Announcements – 8 Oct 2013

1. Exam 2 still going on
    a. ends tomorrow 2 pm, late fee after 2 pm today

Where are we now?

Topics

Kinematics (velocity, acceleration)
Vectors & 2D Motion
Forces & Newton's Laws
Work & Energy
Momentum
Rotations, Torque, and Angular Momentum
Pressure
Fluids & Solids
Temperature, Heat, and Heat Flow
Laws of Thermodynamics
Vibrations & Waves

"Mechanics"

"Thermodynamics"

Part Mechanics,
Part Sound
Part Optics

Conserved quantities

Energy

→ When no non-conservative work done, $E_{bef} = E_{aft}$

Mass

→ If not converted to/from energy ($E=mc^2$),

$$(\text{total mass})_{bef} = (\text{total mass})_{aft}$$

Charge

→ $(\text{total charge})_{bef} = (\text{total charge})_{aft}$

I.e., if some positive charge flows out of a neutral object, it will leave the object with negatively charged

<u>Often conserved (used to balance chemical reactions)</u>

Number of each type of atom

Number of electrons

Etc.

A new conserved quantity… **momentum**

Define $\boxed{\vec{p} = m\vec{v}}$ for each object, then

$$\boxed{\sum \vec{p}_{before} = \sum \vec{p}_{after}}$$ (if no external forces)

Another blueprint equation!

units

$p = mv$

kg $\frac{m}{s}$

Momentum: used for Collision Problems

$+m_1 v_{1i}$ $-m_2 v_{2i}$

after

m_1 v_{1f} m_2

$m_2 v_{2f}$

$-m_1 v_{1f}$

$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$F_{2-1} \leftarrow$ ○ ○ $\rightarrow F_{1-2}$

Derivation of conservation law:

$\Sigma \mathbf{F}_1 = m_1\mathbf{a}_1$ $\Sigma \mathbf{F}_2 = m_2\mathbf{a}_2$

$-F_{21} = m_1 a_1$ $F_{12} = m_2 a_2$

to the left

Newton's 3rd Law: the forces involved in the collision are equal and opposite

If no other forces, then…

$$\vec{\mathbf{F}}_{2\text{-}1} + \vec{\mathbf{F}}_{1\text{-}2} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2$$

$$0 = m_1\Delta\mathbf{v}_1/\Delta t + m_2\,\Delta\mathbf{v}_2/\Delta t$$

Multiply by Δt (which is the same for both)

$$m_1\Delta\mathbf{v}_1 + m_2\,\Delta\mathbf{v}_2 = 0$$

$$m_1(\mathbf{v}_{1\text{ final}} - \mathbf{v}_{1\text{ initial}}) + m_2(\mathbf{v}_{2\text{ final}} - \mathbf{v}_{2\text{ final}}) = 0$$

$$m_1\mathbf{v}_{1\text{ initial}} + m_2\,\mathbf{v}_{2\text{ initial}} = m_1\,\mathbf{v}_{1\text{ final}} + m_2\,\mathbf{v}_{2\text{ final}}$$

… and there you have it!

From warmup

The total momentum of an isolated system of objects is conserved

a. only if conservative forces act between the objects

b. regardless of the nature of the forces between the objects.

From warmup

A truck always has more mass than a roller skate. Does a truck always have more momentum than a roller skate?

a. yes
b. no

Why use conservation of momentum?

it makes some problems easier!

Limitation: Like conservation of energy, conservation of momentum is a "before" and "after" law which doesn't tell you about:

time ________________

If you want to know about time, you have to know acceleration / forces on that object

Another useful equation: of that object

$$\boxed{\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}}$$ "Impulse equation" (focusing on one object)

