

Announcements – 29 Sep 2009

1. Exam 2 is coming up!

- a. Exam begins Thurs, ends next Wednesday (late fee after 1 pm)
- b. Covers Chapters 4 & 5, Homeworks 4-8
 - i. ...but I consider all exams cumulative!
- c. Thursday lecture is 100% exam review
 - i. No reading assignment or warmup quiz
- d. TA-led exam review session:
 - i. Time: Thursday, 6 pm - 7:30 pm
 - ii. Place: ~~probably same as before (Q215 ESC)~~
455 MARB

Some notes about the cumulative nature of the exam:

- You may get problems from exam 1 again
- You will certainly have some problems where you use Newton's 2nd Law to find acceleration then kinematics equations to find more details
- Newton's 2nd Law problems involving springs (today's topic, added to our "bag of tricks") are certainly fair game.

- 2) From TA: ^{Sarah}only email FBDs if emergency (change in policy)
Everyone only gets one emailed FBD (starting now)
- 3) From TA: FBDs from HW 5 graded and in your boxes

Which part of today's assignment was particularly hard or confusing?

How is it that PE of gravity $=mgy$, but PE of a spring is $\frac{1}{2}kx(\text{squared})$? That seems backwards to me.

Soooo many equations!!! Please help us!

4 equations
3 will be given to you!

I always have trouble with the blueprint equations. I know that I'm supposed to put something in the blueprint, but I always put the wrong things in the wrong places.

General comments:

there were a lot of people who had problems with problem 6.5 on the last homework. can you please go over this in class? I think it would be completely unfair to put that question on a test when so many people had such a hard time with it

I like the google group. It's nice to be able to look up hints if you are stuck.

So do you or a TA HONESTLY go and get the homework out of the turn in boxes Saturday night when the ESC closes?

How much more heavily weighted is this test than the last one?

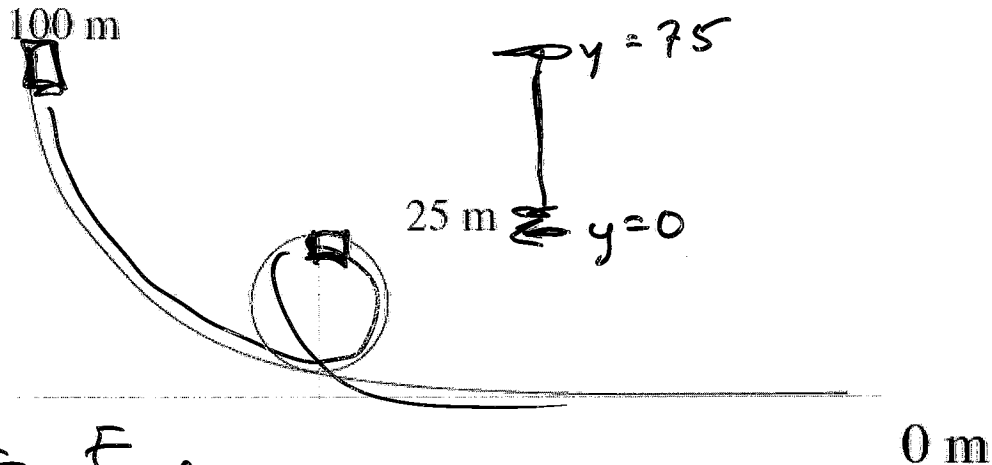
150 pts

60 pts

Clicker quiz (from last time): A 500 kg car starts from rest on a track 100 m above the ground. It does a loop-de-loop that is 25 m from the ground at the top. There is no friction. How fast is it going at the *top* of the loop?

- a. 0-10 m/s
- b. 10-20
- ☒ c. 30-40
- d. 40-50
- e. 50+ m/s

~~$$v_f^2 = v_i^2 + 2a\Delta x$$~~



$$E_{\text{bef}} = E_{\text{aft}}$$

$$mgy = \frac{1}{2}mv^2$$

$$v = \sqrt{2gy} = \sqrt{2(9.8)(75)} = 38 \text{ m/s}$$

Could you do this with N2??

Dr Colton's Guide:

How to solve Conservation of Energy Problems

1. Draw “before” and “after” pictures.

1b. If non-conservative forces: draw a FBD for the object, for the time between “before” and “after”.

