Announcements – Oct 27, 2009

0. TA: pick up your stuff from the boxes
1. **HW 14 due tonight**. **Reminder**: some of your HW answers will need to be written in **scientific notation**. Do this with "e" notation, not with "x" signs.
   a. **6.57E33** → **correct format**
   b. 6.57 x 10^{33} → will be marked wrong

   If you put any spaces or x's in your answer, the computer will mark it wrong.

2. **No homework due Saturday. Happy Halloween!**

3. **Exam 3 starts today!**
   a. Goes to next ~~Wednesday~~ Monday, late fee after 1 pm
   b. Covers Chapters 6, 7 & 8, HW 9-14
      i. …but cumulative, of course
   c. Same format as previous two exams
      i. Study HW, old exams, class problems, warmups, clicker questions, etc.
      ii. No notes, no textbook, no calculator, etc.

4. **Quick quiz:** Without looking at the next pages, write down all the blueprint equations for this exam you can think of. (I thought of 5 or 6)

$\Sigma \vec{F} = m\vec{a} \rightarrow \Sigma \vec{F} = 0$ for equilibrium

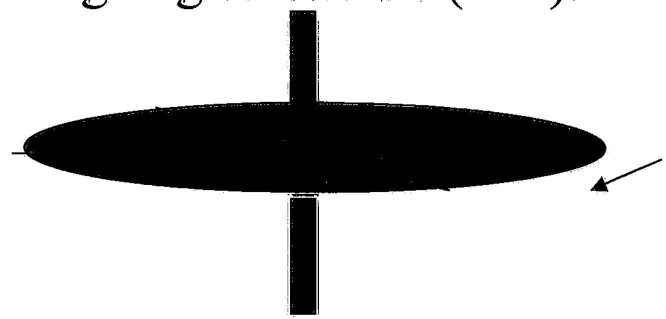
$\Sigma \tau = I\alpha \rightarrow \Sigma \tau = 0$ " "

$\Sigma \vec{P}_{bef} = \Sigma \vec{P}_{aft}$

$\Sigma \vec{L}_{bef} = \Sigma L_{aft}$

$E_{bef} + W_{net} = E_{aft}$

José sits still on frictionless ice, holding a bicycle wheel that's already spinning. View from above it is going **clockwise** (CW).



Question: If he grabs on to the wheel edge firmly and stops it from spinning he will:

a. Start to turn CW (viewed from the top)
b. Start to turn CCW
c. Remain sitting without turning

Clicker quiz: If, instead of stopping the wheel, he carefully turns it over so it is going CCW (viewed from the top), he will start to:

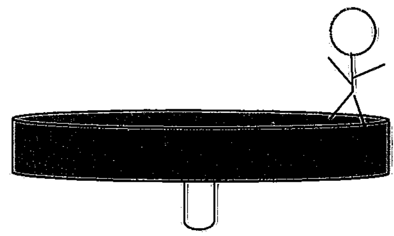
a. Turn CW, but slower than in the previous problem
b. Turn CCW, but slower than in the previous problem
c. Turn CW, but faster than in the previous problem
d. Turn CCW, but faster than in the previous problem
e. Remain sitting without turning

Hint: $(L_{wheel} + L_{man})_{\text{bef}} = (L_{wheel} + L_{man})_{\text{aft}}$

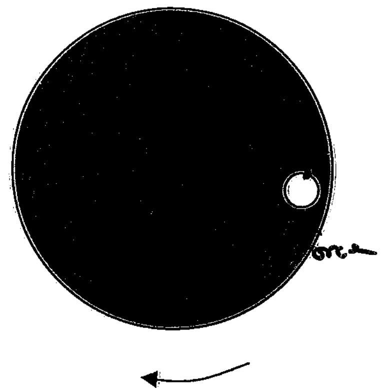
$-\text{something} + 0 = +\text{something} + L_{man}$

$L_{man} = -2\ \text{something}$

A girl is on a spinning merry-go round.



What will happen to the rotational speed ω of the merry-go-round if she…



HINT: Sometimes it's easier to think of the forces (torques) she puts on the merry-go-round, rather than conservation of L.

Clicker quiz 1: Walks towards the center?

a. it slows down
~~b.~~ it stays same speed
c. it speeds up

Clicker quiz 2: Starts running opposite to the spinning so she is at rest vs the ground? (same choices)

C speeds up.