Δtime of collision

Derivation: ΣF = ma = mΔv/Δt; multiple both sides by Δt

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{\Delta v}{\Delta t}$$

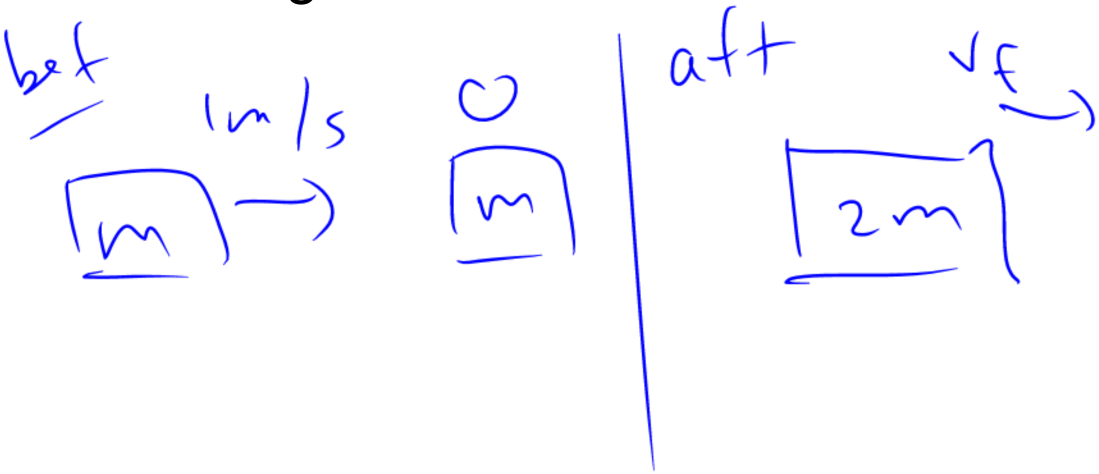
$$\vec{F}\Delta t = \underline{m\Delta\vec{v}}$$

Δp !

When to use?

during

Demo Problem: A cart moving at 1 m/s runs into a second cart (stationary) with the same mass and sticks to it. What velocity do the two stuck together carts now have?

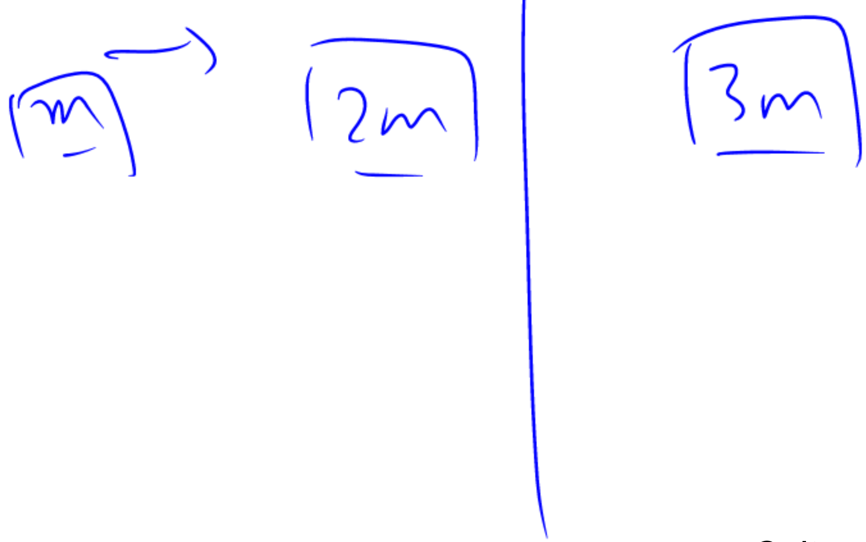


$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\cancel{m}\left(1\,\frac{m}{s}\right) = (2\cancel{m})\,v_f$$

$$v_f = \frac{1}{2}\ \frac{m}{s}$$

Demo Problem: A cart moving at 1 m/s runs into a second cart (stationary) with *twice* the mass and sticks to it. What velocity do the two stuck together carts now have?



$$\cancel{m}\left(1\,\frac{m}{s}\right) = (3\cancel{m})\,v_f$$

$$v_f = \frac{1}{3}\ \frac{m}{s}$$

Demo Problem: A cart moving at 1 m/s runs into a second cart with *twice* the mass and sticks to it. The second cart is moving at 0.5 m/s towards the first one. What velocity do the two (stuck together) carts now have?

Dr Colton's Guide:
How to solve Conservation of Momentum problems

1. Draw initial and final pictures

2. Draw *momentum* or *velocity* vectors (arrows) in each picture

3. Use $\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after}$ as "blueprint equation"

4. Divide into separate x- and y- equations if needed

5. Fill in both sides of blueprint equation(s) using initial and final pictures: one term in equation for each arrow in picture.

6. Reminder: be careful with signs! (Momentum is a **vector**)

The new blueprint

$$\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after}$$

Compare to previous two blueprint equations:

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$E_{before} = E_{after} \qquad \text{(if no non-conservative forces)}$$

Similarities? Differences?

From warmup

Suppose Ralph is floating in outer space with no forces acting on him. He is at rest, so his momentum is zero. Now, he throws a ball. The ball goes one way, and he goes the other way. Before the collision, there was no momentum, and after the collision, there is plenty of momentum! Was momentum conserved?

"Pair share"–I am now ready to share my neighbor's answer if called on.

a. Yes

Worked Problem

In the new sport of “ice football”, a 100 kg defensive end running north at 4 m/s tackles a 75 kg quarterback running *east* at 7 m/s. There’s no friction. What is their combined velocity right after the tackle?

