

Announcements – Oct 20, 2009

0. HINI Announcement from John Tanner

1. **Exam 3** starts one week from today
 - a. Tuesday will be review session
 - b. Exam covers HW 9-14. (HW 14 is due on Tuesday.)
 - c. HW 15 not due until the following Wed, Nov 4.

2. **TA-led evening review session:** To overcome limitations of in-class survey, we will use doodle.com. Everyone inputs which times work for them, it totals up things so that everyone can see which times the most numbers of students can make.
 - a. I ~~will~~ send [†] around the survey link ~~today~~ ^{yesterday}.
 - b. Please **vote on times by tomorrow night**. Then I can announce the decision Thursday in class, and I will still have a couple of days to get the room and TA scheduled.

3. **Today's Goal:** complete the connection between linear and angular quantities
 - a. Distance $\rightarrow \theta$
 - b. Velocity $\rightarrow \omega$
 - c. Acceleration $\rightarrow \alpha$
 - d. Force $\rightarrow \tau$
 - e. Mass $\rightarrow ??$ (today) — Moment ^{of} Inertia
 - f. Energy $\rightarrow ??$ (today) — rotational K.E.
 - g. Momentum $\rightarrow ??$ (next time)

4) Career Expo Info

Which part of today's assignment was particularly hard or confusing?

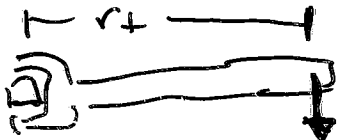
moment of inertia = "rotational inertia"

Whenever the word "moment" is in the name of a quantity, I find it hard to visualize what it is exactly.

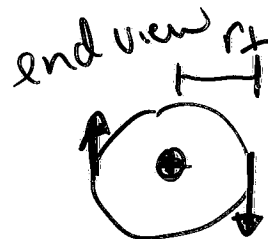
I had a really difficult time understanding table 8.1 regarding moments of inertia for various objects. Where in the world did those fractions come from? Will we be expected to memorize those shapes and their respective fractions for the exam? No

It feels like there's lots of calculus hidden in this class.

Will you explain the "Quick Quiz 8.1" from the seventh edition. From last week's warmup we said that a longer handle will provide more torque. The question reads: Using a screwdriver, you try to remove a screw from a piece of furniture, but can't get it to turn. To increase the chances of success, you should use a screwdriver that (a) is longer (b) is shorter (c) has a narrower handle, or (d) has a wider handle. The answer is d.



General comments:



With regard to late homework...do our first three late homeworks count as the "free" ones or do you take our three highest scoring late homeworks as the "free" ones?