Clicker quiz 3: Slips off when she steps on a frictionless icy part? (same choices)

B same speed

L not conserved
→ external torque from ground

Clicker quiz 4: Throws her shoe off tangentially in the direction she's moving? (same choices)

A slows down

Exam 3 Review

1. Momentum
    a. Definition: $\vec{p} = m\vec{v}$ — <u>not</u> given on exam
    b. Units? $\text{kg}\,\frac{\text{m}}{\text{s}}$
    c. Conservation Law: $\sum \vec{p}_{before} = \sum \vec{p}_{after}$ (if <u>no external forces</u>) — <u>not</u> given on exam
    d. Collision problems
        i. Draw before/after pictures, with velocity vectors
        ii. Write down conservation of momentum "blueprint"
        iii. Divide into components if needed for 2D
        iv. Fill in blueprint; be careful with signs
        v. If elastic, also use velocity reversal equation
        $(v_1 - v_2)_{bef} = (v_2 - v_1)_{after}$ — <u>is</u> given on exam
    e. Impulse: $\vec{F}\Delta t = \Delta\vec{p}$ — <u>is</u> given on exam
    f. Combination problems (e.g. bullet into block of wood)
    g. Center of mass motion
2. Rotational motion
    a. Radians: 2π radians = 360° = 1 rev. — <u>not</u> given on exam
    b. Angular quantities: θ, ω, α
        i. Units? rad $\frac{\text{rad}}{\text{s}}$ $\frac{\text{rad}}{\text{s}^2}$
    c. Connections between linear and rotational motion:
    $s = \theta r$, $v_{tan} = \omega r$, $a_{tan} = \alpha r$ — <u>are</u> given on exam

Obtaining **angular** formulas from old formulas:

<u>None of these</u> given on exam

$$\left.\begin{array}{l} x \to \theta \\ v \to \omega \\ a \to \alpha \end{array}\right\}$$ e.g. definitions/three kinematic equations

$\frac{1}{2}mv^2 \to \frac{1}{2}I\omega^2$ $p=mv \to L=I\omega$

$m \to I$ e.g. rotational kinetic energy, angular momentum

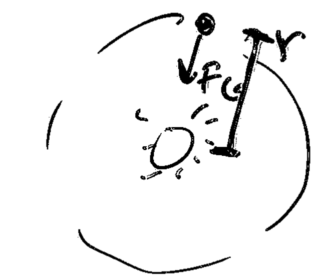
$F \to \tau$ e.g. Newton's 2nd Law for torques $\Sigma F = ma \to \Sigma\tau = I\alpha$

$p \to L$ e.g. conservation of Ang. Mom.

d. Relationship between speed and period: $v = 2\pi r/T$ $\frac{\text{distance}}{\text{time}}$

not given on exam

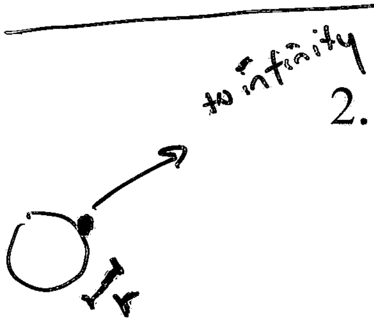
3. Centripetal acceleration, $a_c = v^2/r$ is given on exam
   a. Difference between centripetal and tangential
      i. when the two are combined (e.g. going around corner while speeding up/slowing down)
   b. How to use with N2 blueprint equation: $\Sigma F = ma_c$
      i. Real inward/outward forces on LHS
      ii. v^2/r substituted in on RHS for a_c
   c. "Roller coaster" problems: when does $N = 0$?
   d. Newton's Law of Gravity and orbits planets, satellites, space
      i. Force equation $F = \dfrac{GMm}{r^2}$ is given on exam
      ii. Potential energy equation $PE_g = -\dfrac{GMm}{r}$ is given on exam
      iii. Things which *you* might consider to be formulas (but I don't, so I **won't** give them to you on exam)



1. Quick derivation of satellite orbital velocity:

$$\Sigma F = ma_c \rightarrow \frac{GMm}{r^2} = \frac{mv_{orbit}^2}{r}$$

…or similar derivation of orbital period through $v = 2\pi r/T$. That's Kepler's Third Law)



