Announcements – 13 Oct 2009

1. Exam problem 28—A student pointed out that you get a different answer if you use 9.8 m/s^2 for g than if you use 10 m/s^2 (what you were supposed to use). So, I decided to accept *both* answers. The Testing Center regraded all exams, and you should automatically see this problem marked right if you used 9.8 m/s^2 for g.

2. While you're waiting for class to start, see how many of the review blanks you can fill out on page 2.

Review: <u>Three</u> types of accelerations for rotations

Tangential Accel.: Causes speed to increase or decrease
"a" or "a_{tan}" Causes angular speed to increase or decrease
Therefore, causes: angular acceleration, α

Definitions:

$\theta =$ #radians

$\omega_{ave} = \Delta\theta/\Delta t$ radians/sec

$\alpha_{ave} = \Delta\omega/\Delta t$ rad/sec^2

arc length $s = r\theta$ (meters)

tangential $v = r\omega$

tangential $a = r\alpha$

3 Angular Kinematic Equations $\left\{\begin{array}{l} x \to \theta \\ v \to \omega \\ a \to \alpha \end{array}\right.$

1. $v_f = v_0 + at \to \omega_f = \omega_0 + \alpha t$
2. $x = x_0 + v_0 t + \frac{1}{2}at^2 \to \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
3. $v_f^2 = v_0^2 + 2a\Delta x \to \omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

Centripetal Accel.: Causes a change in direction

Magnitude: $a_c =$ v^2/r Direction: inward

How to use with N2: $\Sigma F_{inward/outward} = ma_c$

$\Sigma F_{inward/outward} = m\frac{v^2}{r}$

Which part of today's assignment was particularly hard or confusing?

Kepler's laws were a little confusing.

What is the M sub E? M_E = mass of the earth

I would like to spend more time on last lecture's material!

What is T (period of the planet)? It was just thrown out there without a lot of definition. how long to go around 1 orbit (a year)

General comments:

So, the planets orbitals are technically elliptical, but are close enough to circular that we can treat them as such mathematically? yes

i have been helping people out on the Google groups but i have never got any bonus points for it and other people are getting them. is there specific help you are looking for students to give other students? or are you just giving points to random helpers.

I watched a mythbuster last night where they were trying to determine if it were possible to do a 360 degree turn around a standard swing set. They didn't do any calculations just experiments. Couldn't they have just determined the velocity needed to go around r (radius of swing) with some of the stuff we went over last time. Could you show us how?

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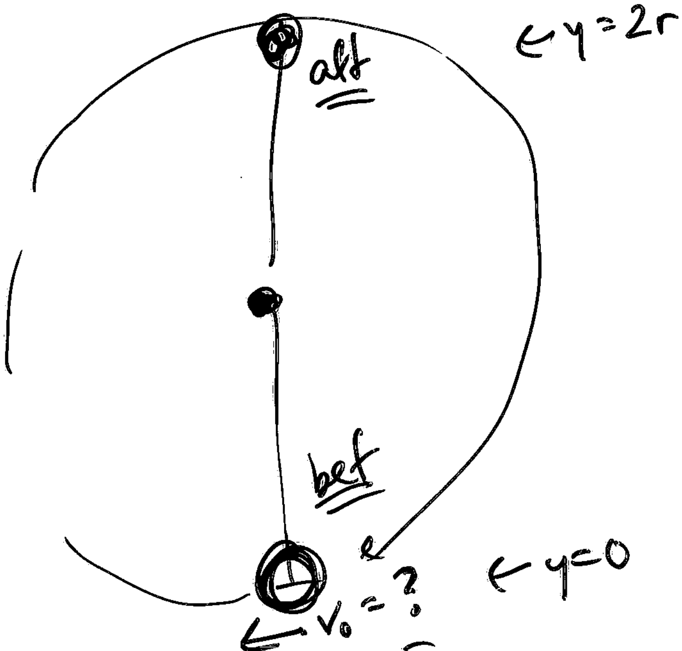