$\sum W$

$$E_{\text{before}} + W_{\text{net}} = E_{\text{after}}$$

Blueprint equation

2. Write down the Conservation of Energy blueprint equation, and fill in the blueprint to get a “real equation”:

- Include all PE and KE terms for each “before” object, on the left hand side
- Include all PE and KE terms for each “after” object, on the right hand side
- Include work on the left hand side for all non-conservative forces (often friction). If needed, use the FBD and N2 to figure out the force.

$$KE = \frac{1}{2} mv^2$$

$$PE_{\text{grav}} = mgy$$

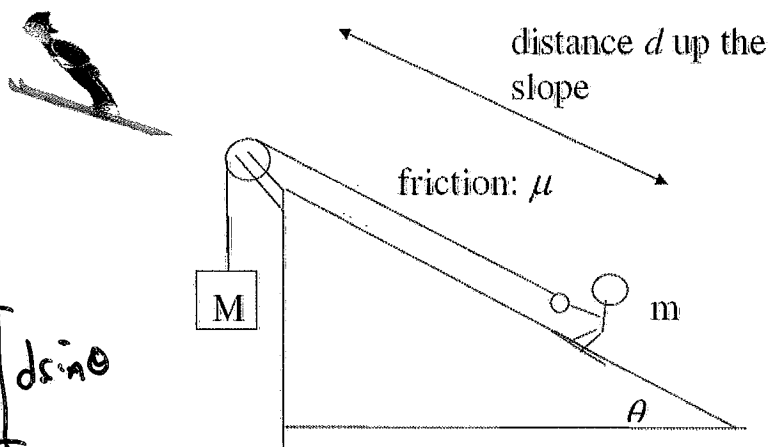
$$W = \mathbf{F}_{\parallel} \Delta \mathbf{x}$$

→ One work term for each force that doesn't have a PE

3. Plug what you know into the “real equation”, and look at what results.

4. Solve the equation for what you're looking for.

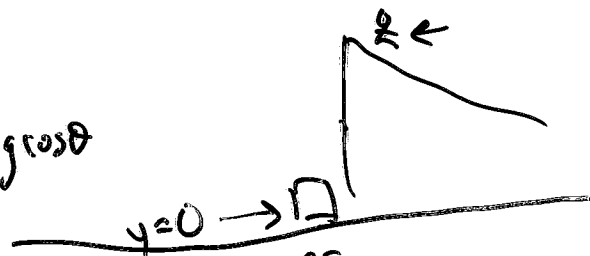
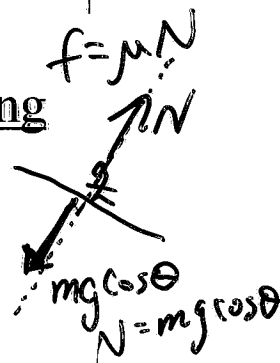
Problem: In terms of M , m , θ , d , and μ , what is his takeoff speed?



Before

During

After



Blueprint:

$$E_{\text{bef}} + W_{\text{net}} = E_{\text{aft}}$$

Fill in:

$$Mgd - f \Delta x = \frac{1}{2} M v_f^2 + \frac{1}{2} m v_f^2 + mg(d \sin \theta)$$

$$Mgd - (\mu mg \cos \theta)(d) = \left(\frac{1}{2} M + \frac{1}{2} m \right) v_f^2 + mg d \sin \theta$$

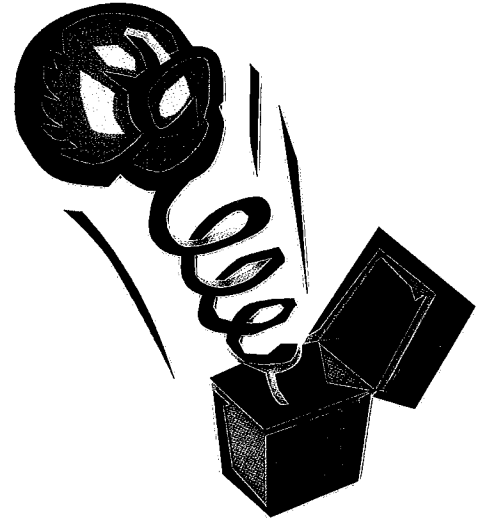
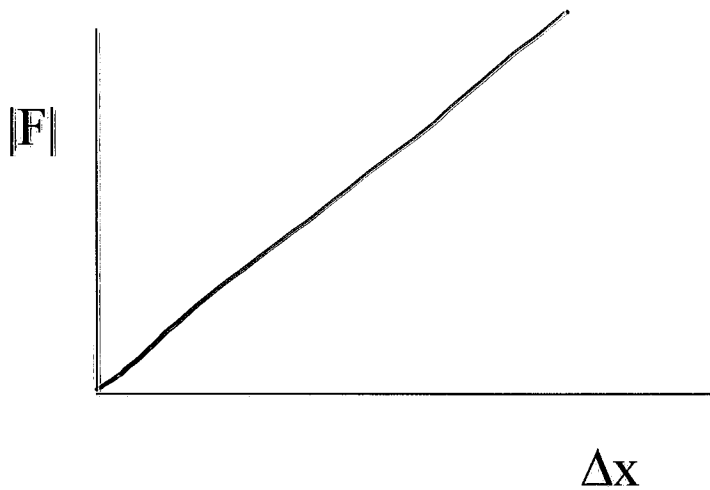
↓
you're looking for