2. Quick derivation of escape velocity:

$$E_{bef} = E_{aft} \rightarrow \underbrace{-\frac{GMm}{r} + \frac{1}{2}mv_{escape}^2}_{PE_{bef} + KE_{bef}} = \underbrace{0}_{E_{after}}$$

→ be clear about which *r* you should use…

4. Torque
   a. Definition $\tau = r_{\perp}F = rF_{\perp}$ — <u>not</u> given on exam
      i. "lever arm" concept
   b. Equilibrium problems: $\Sigma F = 0$, $\Sigma \tau_p = 0$ — <u>not</u> given on exam
   c. Moment of inertia
      i. Mass-like quantity for rotations
      ii. $I_{tot} = I_1 + I_2 + \ldots$ — <u>not</u> given on exam
      iii. Various moments of inertia — <u>are</u> given on exam
         $I = mR^2$ (point mass going in circle)
         $I = 2/5\ mR^2$ (sphere rotating about center)
         $I = mR^2$ (hoop rotating about its axis)
         $I = 1/2\ mR^2$ (disk or cylinder about its main axis)
         $I = 1/12\ mL^2$ (rod about its center)
         $I = 1/3\ mL^2$ (rod about its end)
   d. Newton 2 for rotations: $\Sigma \tau = I\alpha$ — <u>not</u> given on exam

e. Torques and rotation

   i. Combining Newton 2 with angular kinematics

5. Rotational kinetic energy & momentum

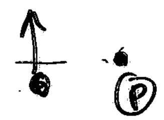
   a. Definitions

      i. $KE_{rot} = \frac{1}{2} I \omega^2$ <u>not</u> given on exam

      ii. The two expressions for L:

      $L = I\omega$ vs. $L = r_{\perp} p = r p_{\perp}$

      (<u>not</u> given)   (<u>is</u> given) "hidden"



   b. Conservation laws

      i. $E_{bef} + W_{net} = E_{aft}$

      ii. $\sum L_{before} = \sum L_{after}$ (if no external torques)

   c. If you do have an external torque, then strange things can happen! ☺

      i. For example, precession

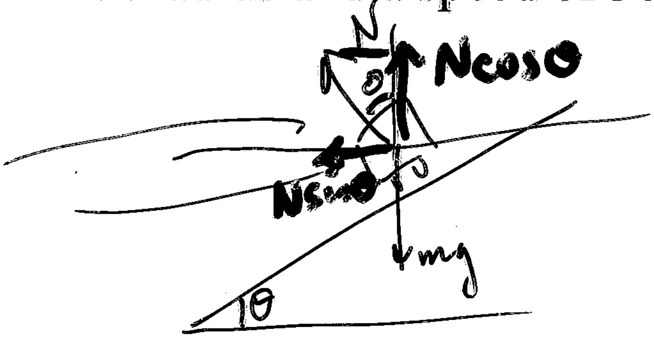
HW 9-6. An 8.29-kg mass moving east at 11.6 m/s on a frictionless horizontal surface collides with a 18.5-kg mass that is initially at rest. After the collision, the first mass moves south at 4.6 m/s. What is (a) the magnitude and (b) the direction of the velocity of the second mass after the collision? (c) What percentage of the initial kinetic energy is lost in the collision?

Answers: 5.59 m/s, 21.6 N of east, 32.4% lost

HW 9-7. A 11.4-g object moving to the right at 21 cm/s makes an elastic head-on collision with a 14.3-g object moving in the opposite direction with some unknown velocity. After the collision, the second object is observed to be moving to the right at 14.6 cm/s. Find the initial velocity of the second object.

Answer: 36 m/s to the left

HW 10-5. An engineer wishes to design a curved exit ramp for a toll road in such a way that a car will not have to rely on friction to round the curve without skidding. She does so by banking the road in such a way that the force causing the centripetal acceleration will be supplied by the component of the normal force toward the center of the circular path. Find the angle (from horizontal) at which the curve should be banked if a typical car rounds it at a 60 m radius and a speed of 30 m/s.



call towards center = positive

$$\boxed{\Sigma F_x = ma_c}$$

$$\Sigma F_y = 0$$

$$N\sin\theta = m\frac{v^2}{r}$$

$$N\cos\theta - mg = 0$$

$$N = \frac{mg}{\cos\theta}$$

$$\cancel{m}g\frac{\sin\theta}{\cos\theta} = \cancel{m}\frac{v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