$$\Sigma F = m a_c$$

$$T + mg = m\frac{v_f^2}{r}$$

set $T = 0$ to get smallest v_f

$$\cancel{m}g = \cancel{m}\frac{v_f^2}{r}$$

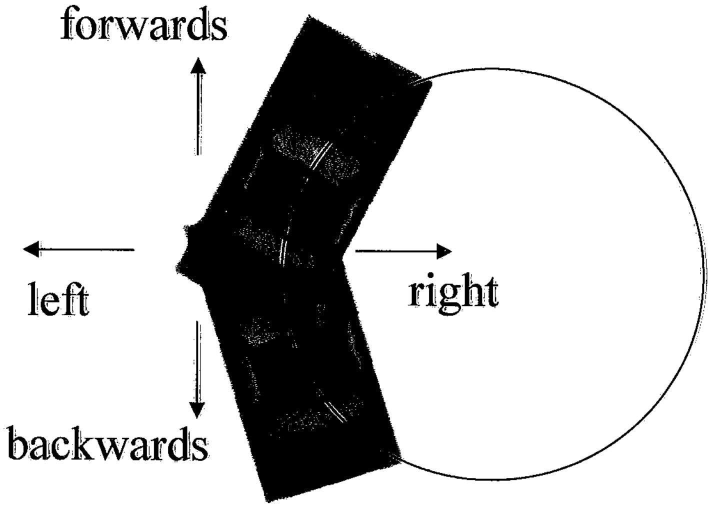
$$v_f = \sqrt{gr}$$

$$E_{bef} = E_{aft}$$

$$KE_{bef} = PE_{aft} + KE_{aft}$$

$$\frac{1}{2}\cancel{m}v_0^2 = \cancel{m}g(2r) + \frac{1}{2}\cancel{m}v_f^2$$

Solve for v_0!



You are on right side of the back seat of the car. The car turns right at constant speed, moving in a circle, and you slide slowly to the left (with friction) before running up against the door.

Clicker quiz: The net horizontal force on you *after you are pressed up against the door* is:

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

Clicker quiz: The net horizontal force on you *while you are sliding* (left) is:

kinetic friction

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

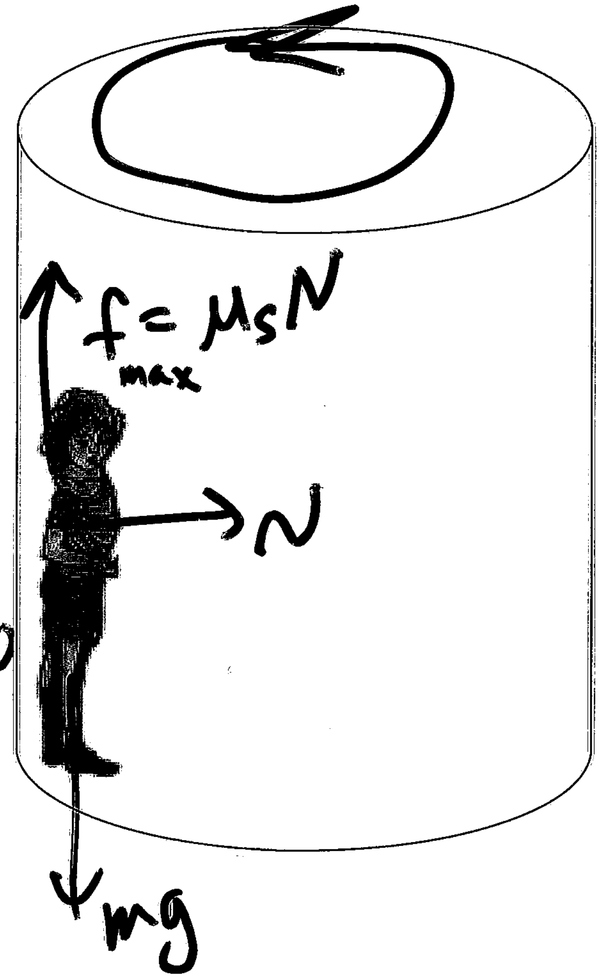
Question: How do answers change if no friction?
Question: What direction is the friction on the car's wheels?

right

μ_s

Floor-dropping ride

If the coefficient of friction is μ, what minimum speed v must you be going before the floor is removed?