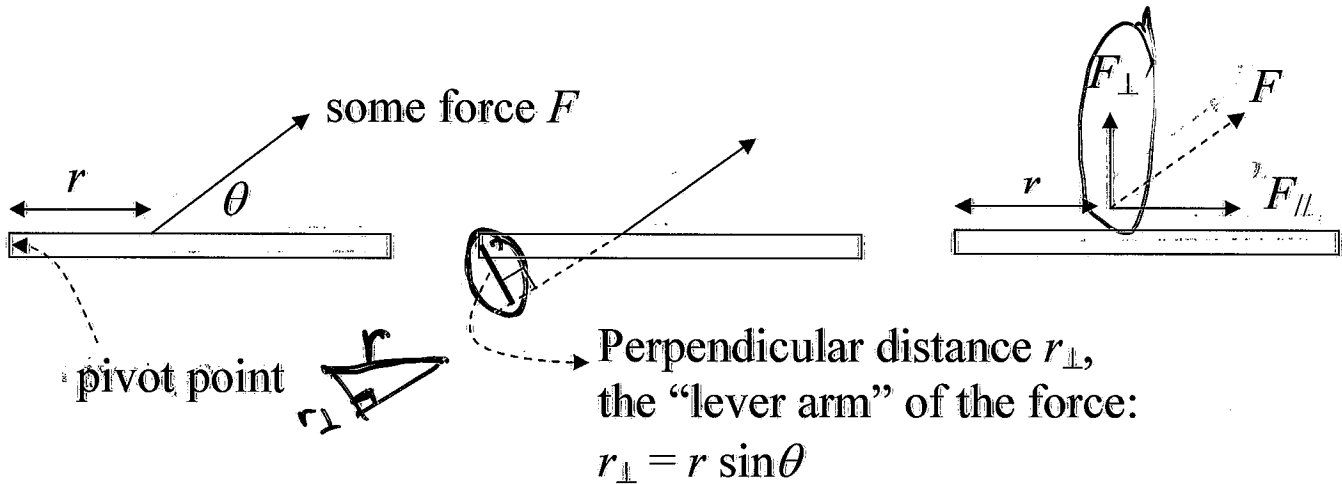
this one

Review of Torques

Definition of torque: (about a point)

$$\tau_p = r_{\perp} F = r F_{\perp} = r F \sin \theta$$

danger!



Positive/negative:

Produces a **clockwise** rotation = **negative**

Produces a **counter-clockwise** rotation = **positive**

Equilibrium

$$\sum F = 0$$

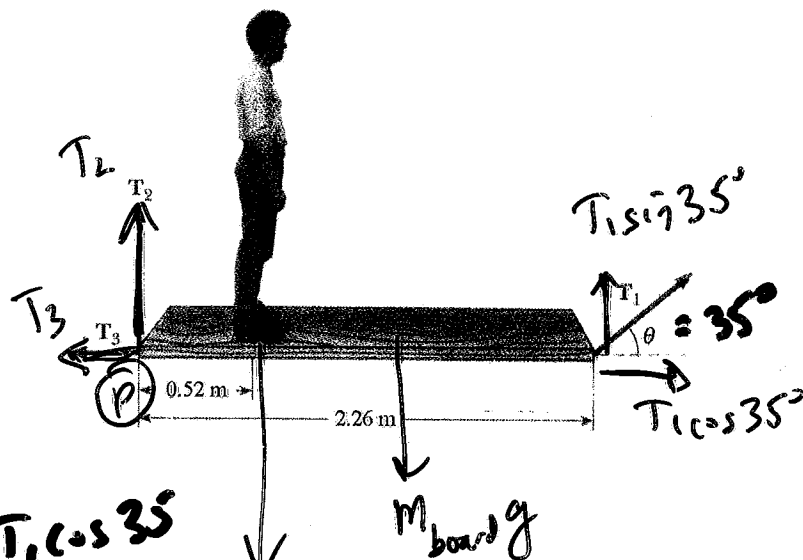
$$\sum \tau_p = 0$$

Translation:

- if an object is not speeding up or slowing down, there is no net force on it
- if an object is not speeding up or slowing down its *rotation*, there is no net *torque* on it.

One more equilibrium problem:

A uniform plank of length 2.26 m and mass 10 kg is balanced by three ropes as indicated in the figure, with $\theta = 35^\circ$. A 75 kg person is standing 0.52 m from the left end. Find the tensions in all three ropes.



$$\sum F_x = 0$$

$$T_1 \cos 35^\circ - T_3 = 0$$

$$T_3 = T_1 \cos 35^\circ = 311.5 \text{ N}$$

$$\sum F_y = 0$$

$$T_2 + T_1 \sin 35^\circ - m_{\text{man}} g - m_{\text{board}} g = 0$$

$$\sum \tau_p = 0$$

solve for

$$T_2 = 615 \text{ N}$$

$$0 + 0 - (m_{\text{man}} g)(0.52 \text{ m}) - (m_{\text{board}} g)(1.13 \text{ m}) + (T_1 \sin 35^\circ)(2.26 \text{ m}) = 0$$

$$+ (75)(9.8)(0.52) + (10)(9.8)(1.13) = T_1 (\sin 35^\circ)(2.26)$$

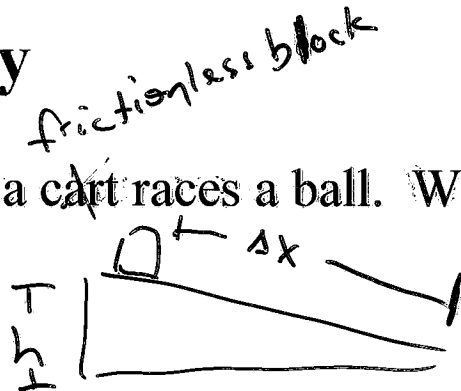
$$T_1 = 380 \text{ N}$$

Answers: 380.3 N, 311.5 N, 614.9 N

Rotational kinetic energy

Clicker quiz: (warmup) Demo... a cart races a ball. Who wins?