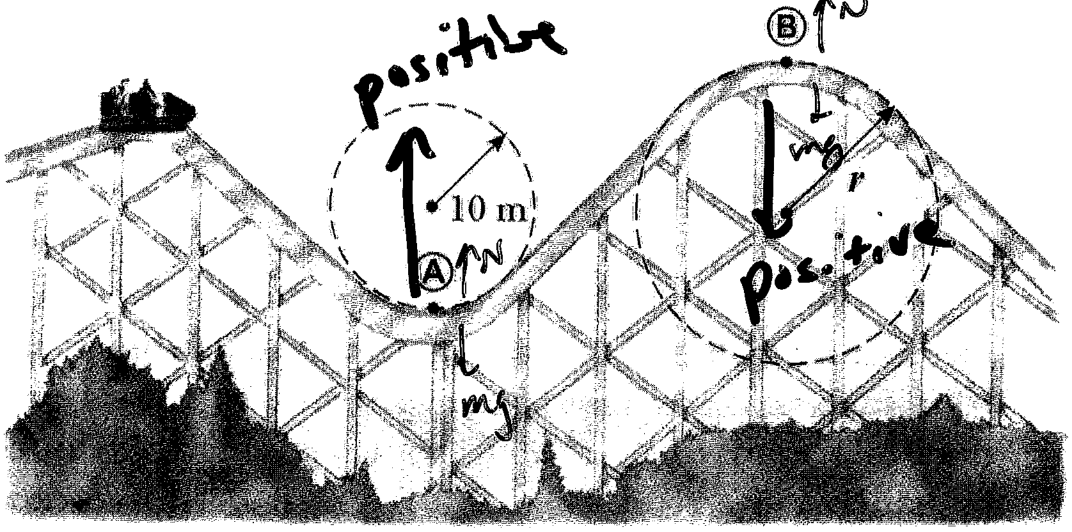
$$\tan\theta = \frac{v^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = \tan^{-1}\left(\frac{30^2}{(9.8)(60)}\right)$$

$$= \boxed{56.8^\circ}$$

Answer: 56.84°

HW 10-7. A roller-coaster vehicle has a mass of 500 kg when fully loaded with passengers. (a) If the vehicle has a speed of 20 m/s at A, what is the magnitude of the force that the track exerts on the vehicle at this point? (b) If the radius of curvature of the track at B is $r = 15$ m, what is the maximum speed the vehicle can have at B in order for gravity to hold it on the track? $N = 0!$



(A) $\Sigma F = ma_c$

$N - mg = \frac{mv^2}{r}$

$N = mg + \frac{mv^2}{r}$

$= (500)(9.8) + \frac{(500)(20^2)}{10\,m}$

$= 24{,}900\,N$

(B) $\Sigma F = ma_c$

$mg - \cancel{N} = \frac{mv^2}{r}$

$g = \frac{v^2}{r}$

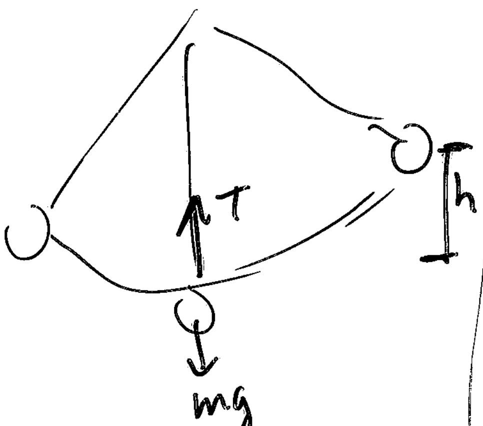
$v = \sqrt{rg}$

$= \sqrt{(15)(9.8)}$

$= 12.1\,\frac{m}{s}$

Answers: 24,900 N; 12.12 m/s

HW11-5. A 0.4 kg pendulum bob passes through the lowest part of its path at a speed of 1.5 m/s. (a) What is the tension in the pendulum cable at this point if the pendulum is 80 cm long? (b) When the pendulum reaches its highest point, what angle does the cable make with the vertical? (c) What is the tension in the pendulum cable when the pendulum reaches its highest point?