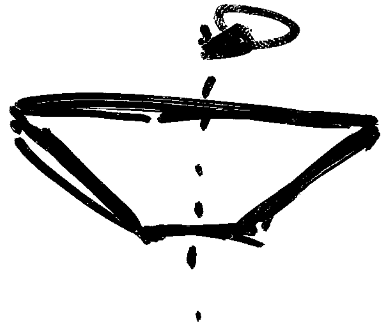
$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

$$N = m\frac{v^2}{r} \qquad \mu_s N - mg = 0$$

$$N = \frac{mg}{\mu_s}$$

$$\frac{\cancel{m}g}{\mu_s} = \cancel{m}\frac{v^2}{r}$$

$$v = \sqrt{\frac{rg}{\mu_s}}$$

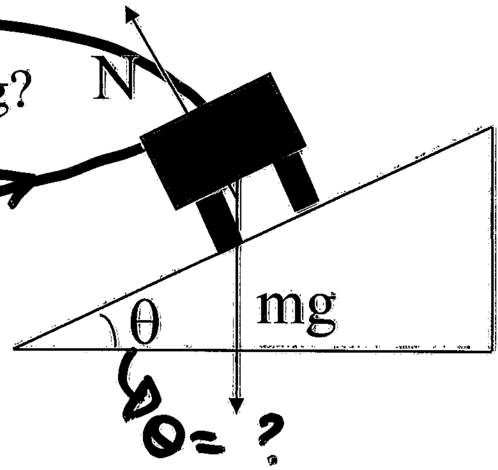


Answer: $\sqrt{\frac{rg}{\mu}}$

Banked roadways

Consider turn with no friction…
What direction will car go if slight banking?
What direction if steep banking?

In between?



So, why do they bank turns?

HW Problem, 10-5 (due tomorrow!): what should the banking angle be so that there is no sideways friction force needed? (given overall turn radius and speed)

Hardest part: which way to draw the axes?? Conflicting advice
Colton: "Make the positive x-axis be along the inclined plane"
✓Colton: "Make the positive x-axis be towards the center of the circle"