75kg, 7m/s

$m_2 \rightarrow$

m_1

100kg

4m/s

after

$v_F = ?$

θ v_y

v_x

175kg

$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\sum p_{x\,bef} = \sum p_{x\,aft}$$

$$(75)(7) = (175)\,v_x$$

$$v_x = 3\ \text{m/s}$$

$$\sum p_{y\,bef} = \sum p_{y\,aft}$$

$$(100)(4) = (175)\,v_y$$

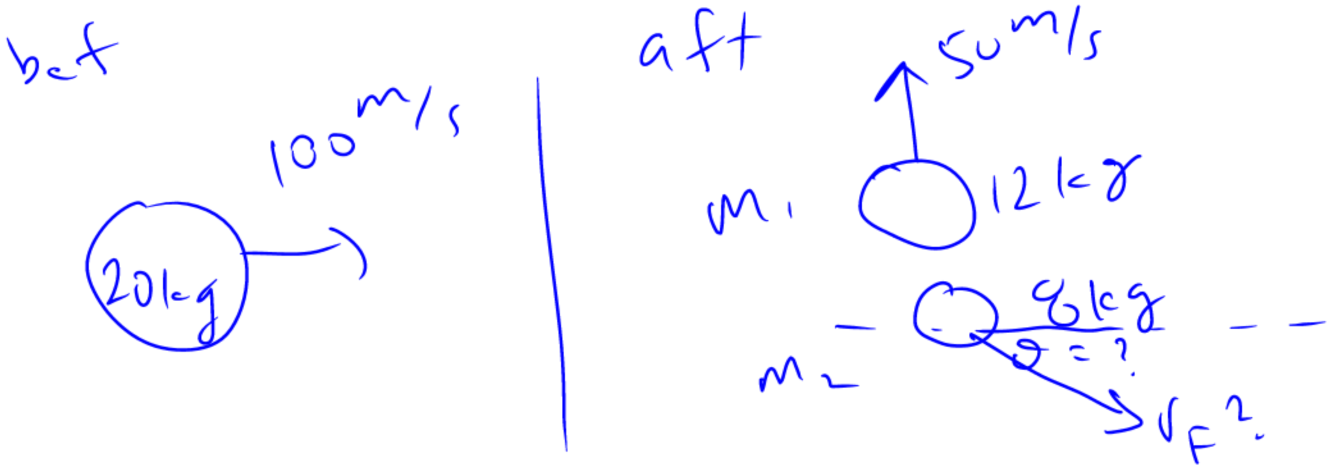
$$v_y = 2.28\ \frac{m}{s}$$

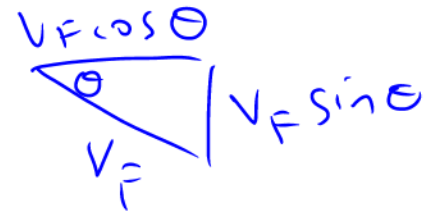
$$v_F = \sqrt{3^2 + 2.28^2} = 3.77\ \frac{m}{s}$$

Answers: v_x = 3 m/s; v_y = 2.28 m/s; v = 3.77 at 37.3° north of east

Worked Problem

An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?





$\Sigma P_{x\,bef} = \Sigma P_{x\,aft}$

$(20)(100) = (8)(v_F \cos\theta)$

$2000 = 8 v_F \cos\theta$

$\Sigma P_{y\,bef} = \Sigma P_{y\,aft}$

$0 = (12)(50) - (8) v_F \sin\theta$

$$\frac{\cancel{8}\,\cancel{v_F} \sin\theta = 600}{\cancel{8}\,\cancel{v_F} \cos\theta = 2000}$$