Plug in given quantities:

Solve for what you want:

$$v_f = \sqrt{\frac{Mgd - \mu mg \cos \theta d - mg d \sin \theta}{\frac{1}{2} M + \frac{1}{2} m}}$$

Springs



Hooke's Law: Proportionality factor k = "spring constant"

$$F_{spring} = -kx$$

(negative sign: force acts opposite displacement: it pulls/pushes back to resting point)

Caution: x must be measured from equilibrium

Springs are conservative:

$$PE_{\text{spring}} = \frac{1}{2} kx^2$$

Derivation: force is varying \rightarrow must use *average* force

$$W = F_{\text{ave}} x$$

Work done to compress or stretch

$$= \left(\frac{1}{2} F_{\text{final}} \right) x$$

Force is linearly varying

$$= \left(\frac{1}{2} kx \right) x$$

$$= \frac{1}{2} kx^2$$

From warmup: You compress a spring 10 cm from its equilibrium position. Then you compress it another 10 cm. In which step did you do more work?

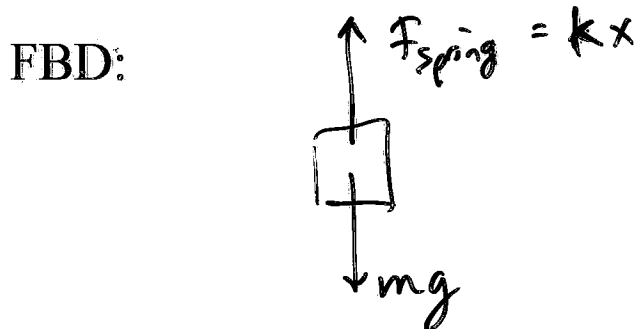
- a. The first 10 cm
- ☒ b. The second 10 cm
- c. Both cases involve the same amount of work

Hint: two ways to think about this

1. Think of change in energy
2. Think of average force during the compression

Problem: Determine the spring constant of a hanging spring.

Method 1: Use forces and N2 (with nonmoving spring)

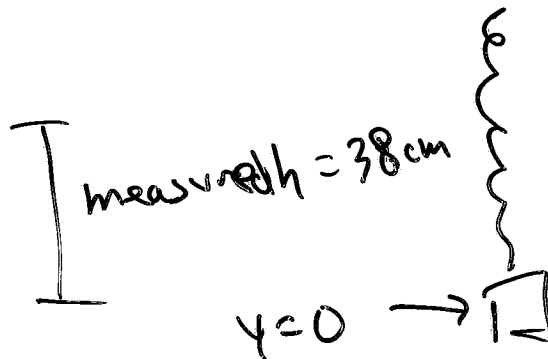
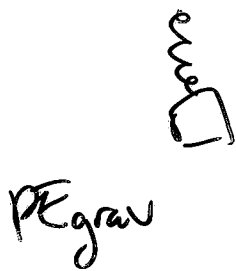


$$\begin{aligned}\sum F &= 0 \\ kx - mg &= 0 \\ kx &= mg \\ k &= \frac{mg}{x} = \frac{(0.5)(9.8)}{.18} \\ &= \boxed{27.22 \text{ N/m}}\end{aligned}$$

measured
18 cm

Method 2: Use energy (suddenly moving spring; compare starting point to lowest point of oscillation)