Conflict resolved:
Advice: choose axes so that $a = 0$ in one direction

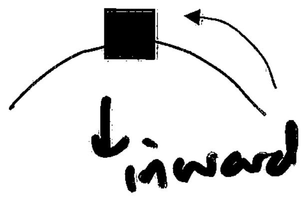
Some component of N is inward

Combined Centripetal and Tangential

Example: Going around a corner while slowing down

Clicker quiz: The centripetal acceleration at this instant is

a. up on page
b. down on page
c. left
d. right
e. zero

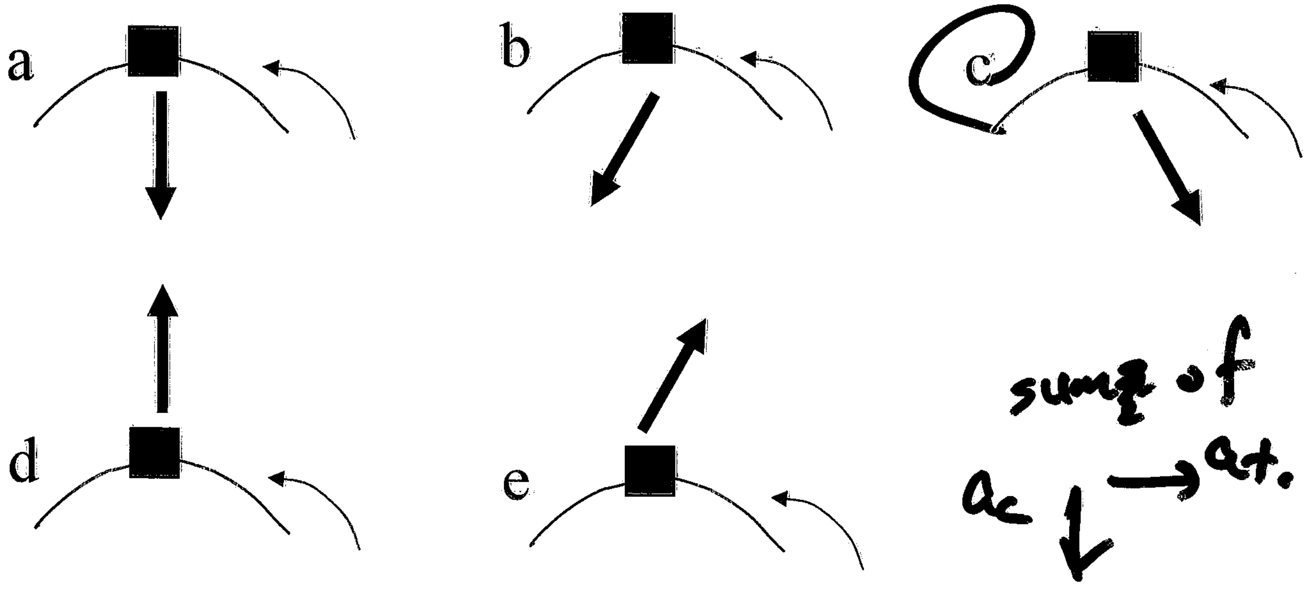


Clicker quiz: The tangential acceleration at that instant is

a. up on page
b. down on page
c. left
d. right
e. zero

going left
+ slowing down
→ accel. is right

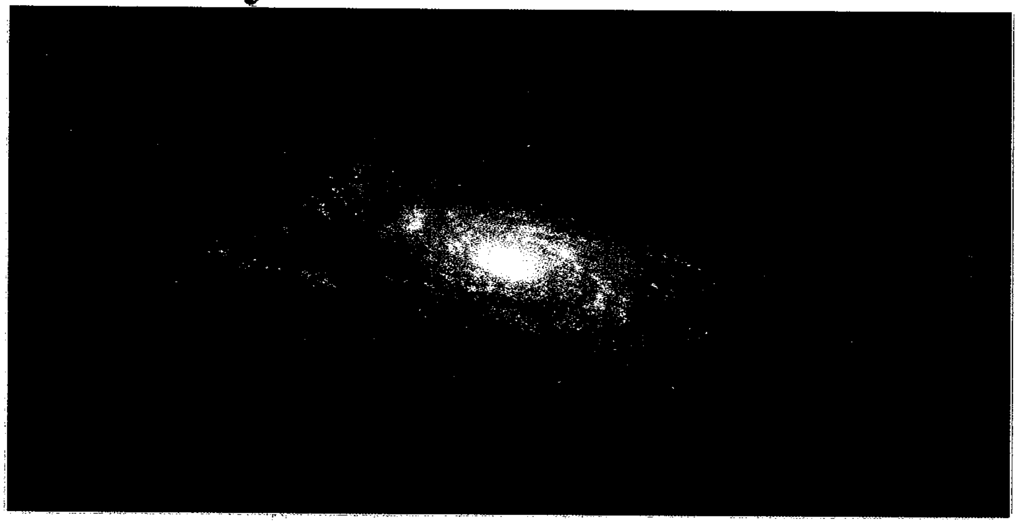
Clicker quiz: Which figure represents the total ***a*** vector?



