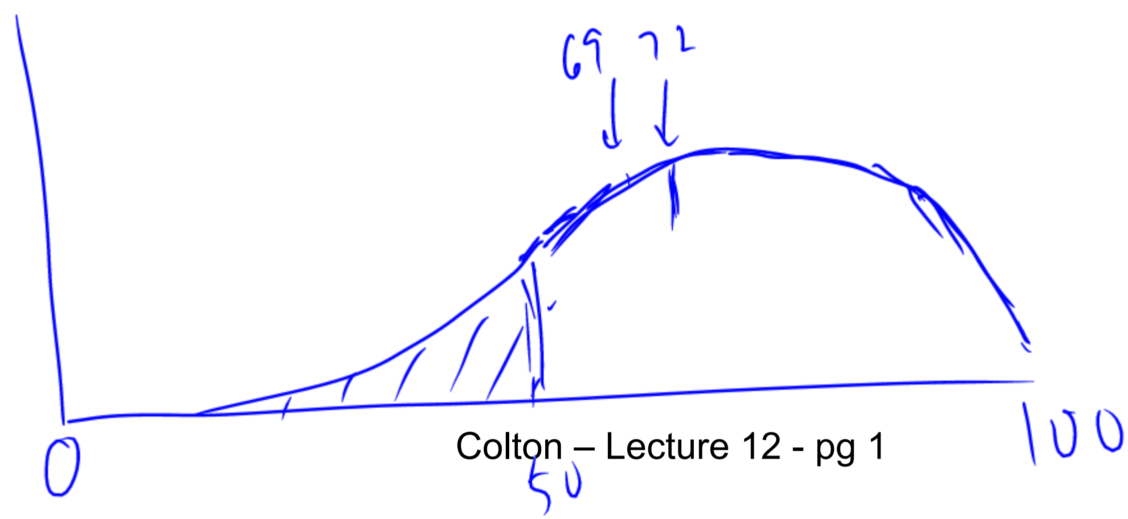
Announcements - 10 Oct 2013

1. **Exam 2 results**
   a. Average Score: Mean = 69 median = 72
   b. Exams will be returned soon (tomorrow afternoon), pick them up in usual place (boxes near N357 ESC)
   c. Solutions will be posted on website soon (tomorrow afternoon)

2. Like last time: if you have questions on the exam, please look over your own exam & the posted solutions to figure things out as best you can, before coming to talk to me or the TA.

Distribution (histogram



Momentum Review

Equations

Definition: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

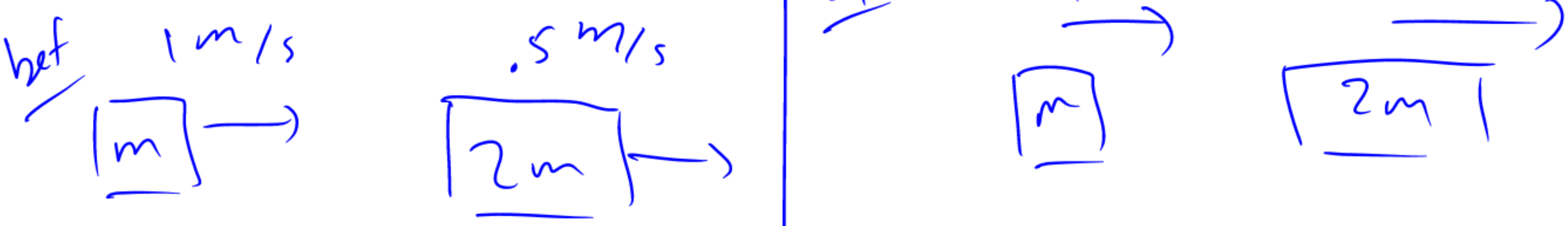
Conservation Law (blueprint): (1) $\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after}$ (if no …. external forces) ← for groups

