Announcements – 6 Oct 2009

1. Exam 2 still going on
   a. ends tomorrow, late fee after 1 pm

2. Next HW due Sat night (HW 9)

3. Don't forget Oct 24 - Deadline to get extra point on extra credits
   a. You automatically get +1 if you turn in extra credit before Oct 24.

Where are we now?

<u>Topics</u>

Kinematics (velocity, acceleration)
Vectors & 2D Motion
Forces & Newton's Laws
Work & Energy
Momentum
Rotations, Torque, and Angular Momentum
} "Mechanics"

Pressure
Fluids & Solids
Temperature, Heat, and Heat Flow
Laws of Thermodynamics
} "Thermodynamics"

Vibrations & Waves → { Part Mechanics, Part Sound, Part Optics

Conserved quantities

Energy
→ When no non-conservative work done, $E_{bef} = E_{aft}$

Mass
→ If not converted to/from energy,
(total mass)$_{bef}$ = (total mass)$_{aft}$

Charge
→ (total charge)$_{bef}$ = (total charge)$_{aft}$
I.e., if some positive charge flows out of a neutral object, it will leave the object with a negative charged

<u>Often conserved (used to balance chemical reactions)</u>
Number of each type of atom
Number of electrons

Etc.

<u>A new conserved quantity… **momentum**</u>

Define $\boxed{\vec{p} = m\vec{v}}$ for each object, then

$$\boxed{\sum \vec{p}_{before} = \sum \vec{p}_{after}}$$ (if no external forces)

Another blueprint equation!

Momentum: used for Collision Problems

Derivation of conservation law:

$\Sigma \mathbf{F}_1 = m_1\mathbf{a}_1$ $\Sigma \mathbf{F}_2 = m_2\mathbf{a}_2$

Newton's 3rd Law: the forces in the collision are __________ and ______________

If no other forces, then…
$\mathbf{F}_{2\text{-}1} + \mathbf{F}_{1\text{-}2} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2$
$0 = m_1\Delta\mathbf{v}_1/\Delta t + m_2\Delta\mathbf{v}_2/\Delta t$
Multiply by Δt (which is the same for both)
$m_1\Delta\mathbf{v}_1 + m_2\Delta\mathbf{v}_2 = 0$
$m_1(\mathbf{v}_{1\ final} - \mathbf{v}_{1\ initial}) + m_2(\mathbf{v}_{2\ final} - \mathbf{v}_{2\ final}) = 0$
$m_1\mathbf{v}_{1\ initial} + m_2\mathbf{v}_{2\ initial} = m_1\mathbf{v}_{1\ final} + m_2\mathbf{v}_{2\ final}$

… and there you have it!

From warmup: The total momentum of an isolated system of objects is conserved
a. only if conservative forces act between the objects
b. regardless of the nature of the forces between the objects.

From warmup. A truck always has more mass than a roller skate. Does a truck always have more momentum than a roller skate?

a. yes
b. no

Why use conservation of momentum?

Limitation: Like conservation of energy, conservation of momentum is a "before" and "after" law which doesn't tell you about: ____________________

If you want to know about _______, you have to know ______

Another useful equation:

$$\boxed{\vec{F}\Delta t = \Delta\vec{p}}$$ "Impulse equation" (if only one force)

Derivation: $\Sigma\mathbf{F} = m\mathbf{a} = m\Delta\mathbf{v}/\Delta t$; multiple both sides by Δt

Demo Problem: A cart moving at 4 m/s runs into a second cart of the same mass and sticks to it. What velocity do the two (stuck together) carts now have?

Demo Problem: A cart moving at 4 m/s runs into a second cart of with *twice* the mass and sticks to it. What velocity do the two (stuck together) carts now have?

Demo Problem: Two carts with the same mass spring apart. If one moves at 4 m/s to the right afterwards, what velocity does the second cart have?