- a. cart *frictionless block*
- b. ball
- c. tie



Review: How fast is cart going at bottom? (Energy)

$$PE_{\text{bef}} = KE_{\text{aft}} + KE_{\text{rot aft}}$$

$$mgh = \frac{1}{2}mv^2 + \text{something}$$

$$v = \sqrt{2gh - \text{something}}$$

How long did it take to get there? (Kinematics)

First find acceleration

$$v_f^2 = v_o^2 + 2a \Delta x$$

$$a = \text{---}$$

$$\text{Then get time} = v_f = v_o + at$$

$$t = \text{---}$$

longer

→ What's different about the **ball**?

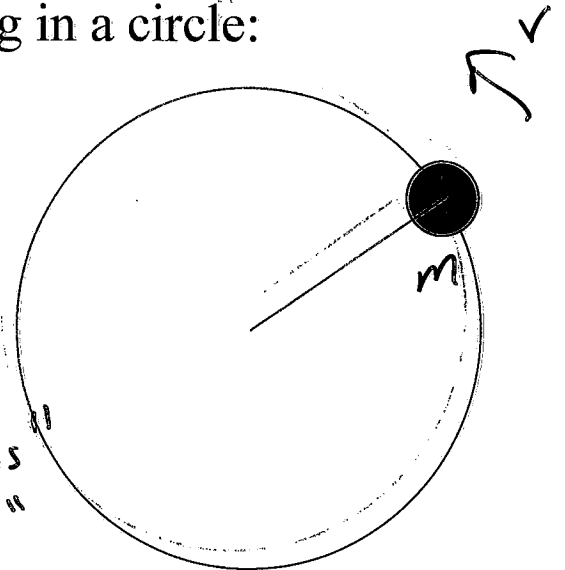
Kinetic energy of a "point mass" rotating in a circle:

$$\frac{1}{2} m v^2$$

Write in terms of ω :

$$\frac{1}{2} m (\omega r)^2$$

$$\frac{1}{2} (\underbrace{m r^2}_{\text{like "mass" call it "I"}}) \omega^2$$



"Moment of inertia"

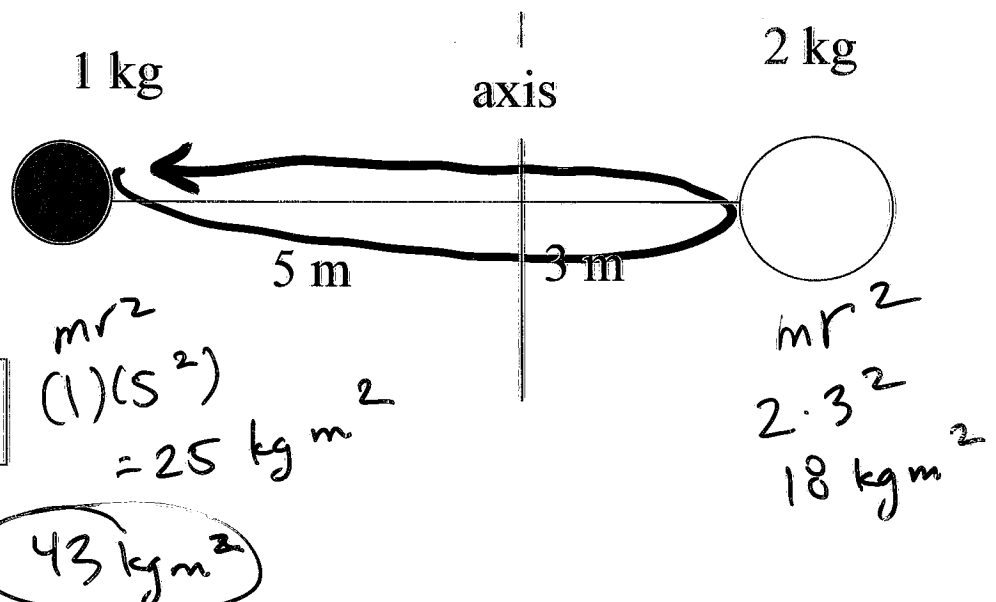
$$I_{pt\ mass} = m r^2 \quad (\text{rotating in a circle}) \quad \text{units: } \text{kg} \cdot \text{m}^2$$