"Impulse equation": $\vec{\mathbf{F}}_{\mathbf{net}} \Delta t = \Delta \vec{\mathbf{p}}$ (applied to … an individual) object

Velocity reversal: (2) $(v_1 - v_2)_{before} = (v_2 - v_1)_{after}$ (if … elastic)

Worked Problem

A cart moving at 1 m/s bounces elastically off of a second cart of twice the mass which is moving at 0.5 m/s in the same direction. What velocity does each cart now have?



$$\Sigma \vec{p}_{bef} = \Sigma \vec{p}_{aft}$$

$$(m)(1) + (2m)(.5) = m v_1 + 2m v_2$$

$$v_1 + 2v_2 = 2$$

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$1 - .5 = v_2 - v_1$$

$$v_2 - v_1 = .5$$

$$v_2 = .5 + v_1$$

$$v_1 + 2(.5 + v_1) = 2$$

$$v_1 + 1 + 2v_1 = 2$$

$$3v_1 = 1$$

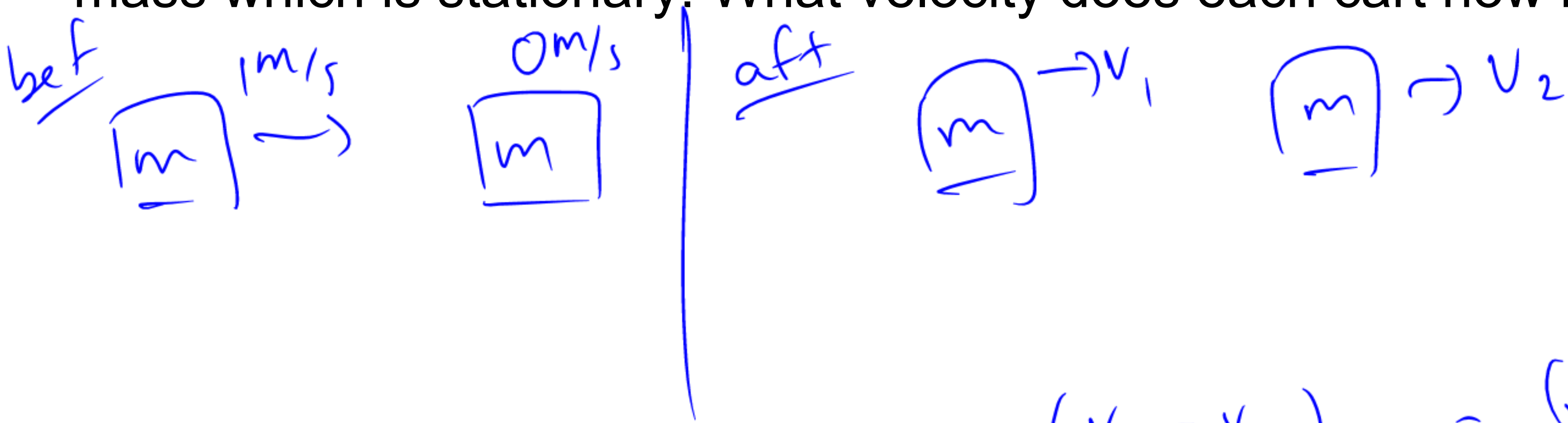
$$v_1 = 1/3 \text{ m/s}$$

$$v_2 = .5 + v_1 = .833 \text{ m/s}$$

Answer: v_1 = 0.333 m/s; v_2 = 0.833 m/s

Worked Problem/Demo

A cart moving at 1 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have?