$\tan\theta = 600/2000$

$\theta = 16.7^\circ$

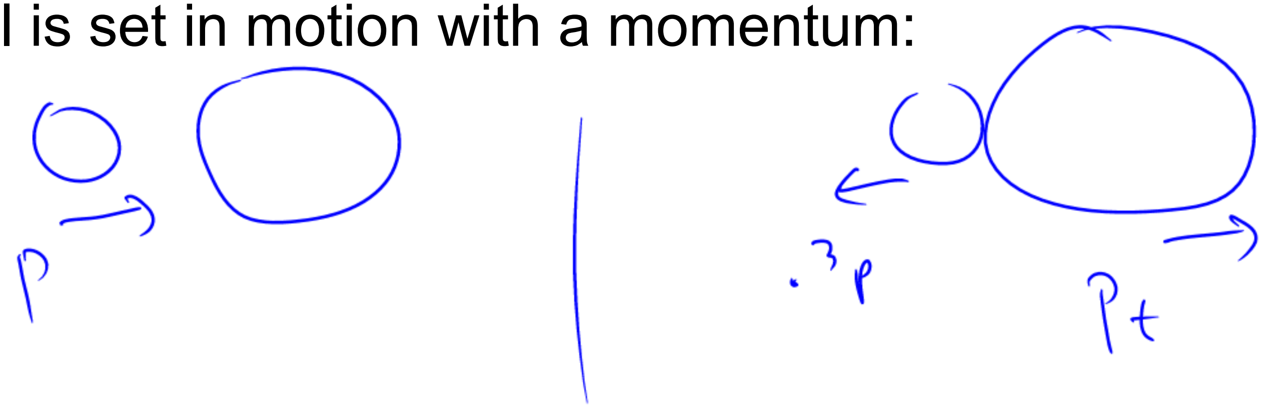
$v_F = 261\ \text{m/s}$

Answers: v_x = 250 m/s; v_y = -75 m/s; v = 261 m/s at 16.7° south of east

From warmup, do as clicker quiz

A ping-pong ball moving forward with a momentum *p* strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

a. greater than *p*
b. less than *p*
c. equal to *p*



$\Sigma P_{bef} = \Sigma P_{aft} \rightarrow$ $\quad p = -.3p + P_t$

$P_t = 1.3p$

What about if ping-pong ball "thuds" and falls flat?

$P_t = p$

Demo: Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?

Question

"elastic"

Is energy conserved in collisions? All? Some? None?

Special Case: "Elastic" Collisions

In some special collisions, energy is also conserved!

Elastic collisions: no lost kinetic energy

→ they are "bouncy"

(but not all bouncy-looking collisions are elastic… don't assume)

Inelastic collisions:

→ KE not conserved

Perfectly inelastic collisions:

→ stick together → most lost KE

Dr. Colton's guide, cont.

#7. If it's an elastic collision then…

~~$\Sigma KE_{before} = \Sigma KE_{after}$~~

~~→ This is in addition~~ to $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

The two equations can be put together to give:

$$\boxed{(v_1 - v_2)_{\mathbf{bef}} = (v_2 - v_1)_{\mathbf{aft}}}$$

used in addition to cons. of mom. for elastic collisions

"velocity reversal"

Careful with signs! "Right = positive, left = negative" still applies

Derivation:

<u>Cons. mom</u>

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 \left(v_{1i} - v_{1f} \right) = m_2 \left(v_{2f} - v_{2i} \right)$$

<u>Cons. energy</u>

$$\tfrac{1}{2} m_1 v_{1i}{}^2 + \tfrac{1}{2} m_2 v_{2i}{}^2 = \tfrac{1}{2} m_1 v_{1f}{}^2 + \tfrac{1}{2} m_2 v_{2f}{}^2$$

$$m_1 \left(v_{1i}{}^2 - v_{1f}{}^2 \right) = m_2 \left(v_{2f}{}^2 - v_{2i}{}^2 \right)$$

$$m_1 \left(v_{1i} + v_{1f} \right)\left(v_{1i} - v_{1f} \right) = m_2 \left(v_{2f} + v_{2i} \right)\left(v_{2f} - v_{2i} \right)$$

Divide the two equations.

$$\frac{\cancel{m_1} \left(v_{1i} + v_{1f} \right) \cancel{\left(v_{1i} - v_{1f} \right)}}{\cancel{m_1} \cancel{\left(v_{1i} - v_{1f} \right)}} = \frac{\cancel{m_2} \left(v_{2f} + v_{2i} \right) \cancel{\left(v_{2f} - v_{2i} \right)}}{\cancel{m_2} \cancel{\left(v_{2f} - v_{2i} \right)}}$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

Demo Problem

A cart moving at 1 m/s bounces elastically off of a second cart of twice the mass which is moving at 0.5 m/s in the same direction. What velocity does each cart now have?

Answer: v_1 = 0.33 m/s; v_2 = 0.83 m/s

Demo Problem

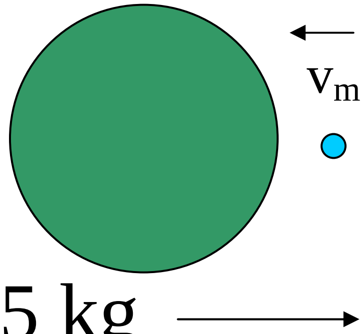
A cart moving at 1 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have?

Demo: Newton’s cradle

Demo problem:

Elastic collision between big M and small m

Bowling ball and a marble! Marble is at rest.



v_m going towards bowling ball, 15 g

v_b, 5 kg

What are final speeds?

Simplification: $v_{\text{bowling ball final}} \approx v_{\text{bowling ball initial}}$

Demo: "Velocity amplifier"