Kinetic energy in terms of I and ω :

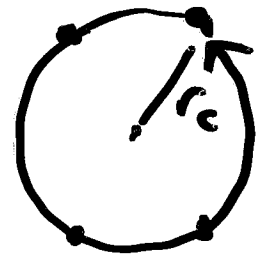
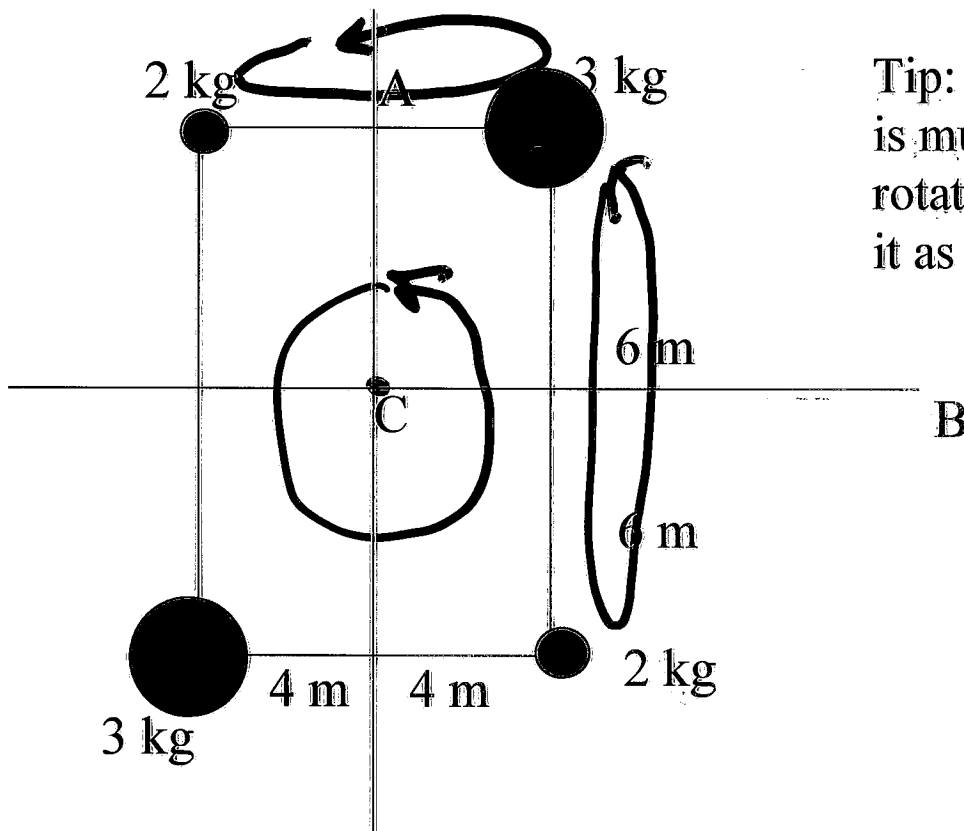
$$KE_{rot} = \frac{1}{2} I \omega^2$$

$$KE = \frac{1}{2} m v^2$$

Moment of inertia for two masses? (connected with a rod)



$$I = I_1 + I_2 + \dots$$



Clicker quiz: Does I change when you rotate about axis A vs. axis B?