(a) $\Sigma F = ma_c$

$T - mg = \frac{mv^2}{r}$

$T = mg + \frac{mv^2}{r}$

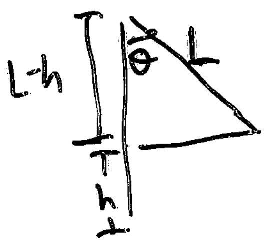
$= (.4)(9.8) + \frac{(.4)(1.5^2)}{(.8)}$

$= \boxed{5.05N}$

(b) $E_{bef} = E_{aft}$

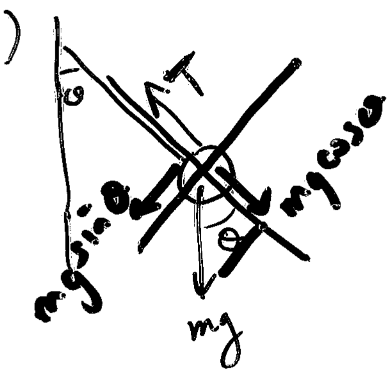
$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$h = \frac{1}{2}\frac{v^2}{g} =$ ____



$\cos\theta = \frac{L-h}{L} \rightarrow \theta =$ ____

(c)



$\Sigma F_{inward/outward} = ma_c$

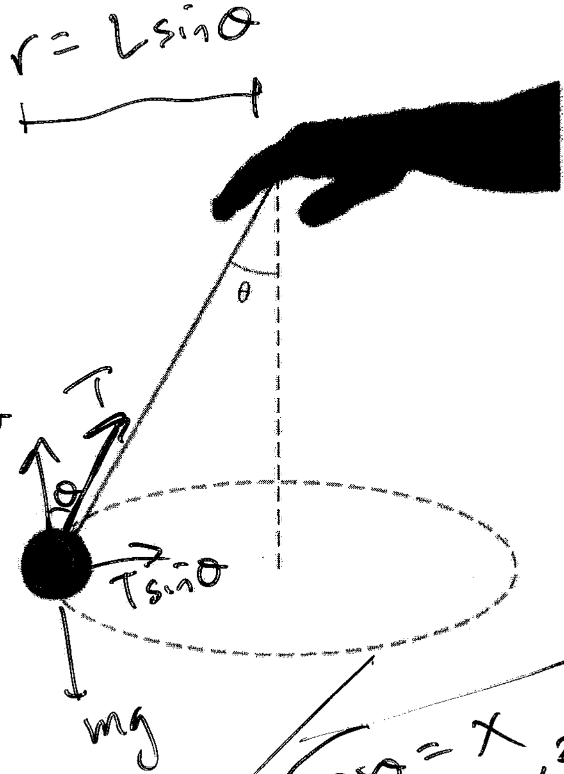
$= \frac{mv^2}{r}$

$T - mg\cos\theta = 0$

$T = mg\cos\theta$

Answers: 5.045 N; 31.07°; 3.358 N

HW 11-6. A 0.537-kg ball that is tied to the end of a 1.83-m light cord is revolved in a horizontal plane with the cord making a 28.6° angle with the vertical. (a) Determine the ball's speed. (b) If instead the ball is revolved so that its speed is 4.24 m/s, what angle does the cord make with the vertical? *Hint*: $\sin^2\theta = 1 - \cos^2\theta$.



$\Sigma F_x = ma_c$

$\Sigma F_y = 0$

$T\sin\theta = m\frac{v^2}{r}$

$T\cos\theta - mg = 0$

$T = \frac{mg}{\cos\theta}$

$\cancel{m}g\frac{\sin\theta}{\cos\theta} = \cancel{m}\frac{v^2}{(L\sin\theta)}$

$v = \sqrt{Lg\frac{\sin^2\theta}{\cos\theta}}$

$= 2.16\ \frac{m}{s}$

$\boxed{\begin{array}{l}\cos\theta = x \\ \sin^2\theta = 1 - x^2\end{array}}$

(b) $v^2 = Lg\frac{\sin^2\theta}{\cos\theta}$

$xv^2 = Lg(1 - x^2)$

$xv^2 = Lg - Lgx^2$

$\underbrace{Lg}_{A}x^2 + \underbrace{v^2}_{B}x\ \underbrace{-Lg}_{C} = 0$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