sum of $a_c \downarrow \rightarrow a_t$

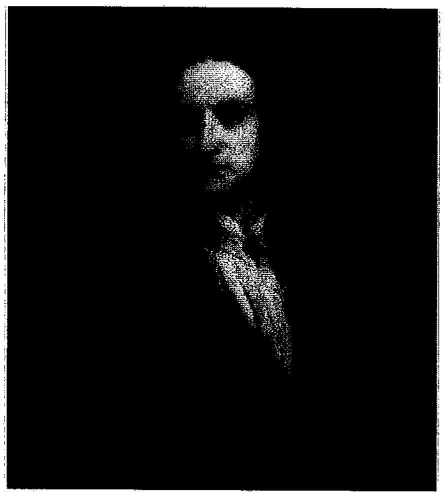
$= \searrow$

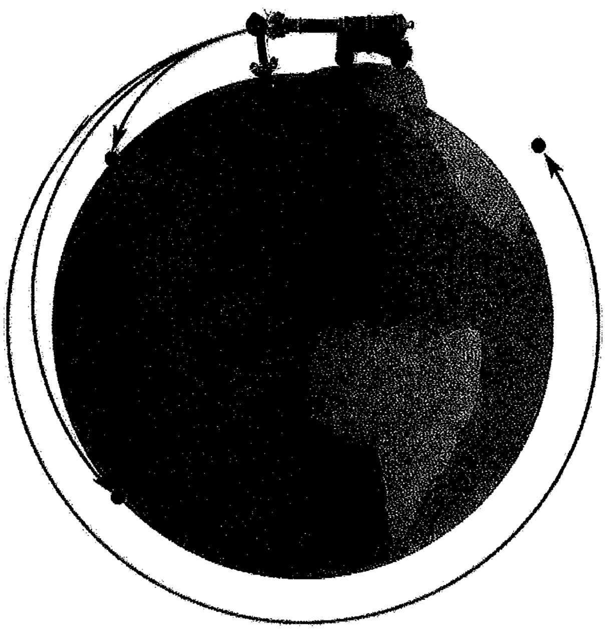
On to Gravity!!



Classical physics was invented to understand the motion of the planets

Newton's thoughts about the moon's orbit and projectile motion, c. 1670:





Parabola of projectile turns into a circle.

The apple, the cannonball, and the Moon

→ all are in free fall

Newton's Law of Gravity:

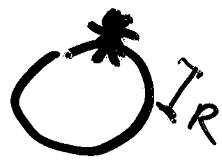
All masses attract all other masses!

$$\boxed{F_G = G\frac{mM}{r^2}}$$

r measured from center to center

(sometimes with negative sign)

Proportionality constant: $G = 6.674 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$



Near the surface of the earth:

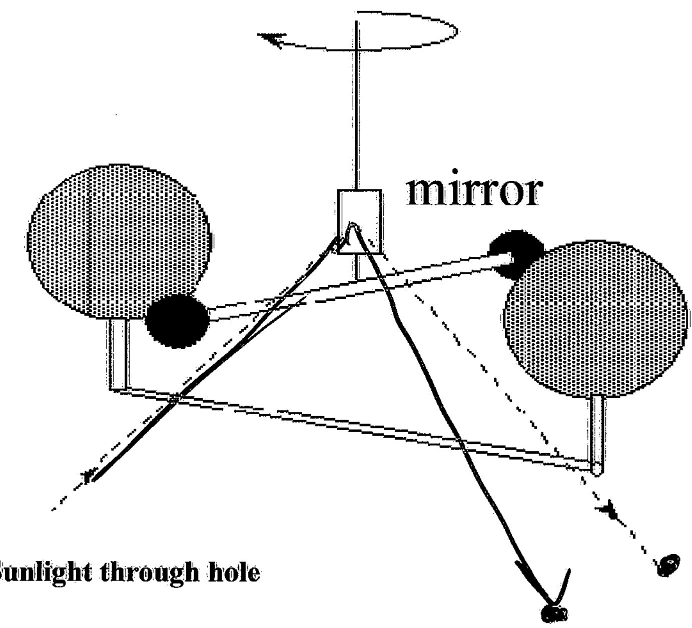
$R_{Earth} = 6371$ km
$M_{Earth} = 5.974 \times 10^{24}$ kg

$$F_G = \frac{(6.67\cdot 10^{-11})\, m\, (5.97\cdot 10^{24})}{(6371\cdot 10^{3})^2}$$

$$= m\underbrace{\left(9.8\ \frac{m}{s^2}\right)}_{\text{"g"}}$$

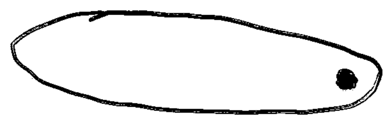
1783: first measurement of forces between "regular" masses, by Cavendish.