- a. About axis A has larger I
- ☒ b. About axis B has larger I
- c. They have the same I

What about axis C? (C is into the page)

biggest

$$r = \sqrt{4^2 + 6^2}$$

$$> 6$$

Demo: variable “I-rotator”

Rotating “extended” objects

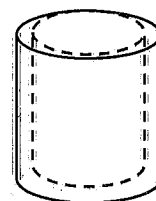
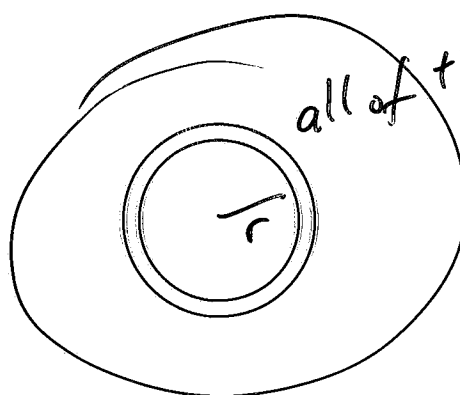
Must add up mr^2 for each bit of mass in the object

Which bits of mass contribute the most to I ?

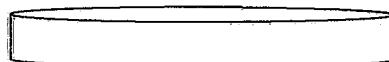
far from axis of rotation

Which of these objects will have the largest I ?

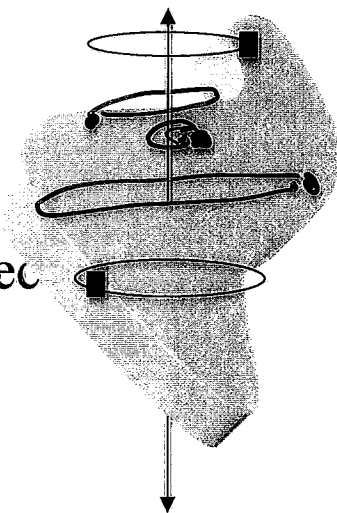
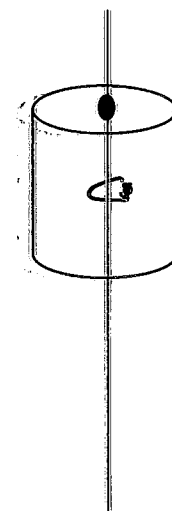
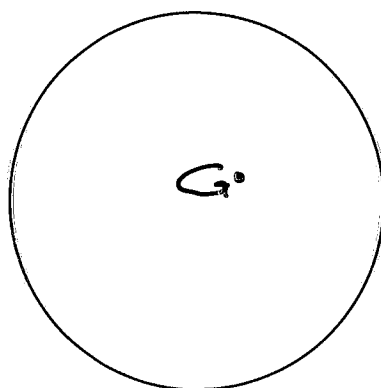
Hoop/cylindrical shell



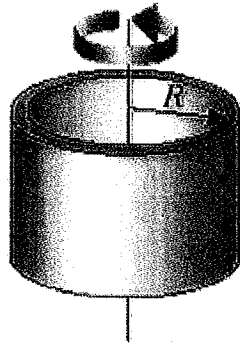
Solid disk/cylinder



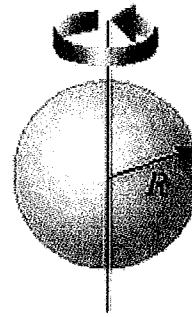
Solid sphere



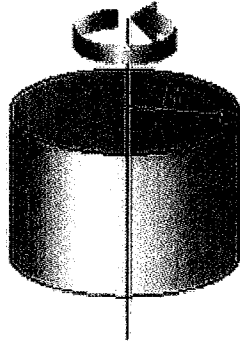
Hoop or thin
cylindrical shell
 $I = MR^2$



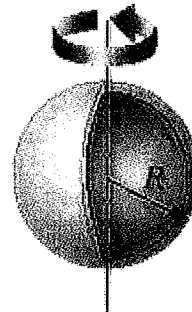
Solid sphere
 $I = \left(\frac{2}{5}\right)MR^2$



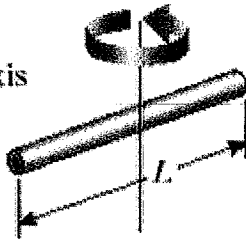
Solid cylinder
or disk
 $I = \left(\frac{1}{2}\right)MR^2$