$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$m(1) + 0 = m v_1 + m v_2$$

$$v_1 + v_2 = 1$$

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$1 - 0 = v_2 - v_1$$

$$v_2 - v_1 = 1$$

$$+ \ (v_2 + v_1 = 1)$$

$$2v_2 + 0 = 2$$

$$v_2 = 1$$

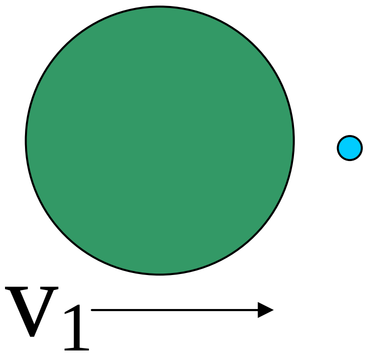
$$v_1 + 1 = 1$$

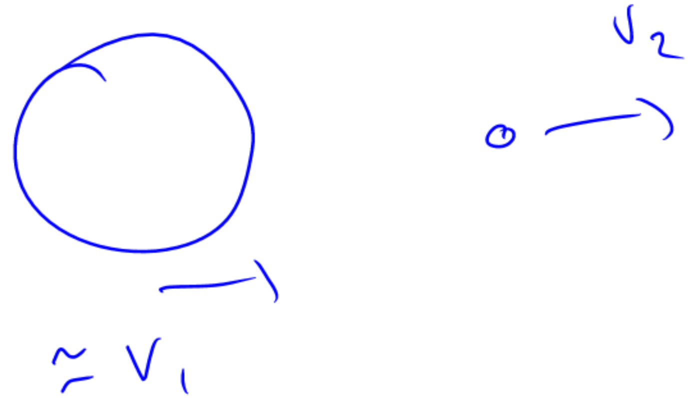
$$v_1 = 0$$

Demo: Newton's cradle

Worked Problem/Demo

Bowling ball and a marble. Marble is at rest. Elastic.





If $v_{\text{bowling ball final}} \approx v_{\text{bowling ball initial}}$,
what does velocity reversal equation imply about $v_{\text{marble final}}$?

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$v_1 - 0 = v_2 - v_1$$

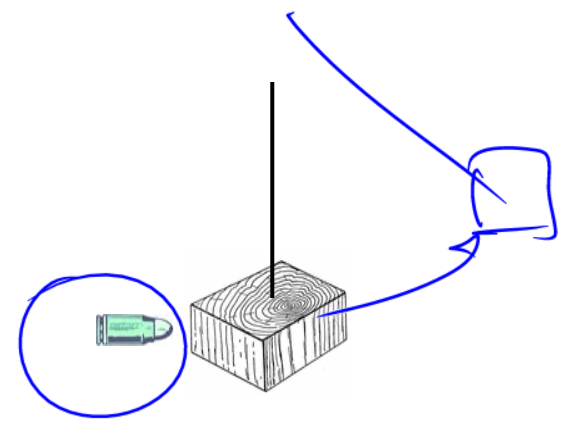
$$v_2 = 2v_1$$

Demo: "Velocity amplifier"

Multi-step problems

→ Collision followed by something else

"Ballistic pendulum". A bullet of mass *m* and speed *v* embeds in a block of wood of mass *M* hanging from a string. How high do they rise?



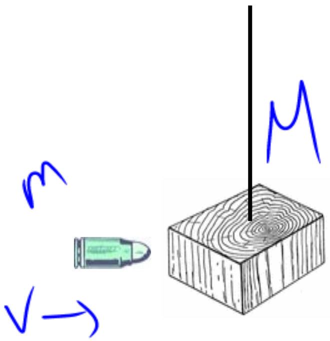
How not to do: $$\frac{1}{2}mv^2 = (m+M)gh$$

How to do:

1. **Collision part: *p* is conserved** (but KE is not!)
   i. This gets you the velocity right after the collision
2. **Motion part: Energy is conserved** (but *p* is not!)
   i. This gets you the height based on velocity

cons. mom. — cons. energy

Before — **After/Before** — **After**



v_{comb}

h

$\Sigma \vec{p}_{bef} = \Sigma \vec{p}_{aft}$

$mv = (m+M)v_{comb}$

$v_{comb} = \dfrac{mv}{m+M}$

$E_{bef} + \cancel{W} = E_{aft}$

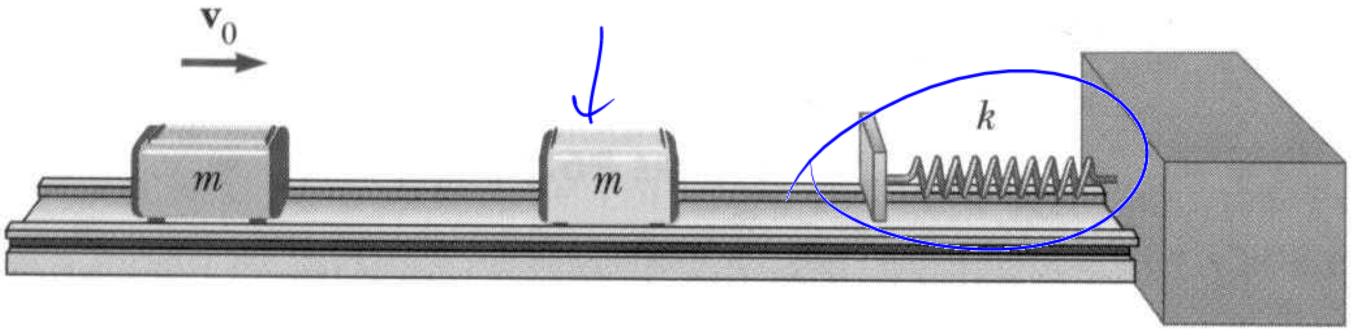
$\frac{1}{2}(m+M)v_{comb}^2 = (m+M)gh$

$h = \frac{1}{2}\dfrac{v_{comb}^2}{g}$

$h = \frac{1}{2}\dfrac{\left(\frac{mv}{m+M}\right)^2}{g}$

Answer: $h = \left(\dfrac{m}{m+M}\right)^2 \dfrac{v^2}{2g}$

HW 12-1: Very similar!



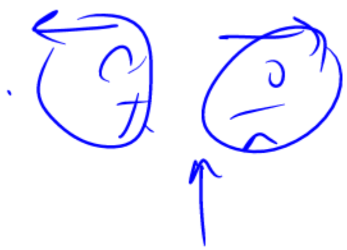
Center of Mass

What is the center of mass?

center of object
of group of objects

What forces can change the motion of the center of mass?

external

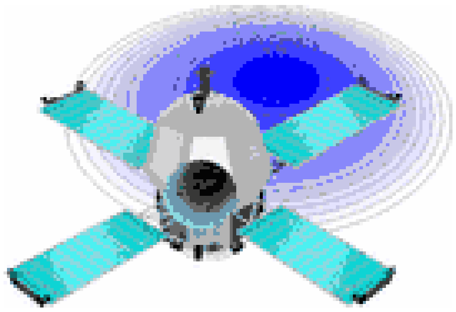


How does the center of mass move?

like a "pt object"

Demos: Foam object

Circular Motion



Demo: Bicycle wheel

Complicated motion of rotating body:
Different r, v, a’s for different parts

But same __angular velocity__

or rpm

From warmup

Which has greater linear speed, a horse near the outside rail of a merry-go-round or a horse near the inside rail?

a. outside horse
b. inside horse
c. both the same

Do revolutions relate to angles?

Question: Which angle is greatest:

a. 30 revolutions
b. 30^{o}
c. 30 radians

1 rev = 360°

30 rev = 30×360°

1 rad ≃ 60° roughly!

180° in π radians

or 2π rad in 360° or 1 rev.