Dr Colton's Guide:
How to Solve Conservation of Momentum Problems

1. Draw initial and final pictures
2. Draw *momentum* or *velocity* vectors (arrows) in each picture
3. Use $\sum \vec{p}_{before} = \sum \vec{p}_{after}$ as "blueprint equation"
4. Divide into separate x- and y- equations if needed
5. Fill in both sides of blueprint equation(s) using initial and final pictures: one term in equation for each arrow in picture.
6. Reminder: be careful with signs! (Momentum is a **vector**)

Compare to previous two blueprint equations:

$$\sum \vec{F} = m\vec{a}$$

$$E_{before} = E_{after}$$ (if no non-conservative forces)

Similarities? Differences?

Problem: In the new sport of "ice football", a 100 kg defensive end running north at 4 m/s tackles a 75 kg quarterback running *east* at 7 m/s. There's no friction. What is their combined velocity right after the tackle?

Answers: $v_x = 3$ m/s; $v_y = 2.28$ m/s; v = 3.77 at 37.3° north of east

Problem: An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

Answers: v_x = 250 m/s; v_y = -75 m/s; v = 261 m/s at 16.7° south of east

From warmup: Suppose Ralph is floating in outer space with no forces acting on him. He is at rest, so his momentum is zero. Now, he throws a ball. The ball goes one way, and he goes the other way. Before the collision, there was no momentum, and after the collision, there is plenty of momentum! Was momentum conserved?

Answer from the class:

From warmup, do as clicker quiz: A ping-pong ball moving forward with a momentum *p* strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

a. greater than *p*
b. less than *p*
c. equal to *p*

What about if ping-pong ball "thuds" and falls flat?

Demo: Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?

Question: Is <u>energy</u> conserved in collisions? All? Some? None?

Special Case: "Elastic" Collisions

In some special collisions, energy is also conserved!

Elastic collisions: no lost kinetic energy

→ they are "bouncy"

(but not all bouncy-looking collisions are elastic… don't assume)

Inelastic collisions:

Perfectly inelastic collisions:

Dr. Colton's guide, cont.

#7. If it's an elastic collision …

$\Sigma KE_{before} = \Sigma KE_{after}$

→ This is in addition to $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

The two equations can be put together to give:

$$\boxed{(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}}$$

used in addition to cons. of mom. for elastic collisions

Careful with signs! "Right = positive, left = negative" still applies

Derivation:

Cons. mom

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$m_1 \left(v_{1i} - v_{1f}\right) = m_2 \left(v_{2f} - v_{2i}\right)$$

Cons. energy

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$
$$m_1 \left(v_{1i}^2 - v_{1f}^2\right) = m_2 \left(v_{2f}^2 - v_{2i}^2\right)$$
$$m_1 \left(v_{1i} + v_{1f}\right)\left(v_{1i} - v_{1f}\right) = m_2 \left(v_{2f} + v_{2i}\right)\left(v_{2f} - v_{2i}\right)$$

Divide the two equations.

$$\frac{\cancel{m_1} \left(v_{1i} + v_{1f}\right) \cancel{\left(v_{1i} - v_{1f}\right)}}{\cancel{m_1} \cancel{\left(v_{1i} - v_{1f}\right)}} = \frac{\cancel{m_2} \left(v_{2f} + v_{2i}\right) \cancel{\left(v_{2f} - v_{2i}\right)}}{\cancel{m_2} \cancel{\left(v_{2f} - v_{2i}\right)}}$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$
$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

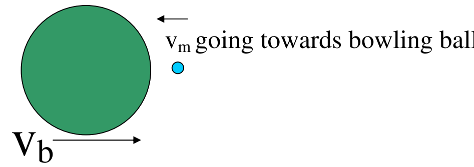
Demo Problem: A cart moving at 4 m/s bounces elastically off of a second cart of twice the mass which is moving at 2 m/s in the same direction. What velocity does each cart now have?

Demo Problem: A cart moving at 4 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have? **Demo:** Newton's cradle

Answer to first one: $v_1 = 1.33$ m/s; $v_2 = 3.33$ m/s

Demo problem: Elastic collision between very large and very small mass

Bowling ball and a marble! Marble is at rest.



What are final speeds?

Hint: $v_{\text{bowling ball final}} \approx v_{\text{bowling ball initial}}$

Demo: "Velocity amplifier"