"Weighing the world"

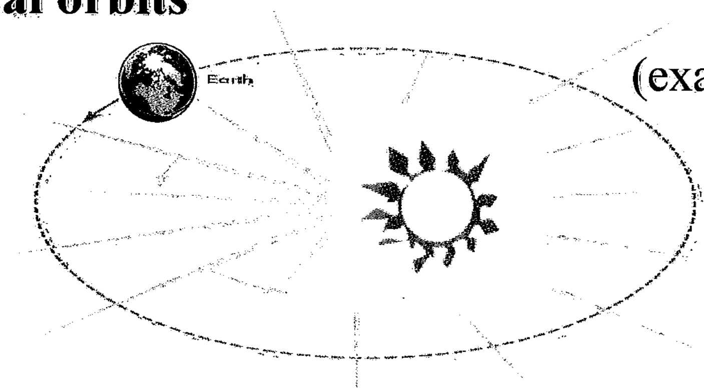


How did Newton know it was **inverse square**?

<u>Kepler's laws</u> (about 1600) came from observations of the planets in our solar system:



1. **Elliptical orbits**



(exaggerated ellipse)

2. **Equal areas in equal times**: fastest close to Sun

3. $\mathbf{T^2 \sim r^3}$

$T^2 = k r^3$ (T = "orbital period" = planet's year)

All three can be exactly predicted using Newton's Second Law together with Newton's Law of Gravity!

From warmup: Which is not one of Kepler's laws?

a. Planets all move in the same plane
b. Planets move in elliptical orbits
c. Equal areas swept out in equal time: faster closer to sun
d. The period of orbit increases as r increases

$T =$ period

Worked Problem: Figure out what the proportionality constant is in Kepler's Third Law in terms of G and the mass of the sun. Assume a circular planetary orbit.

$$\Sigma F = m a_c$$

$$F_g = \frac{mv^2}{r}$$

$$G\frac{\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{2\pi r}{T}$$

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\boxed{T^2 = \left(\frac{4\pi^2}{GM}\right) r^3_{orbital}}$$

$$k = \frac{4\pi^2}{GM_{sun}}$$

Problem: How long is Jupiter's year? ($r_{Jupiter} \approx 5.2\, r_{Earth}$)

to Earth's orbit

$$\frac{T_{Jup}^2 = k\, r_{Jup}^3}{T_{Earth}^2 = k\, r_{Earth}^3}$$

$$\frac{T_{Jup}^2}{(1\,\text{year})^2} = \frac{(5.2\, \cancel{r_{Earth}})^3}{(\cancel{r_{Earth}})^3}$$

$$\boxed{T_{Jup} = \sqrt{5.2^3}\ \text{years}}$$

$$= \underline{11.9\ \text{years}}$$

Question: Does Kepler's 3rd apply to satellites orbiting the <u>earth</u>?

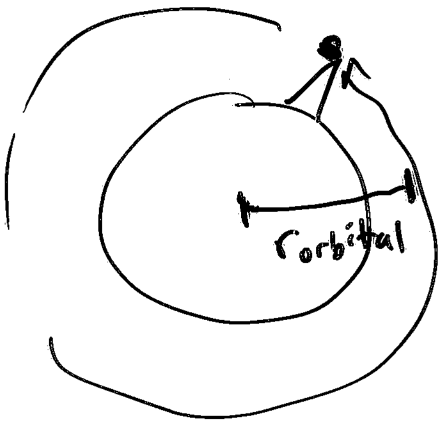
yes → $k = \frac{4\pi^2}{GM_{earth}}$

Answers: $k = 4\pi^2/GM$; 11.86 years

Orbital Velocity

On the moon (no air friction) someone *could* get into orbit by being fired horizontally off the highest mountain.

How fast would you have to shoot that person?



$$\Sigma F = m a_c$$

$$\frac{G M \cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$$

$$V_{orb} = \sqrt{\frac{G M_{moon}}{r_{orbital}}}$$

How long would it take him to go around once?

"orbital period"

$$V = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{V} = \frac{2\pi r_{orbital}}{V_{orbital}}$$

Answers: v=sqrt(GM/r), 2πr/v

Circular orbits

For each v, only one r will work
For each r, only one v will work!