Definition of radian

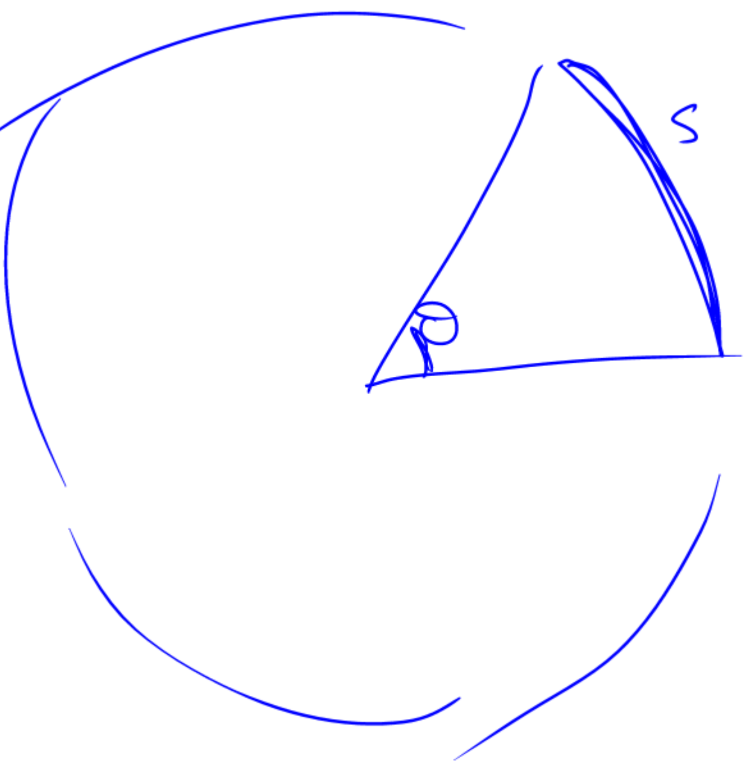
How many radians in one circumference?

How many radians in 360°?

→ I will not give you these conversion factors on exam!

How many radians in an arc of length "s"?

→ I <u>will</u> give you this equation on exam



$$\frac{s}{\text{circum}} = \frac{\theta}{360^\circ}$$

$$\frac{s}{2\pi r} = \frac{\theta}{360^\circ}$$

$$s = \theta \cdot r \quad \frac{2\pi}{360^\circ}$$

↑ in deg

$$s = \theta r$$

↑ in rad

What is angular speed? (aka angular velocity)

$$\omega = \frac{\text{rad}}{\text{s}} = \frac{\Delta\theta}{\Delta t}$$

Clicker quiz: The symbol ω, used for angular velocity, is pronounced:

a. "al-pha"
b. "double-you"
c. "gam-ma"
d. "om-e-ga"
e. "pi"

From warmup

Which has greater *angular* speed, a horse near the outside rail of a merry-go-round or a horse near the inside rail?

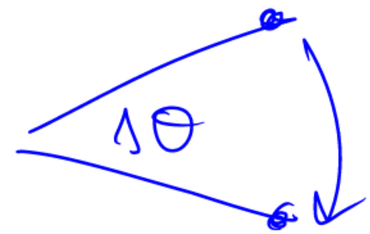
d. outside horse
e. inside horse
f. both the same

Angular quantities

displacement $\Delta\theta = \theta_f - \theta_i$

average velocity $\omega_{ave} = \dfrac{\Delta\theta}{\Delta t}$

average acceleration $\alpha_{ave} = \dfrac{\Delta\omega}{\Delta t}$



Units?

rad

rad/s

rad/s^2

Kinematic equations (for constant *angular* acceleration)

Substitutions:

$$x \rightarrow \theta$$
$$v \rightarrow \omega$$
$$a \rightarrow \alpha$$

omega
alpha

Regular kinematic	Angular kinematic
Definition: $v_{ave} = \langle v \rangle = \frac{\Delta x}{\Delta t}$	$\omega_{ave} = \langle \omega \rangle = \frac{\Delta \theta}{\Delta t}$
Definition: $a_{ave} = \langle a \rangle = \frac{\Delta v}{\Delta t}$	$\alpha_{ave} = \langle \alpha \rangle = \frac{\Delta \omega}{\Delta t}$
For constant *a*:	For constant α
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0{}^2 + 2a(x - x_0)$	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

