration, $a_c = v^2/r$
   a. Difference between centripetal and tangential
      i. …and when the two are combined going in a circle + speeding up
   b. Newton's Law of Gravity and orbits
      i. Force equation
      ii. Potential energy equation
      iii. Orbital velocity and/or escape velocity
   c. "Roller coaster" problems
      i. Normal force = 0 when you "fly out of your seat"



$\Sigma F = ma_c = m\frac{v^2}{r}$ set $N=0$

4. Torque
   a. Definition
      i. "lever arm" concept, $r_\perp$
   b. Equilibrium problems: $\Sigma F = 0$, $\Sigma\tau = 0$
   c. Moment of inertia
      i. Equation for I for various situations will be given
   d. Newton's 2nd Law for rotations: $\Sigma\tau = I\alpha$
   e. Torques and rotation
      i. Combining Newton 2 with kinematics

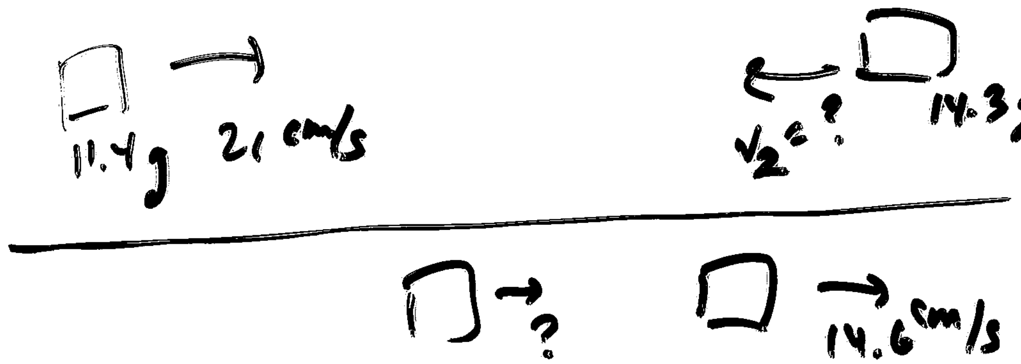


5. Rotational kinetic energy & momentum
   a. Definitions $KE = \frac{1}{2}I\omega^2$ $L = I\omega$
   b. Conservation laws
   c. The two expressions for L:

$$L = r_\perp p \quad \text{and} \quad L = I\omega$$

Some HW problems (missed by many):

HW 10 Problem 2. A 11.4-g object moving to the right at 21 cm/s makes an elastic head-on collision with a 14.3-g object moving in the opposite direction with some unknown velocity. After the collision, the second object is observed to be moving to the right at 14.6 cm/s. Find the initial velocity of the second object.



$$\Sigma P_{bef} = \Sigma P_{aft}$$

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

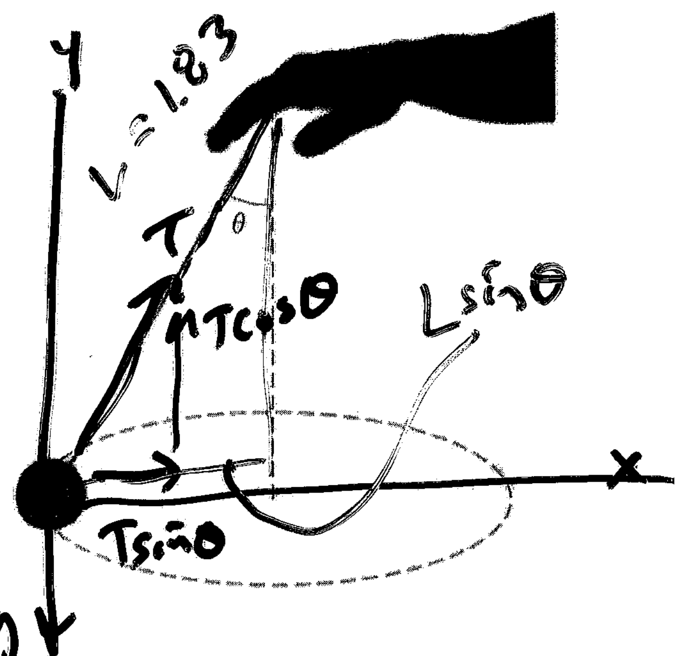
$$(.0114)(.21) - v_2(.0143) = (.0114)v_{1f} + (.0143)(.146)$$

$$.21 - (-v_2) = .146 - v_{1f}$$

Solve for v_{1f} → plug in

Answer: 36 m/s to the left

HW 11 Problem 6. A 0.537-kg ball that is tied to the end of a 1.83-m light cord is revolved in a horizontal plane with the cord making a 28.6° angle with the vertical. (a) Draw a free-body diagram of the ball. (b) Determine the ball's speed. (c) If instead the ball is revolved so that its speed is 4.24 m/s, what angle does the cord make with the vertical? (d) If the cord can withstand a maximum tension of 8.76 N, what is the highest speed at which the ball can move?



$\Sigma F_x = ma_c = \frac{mv^2}{r}$

$T\sin\theta = \frac{mv^2}{r}$

$\Sigma F_y = 0$

$T\cos\theta - mg = 0$

$T = mg/\cos\theta$

$mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{L\sin\theta}$

(b) $v = \sqrt{gL\frac{\sin^2\theta}{\cos\theta}}$

$v^2 = \frac{gL(1-\cos^2\theta)}{\cos\theta}$ let $\cos\theta = x$

(c) same equations!

$v^2 x = gL - gLx^2$

$gLx^2 + v^2x - gL = 0$

→ use quadratic form

→ take $\cos^{-1}$

(d) Solve for θ

Solve for v

Answers: 2.16 m/s, 51.9°, 4.37 m/s

Some conceptual quizzes

Clicker quiz: An elastic collision means:

a. the objects deform when they collide
b. each object keeps its kinetic energy when they collide
c. the total kinetic energy of the objects stays the same
d. both b and c

Clicker quiz: Newton's second law ($\Sigma F=ma$) for rotational motion is:

a. $\Sigma\tau = I\alpha$
b. $\Sigma\tau = I\omega$
c. $\Sigma\tau = L\alpha$
d. $\Sigma\tau = L\omega$

Clicker quiz: You go around a curve in your car at constant speed. The tangential acceleration of the car is zero.

a. True
b. False

Clicker quiz: The net horizontal force on the car is:

a. Toward the center of the circle
b. Away from the center of the circle
c. Tangent to the circle, in the direction of travel
d. Tangent to the circle, opposite the direction of travel

More Conceptual Questions

Clicker quiz. A boy is at the stern (back) of a sailboat with a bunch of beanbags. The wind has stopped. If the boy throws the beanbags against the sail with sufficient velocity, he can get the boat to move forward.

a. True
b. False

Clicker quiz. Two snowballs are thrown at each other. One is 2 kg traveling to the right at 3 m/s. The second is 0.5 kg traveling to the left at 8 m/s. After they collide they stick together. In this collision, total kinetic energy was conserved.

a. True
b. False

Clicker quiz: Their velocity after they stick is:

a. to the right
b. to the left

Clicker quiz: A large solid steel sphere and a small steel hoop are rolled down an inclined plane. Which reaches the bottom first?

a. Sphere I is $\frac{2}{5}mr^2$
b. Hoop

$I_{hoop} = mr^2$

c. Tie
d. Need to know masses
e. Need to know diameters