before



after

PE_spring

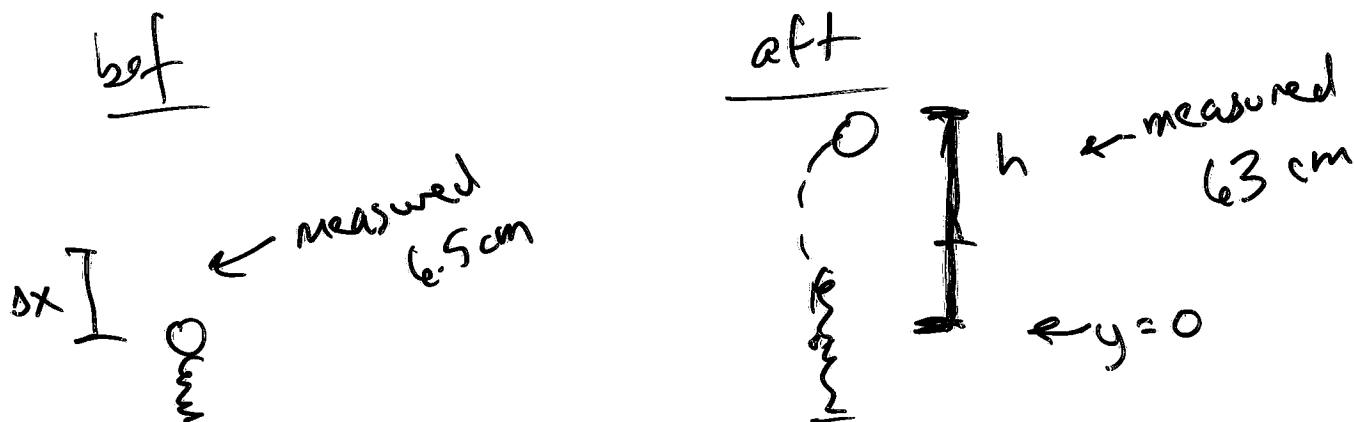
$$E_{\text{bef}} = E_{\text{aft}}$$

$$\begin{aligned}mgh &= \cancel{mgh} + \frac{1}{2}kx^2 \\ mgh &= \frac{1}{2}kh^2\end{aligned}$$

$$\begin{aligned}k &= \frac{2mg}{h} \\ &= \frac{2(0.5)(9.8)}{.38} \\ &= \boxed{25.8 \text{ N/m}}\end{aligned}$$

Problem: Determine the spring constant of the spring inside the "vertical cannon cart"

Method 3: Spring PE converted to gravity PE



$$E_{\text{eff}} = E_{\text{aft}}$$

$$\frac{1}{2} k s_x^2 = m g h$$

$$k = \frac{2 m g h}{(s_x)^2}$$

mass:
estimated
15 g

$$= \frac{2 (.015 \text{ kg}) (9.8 \text{ m/s}^2) (.63 \text{ m})}{(.065 \text{ m})^2}$$

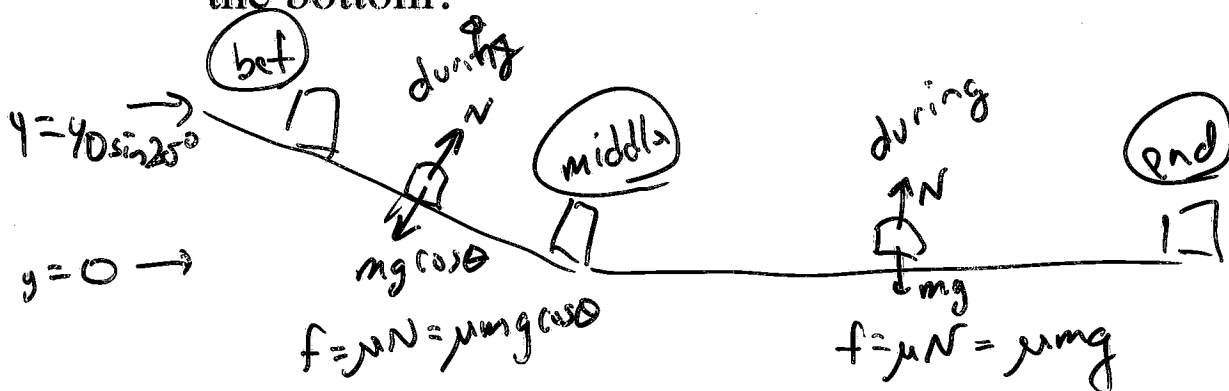
$$= 43.8 \text{ N/m}$$



Worked Problem: Fred, 50 kg (including ice), goes ice-blocking on the grass. Starting from rest he rides 40 m down a hill which has a 20° slope. $\mu_k = 0.2$ between the ice and grass.

(a) What is his speed at the bottom?