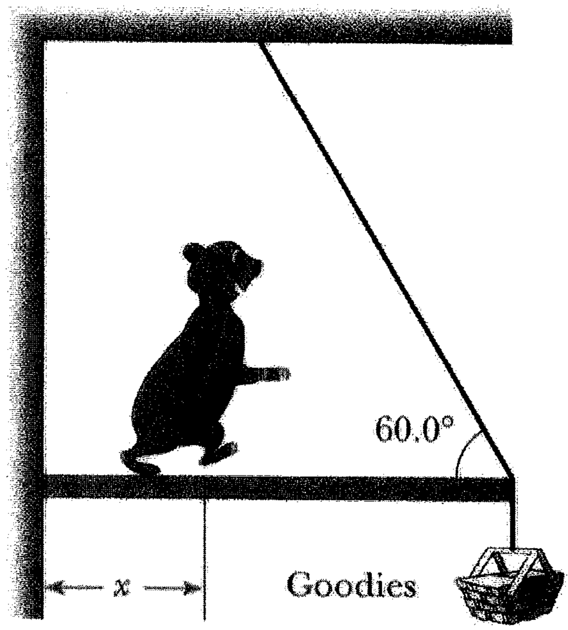
$\theta = \cos^{-1}(x)$

Answers: 2.16 m/s, 51.9°

HW12-4. An 8 m, 300 N uniform ladder rests against a smooth (frictionless) wall. μ_s between the ladder and the ground is 0.6. The ladder makes a 50° angle with the ground. A 600 N person is standing on the ladder a distance *d* from the bottom end of the ladder. How far up the ladder (distance *d*) can the person climb before the ladder begins to slip?

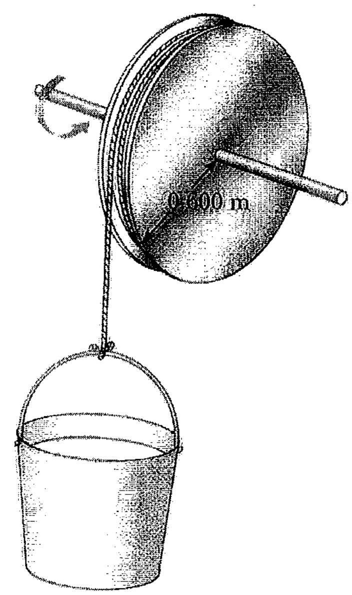
Answer: 6.58 m

HW12-5. A hungry 728 N bear walks out on a beam in an attempt to retrieve some "goodies" hanging at the end (see figure). The beam is uniform, weighs 216 N, and is 5.17 m long; the goodies weigh 82 N. (a) When the bear is at x = 1.00 m, find the tension on the wire and the magnitude of the hinge force where the beam is connected to the wall. (b) If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks?



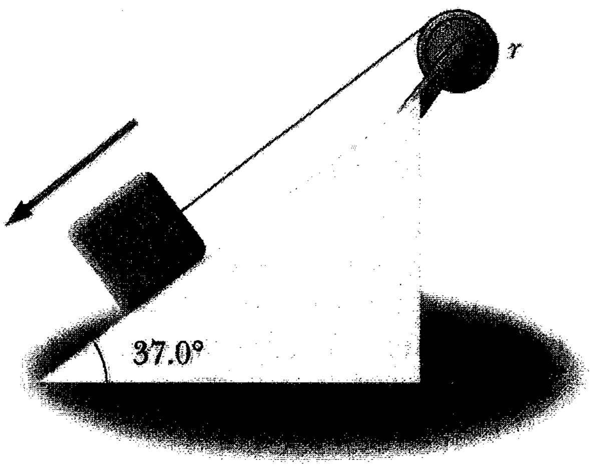
Answers: 382 N, 721 N; 4.19 m

HW13-2. A cylindrical 6 kg reel with a radius of 0.6 m starts from rest and speeds up as a 4 kg bucket falls into a well, making a light rope unwind. The bucket starts from rest and falls for 4 sec. (a) What is the linear acceleration of the falling bucket? (b) How far does it drop? (c) What is the angular acceleration of the reel?