Clicker quiz: A satellite in a higher orbit will be going

a. faster
b. slower

than a satellite in a lower orbit

Real satellites:

http://science.nasa.gov/RealTime/JTrack/3d/JTrack3d.html

(Hubble)
International space station, 340.5 km above surface of Earth
(R_e = 6,371 km) 7.707 km/s

Geostationary orbit, 35,786 km above surface
3.075 km/s

Moon, average R = 381,715 km
1.022 km/s

How long does it take ISS to orbit?

$$v = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{2\pi r_{orb}}{v_{orb}} = 91.2\text{ min}$$

Answer: 91.2 min

From warmup: The Moon does not fall to Earth because:

a. the gravitational pull of the Earth on the moon is weak
b. the moon has a sufficiently large orbital speed
c. the gravitational pull of the sun keeps the moon up
d. the moon has less mass than Earth
e. none of the above

Clicker quiz: You are on planet Xarthon, which has a mass of 2× that of the earth and a radius 2× as big. If you throw a ball at the surface, and you will find that $g_{Xarthon}$ is ____ g_{earth}

a. larger than
b. smaller than
c. the same as

$$F_g = \frac{GmM}{r^2} \quad \left(\text{also} = mg\right)$$

surface acceleration

$$g = \frac{GM_{planet}}{R^2_{planet}}$$

Clicker quiz: Satellites in higher orbits are travelling slower, so to "shoot" a satellite from the surface of the earth into a high orbit (i.e. with a cannon), you would provide it with

a. more
b. less

don't forget PE

initial kinetic energy than for a satellite in a low orbit

Gravitational PE

Need new $\mathbf{PE_{gravity}}$ **PE = mgy** just won't work…
Force isn't "mg" any more!

Using calculus to calculate work done against (non-constant) gravitational force…

$$\boxed{PE_G = -\frac{GMm}{r}}$$

from center of earth

Here the negative sign is critically important!! (not a vector direction)

Before: PE = 0 when $y = 0$ (free to choose)

With new equation: PE = 0 when $r = \infty$ (not free)

From warmup: Ralph wonders how can PE = -GMm/r "reduce to PE = mgy close to the surface of Earth"?? It's negative!

Answers from the class:

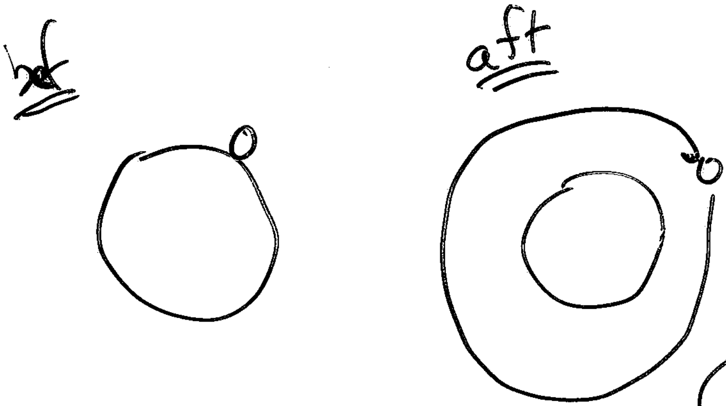
235-------------------------

This is because if an object falls because of gravity, its change is a negative potential energy as it fall closer to the earth and the KE increases as is goes toward the earth.

584-------------------------

My dear friend Ralph, both equations prove that the change in the object's potential energy is negative. But one of the equations is negative because you cannot change any of the set values in this equation, whereas in the other equation one can adjust *h* according to a specified reference level that automatically makes the equation positive.

Worked problem: How much energy would you have to provide in order to "shoot" a 100 kg satellite into a near orbit like the ISS, 6712 km from center of earth? (Assume via initial KE)



$$E_{bef} = E_{aft}$$

$$KE_i - \frac{GMm}{R_{Earth}} = \frac{1}{2} m v_{orb}^2 - \frac{GMm}{R_{orbit}}$$

$$KE_i = 3.3 \cdot 10^9 \text{ J}$$

…into a much farther geostationary orbit? (42,157 km)

…to an orbit at the moon's distance (381,715 km)

Answers: 3.29E9 J, 5.79E9 J, 6.21E9 J