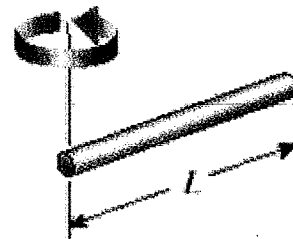
Thin spherical
shell
 $I = \left(\frac{2}{3}\right)MR^2$



Long thin rod
with rotation axis
through center
 $I = \frac{1}{12}ML^2$



Long thin rod
with rotation axis
through end
 $I = \frac{1}{3}ML^2$



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From warmup. Moment of inertia is biggest for:

- a. compact objects
- b. objects that are spread out**
- c. neither; doesn't depend on shape

Demo: Long & short “I-bars”

Clicker quiz: Which kind of rolling object will be moving the fastest at the bottom of an incline?

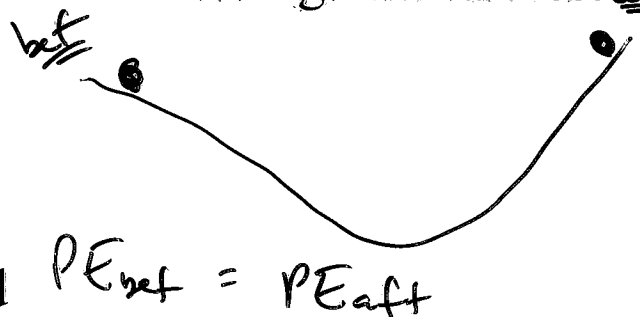
- a. Hoop
- b. Solid disk
- ☒ c. Sphere
- d. They will all tie
- e. Can't tell; it depends on size and mass

→ Which object will get there first?

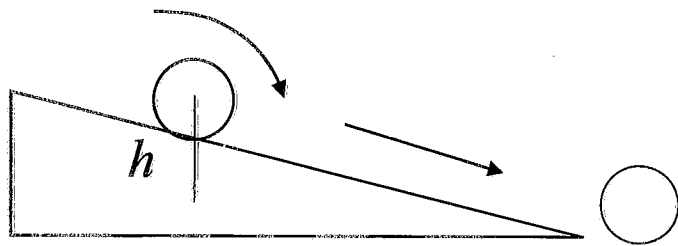
Demo: racing objects down incline

Clicker quiz: If they continued on, which would go the farthest up a hill on the other side?

- a. Hoop
- b. Solid disk
- c. Sphere
- ☒ d. All the same height at the end



Worked Problem: An object with moment of inertia I rolls down a height h without slipping. Find the speed at bottom.



$$\omega = \frac{v}{R}$$

$$PE_{\text{bef}} = KE_{\text{aft}} + KE_{\text{rot aft}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

center of mass

$\left(\frac{v}{R}\right)^2$ tangential

$$mgh = \left(\frac{1}{2}m + \frac{1}{2}\frac{I}{R^2}\right)v^2$$

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{R^2}}}$$

$$\sqrt{\frac{gh}{\frac{1}{2} + \frac{1}{2}\frac{I}{mR^2}}}$$

Answer: $v = \sqrt{\frac{2gh}{1 + I/mR^2}}$

Newton's second law for rotation

$$\sum \tau_p = I\alpha$$

still also have $\sum \vec{F} = m\vec{a}$

... but acceleration of what?
center of mass

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

From warmup. Angular acceleration will definitely increase if:

- a. torque is decreased and momentum of inertia is decreased
- b. torque is decreased and momentum of inertia is increased
- ☒ c. torque is increased and momentum of inertia is decreased
- d. torque is increased and momentum of inertia is increased