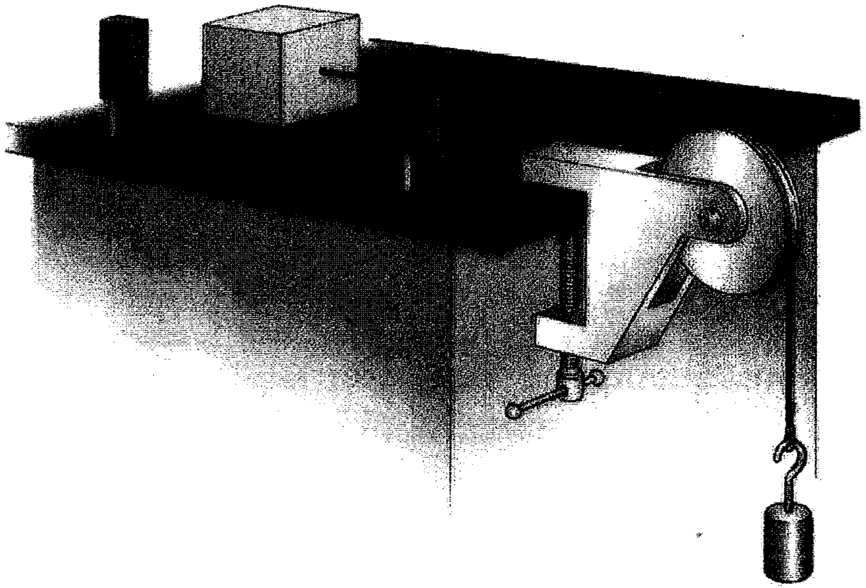
Answers: 5.6 m/s^2; 44.8 m; 9.33 rad/s

HW13-4. A 12 kg object is attached to a cord that is wrapped around a wheel of radius r = 11 cm. The acceleration of the object down the frictionless incline is measured to be 1.5 m/s^2. Determine (a) the tension in the rope, (b) the moment of inertia of the wheel, and (c) the angular speed of the wheel 2 seconds after it begins rotating, starting from rest.



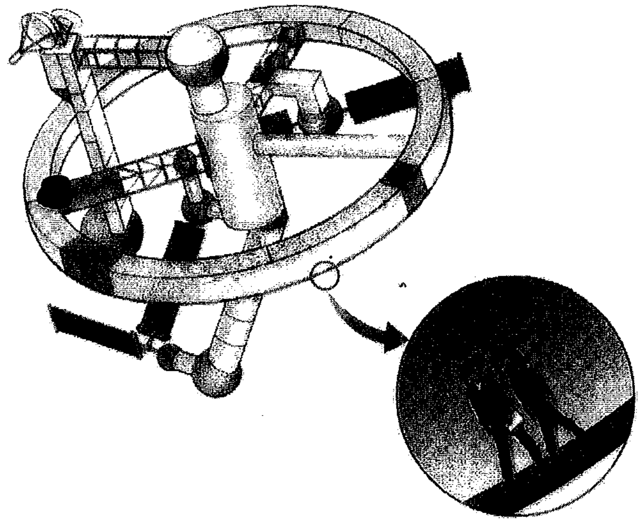
Answers: 70.77 N; 0.5709 kg·m; 27.27 rad/s

HW 13-6. The sliding block has a mass of 0.3 kg, the falling block has a mass of 0.4 kg, and the pulley is a uniform solid cylinder with a mass of 0.5 kg and an outer radius of 3 cm. μ_k between the block and the horizontal surface is 0.2. The block has a velocity of 1.2 m/s when it passes through a photogate. (a) Find its speed when it passes the second photogate, 0.7 m away. (b) Find the angular speed of the pulley at the same point.



Answers: 0.7573 m/s; 25.24 rad/s

HW 14-5. A space station has a radius 118 m and a moment of inertia of 4.77E08 kg·m^2. 150 people are on the rim, and the station is rotating so that they experience an apparent acceleration of 1 g. The people add to the total angular momentum of the system. When 100 people move to the center of the station, the angular speed changes. What apparent acceleration do the 50 remaining people feel? Assume m = 65 kg for all the inhabitants.



$\Sigma \tau = 0$

$N = mg$

$\Sigma F = ma_c$

$N = \frac{mv^2}{r}$

$\cancel{m}g = \frac{\cancel{m}v^2}{r}$

$v = \sqrt{rg}$

$L_{bef} = L_{aft}$

$(I_{station} + \underbrace{I_{people}}_{150mR^2})\,\omega_0 = (I_{station} + \underbrace{I_{people}}_{50mR^2})\,\omega_f$

$\omega_0 = \left(\frac{v_0}{r}\right)$, $\omega_f = \left(\frac{v_f}{r}\right)$

solve for v_f

then use to get g_{new}

Answer: 13.5 m/s^2