(b) How far will he go horizontally after he reaches the bottom?



$$\begin{aligned}
 \text{a) } E_{\text{ref}} + W_{\text{net}} &= E_{\text{middle}} \\
 mgy - f \Delta x &= \frac{1}{2}mv^2 \\
 mg(40 \sin 20^\circ) - [\mu mg \cos 20^\circ]40 &= \frac{1}{2}mv^2 \\
 v &= \sqrt{2 \left[9.8(40 \sin 20^\circ) - 0.2(9.8) \cos 20^\circ(40) \right]} \\
 &= \boxed{10.99 \text{ m/s}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } E_{\text{middle}} + W_{\text{net}} &= E_{\text{end}} \\
 \frac{1}{2}mv^2 - f \Delta x &= 0 \\
 \frac{1}{2}mv^2 - \mu mg \Delta x &= 0 \\
 \Delta x &= \frac{\frac{1}{2}v^2}{\mu g} = \boxed{30.8 \text{ m}}
 \end{aligned}$$

Answers: 10.99 m/s, 30.82 m

Power!

The rate at which energy is produced or consumed

$$P = \frac{\Delta E}{\Delta t}$$

SI units: 1 Watt = 1 J/s = 1 N·m/s = 1 kg·m²/s³ (yuck!)

Unit conversion: 1 horsepower (hp) = 746 W

Or... (equivalently)

Power is the rate at which work is being done

Another useful formula, if constant velocity:

$$P = F_{//} v$$

Derivation:

$$\begin{aligned} P &= \frac{\Delta E}{\Delta t} \\ &= \frac{W_{net}}{\Delta t} \\ &= \frac{F_{//} \Delta x}{\Delta t} \\ &= F_{//} \cdot v \end{aligned}$$



Image credit: Wikipedia. (Northern end of Stelvio Pass, Italy)

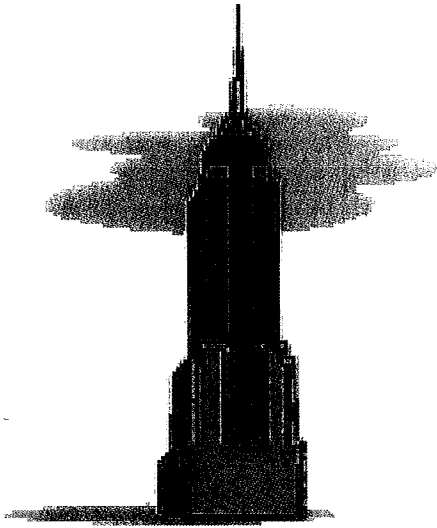
From warmup: Switchbacks on mountain roads (consider only work done against gravity):

- a. increase the work needed to go up a mountain
- b. decrease the work needed to go up a mountain
- Ⓒ keep the work needed the same

$$P = \frac{\text{energy}}{\text{time}}$$

From warmup: Switchbacks on mountain roads (consider only work done against gravity):

- a. increase the power needed to go up a mountain
- Ⓑ decrease the power needed to go up a mountain
- c. keep the power the same



Empire state building:

Height: 1,250 feet, 443 meters

Stories: 102

There are 1,575 steps from the building's lobby to the 86th floor (374 m). Paul Crake holds the record for racing these steps in 10 minutes, 15 seconds. = 615 sec

What average power did he expend against gravity?

(Assume $m=80$ kg)

From work:

$$P = \frac{W_{\text{net}}}{\text{time}} = \frac{mgy}{\text{time}} = \frac{(80)(9.8)(374)}{615} = 476 \text{ W}$$

From velocity:

$$P = F \cdot v = (mg) \left(\frac{374 \text{ m}}{615 \text{ sec}} \right)$$

= the same!

Answer: 476 W, 0.638 hp

Clicker quiz: A car weighing 3000 N moves at a speed of 30 m/s on level ground. To do this, it pushes backwards on the road with a 5000 N force. What is the power output of the car engine?

- a. 0 kW
- b. 60 kW
- c. 90 kW

- ☒ d. 150 kW
- e. 240 kW

$$P = F \cdot v$$

↑
parallel!
= 5000N

→ Where does this power go? If the car moves at constant speed, it's not used to accelerate the car.

From warmup: Ralph sees that his car's engine is rated at 100 hp. He thinks, "Cool, this means if I ever get in a tug of war with 90 horses, I will win!" Is he thinking about this correctly?

Answer from the class:

988-----

Tell Ralph that horsepower is a measure of Power, or the rate at which work is done. ~~This is not to say that~~ his car can do more work than 90 horses, ~~or even 50 for that matter.~~ Although his car can ^{Time} ^{do work} ~~exert force~~ at a faster rate, the force which it exerts is certainly considerably less than that of the combined efforts of horses.

Many incorrect answers: "horsepower has nothing to do with horses"