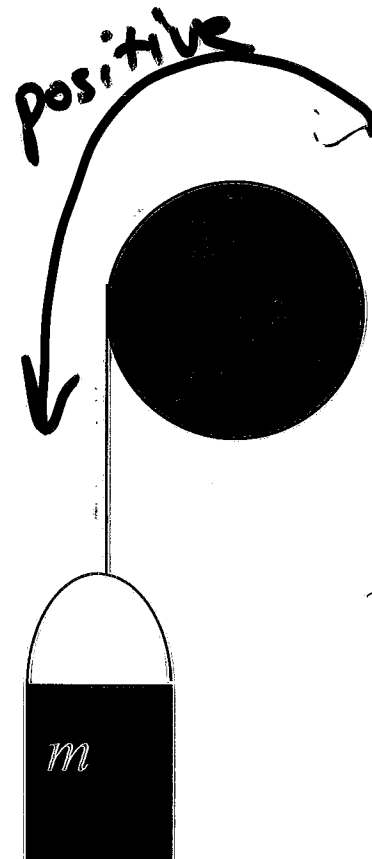
From warmup. Ralph heard his instructor say "Moment of inertia plays the same role in rotational motion that mass does in linear motion." This confuses him. What does it mean?

Answer from the class:

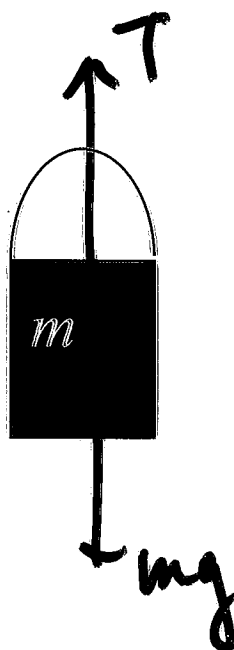
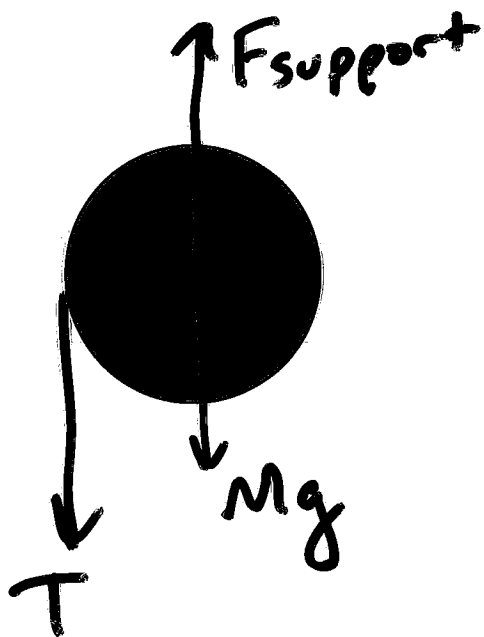
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Moment of inertia is similar to mass in that they both resist change. Mass resists linear changes and moment of inertia resists rotational changes.

Worked problem: A falling mass starts a cylinder rotating (not a “massless pulley”). What is the acceleration of m ?



Start with FBDs:



Write equations...

Cylinder

$$\sum \tau_p = I \alpha$$

$$0 + 0 + (T)(R) = I \alpha$$
$$TR = \left(\frac{1}{2}MR^2\right)\alpha$$

from table

Make a connection between α and a : $\alpha = \frac{a}{R}$

$$T \cdot R = \frac{1}{2}MR^2 \left(\frac{a}{R}\right)$$

$$T = \left(\frac{1}{2}Ma\right)$$

$$mg - \frac{1}{2}Ma = ma$$

$$mg = \frac{1}{2}Ma + ma$$

$$mg = \left(\frac{1}{2}M + m\right)a$$

Alternate method:

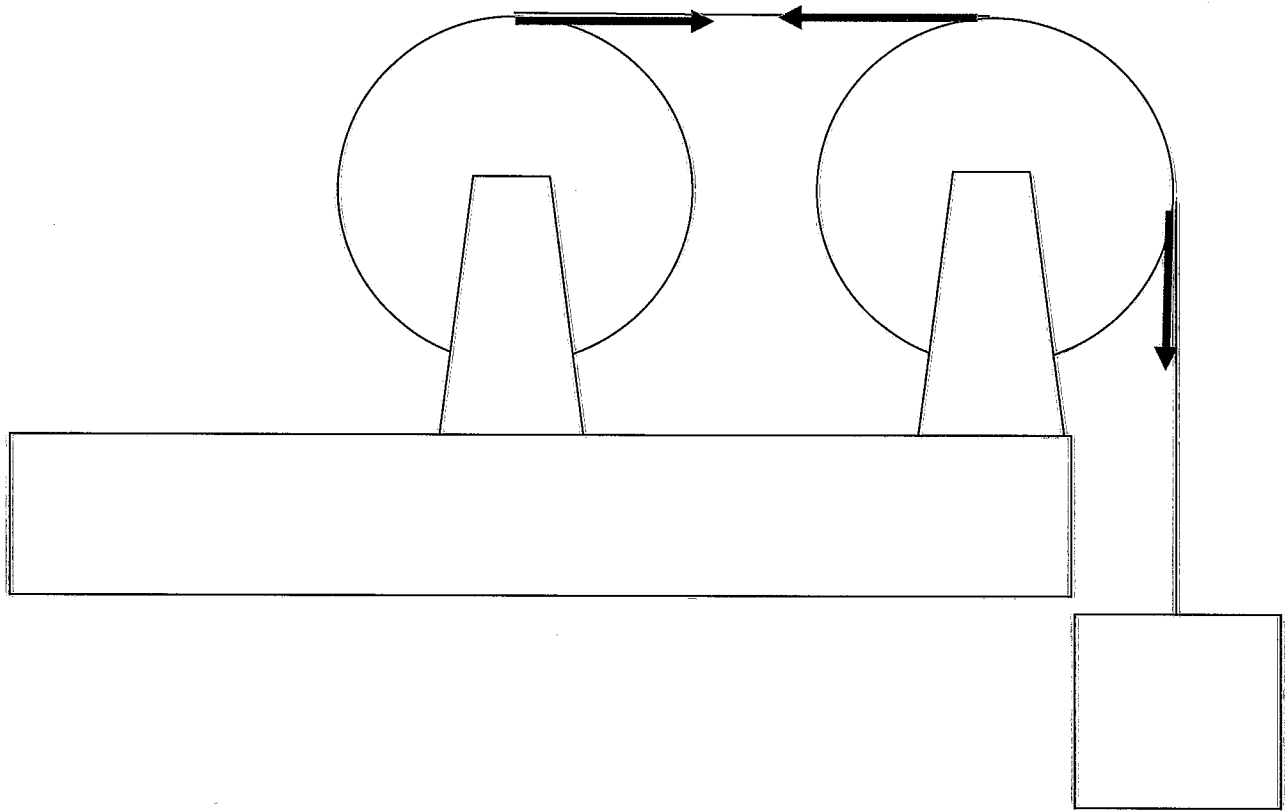
$$a = \frac{mg}{\frac{1}{2}M + m}$$

Answer: $a = \frac{m}{m + M/2}g$

Pail

$$\sum F = ma$$

$$mg - T = ma$$



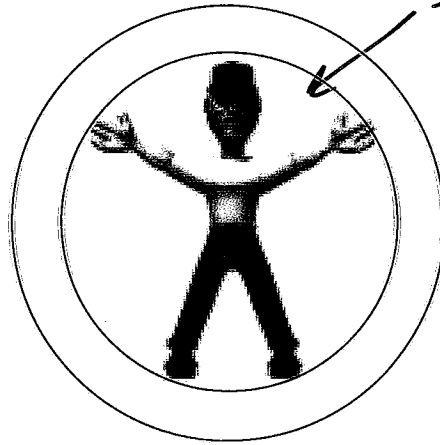
Clicker quiz: The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is no slipping of the rope on either wheel; both wheels accelerate at the same rate. The tension in the rope is

- a. Largest between the disks
- b. Largest above the mass
- c. The same in both places.

What's the difference with our old “massless pulleys”?

Clicker quiz: Mary and Fred are rolling a large tire down a hill. Mary says it will go faster if Fred gets inside the tire as shown and rolls down with it. Fred's not sure. What do you think?

- ☒ a. It will go faster
- b. It will go slower
- c. It will take the same time



*this is closer to
a solid
cylinder*

Let's do the experiment!