Note: I will not give you the angular kinematic equations on exam

Angular motion of the whole object vs. motion of a spinning **point**

Angular displacement $\Delta\theta$ vs "distance around circumference", *s*

Angular velocity ω vs tangential speed *v*

Angular acceleration α vs tangential acceleration *a*

Important: You must use radians if you want to use those equations

Some Guidance, aka. What I Do

1. Pretend a problem involves regular distances & velocities, and figure out how you would solve it
2. Then use the corresponding angular equations

Worked Problem

Friction slows down a 5 cm diameter spinning top with angular deceleration of 2 rad/s^2. It was initially spinning at 50 rad/s.

1. How many revolutions will it turn before stopping?

"Translated problem": Friction slows down a block, a = -2 m/s^2. It was initially travelling at 50 m/s. How far will it go before stopping?

Answer: 99.47 revs

Worked Problem, cont.

Friction slows down a 5 cm diameter spinning top with angular deceleration of 2 rad/s^2. It was initially spinning at 50 rad/s.

2. How long will that take?

3. How fast was a point on the rim initially going?

Answers: 25 s, 1.25 m/s

Centripetal vs. Tangential

Ball on string: **Demo**

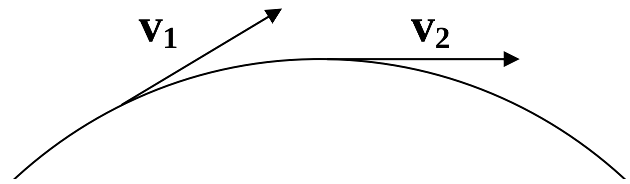
Is the ball accelerating?

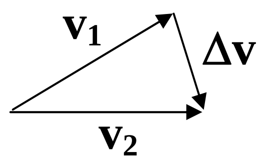
Speeding up/slowing down: direction of acceleration is ________________

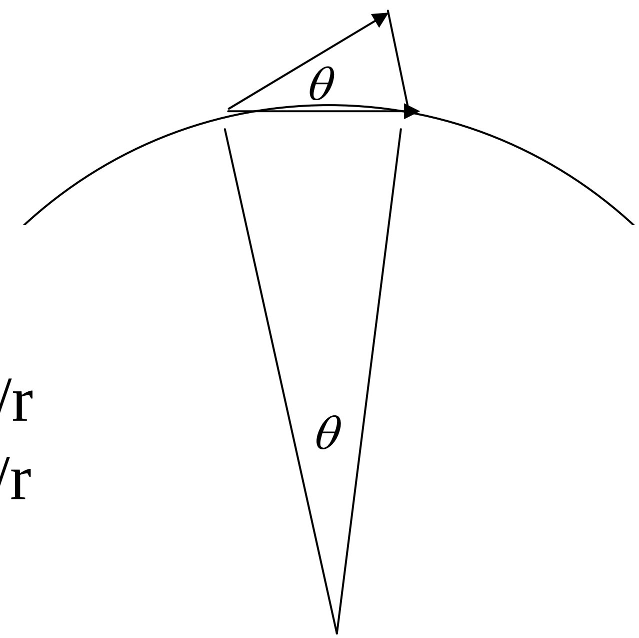
What if the ball is not speeding up/slowing down?

Centripetal (turning) acceleration: $a_c = \frac{v^2}{r}$

Derivation:







$$\vec{\mathbf{a}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Similar triangles: $\Delta v/v = \Delta s/r$

$\Delta v = v\,\Delta s/r$

Piece together:

$$\frac{\Delta v}{\Delta t} = \frac{\left(v\Delta s/r\right)}{\Delta t}$$

$$= \left(\frac{v}{r}\right)\frac{\Delta s}{\Delta t}$$

$$= \frac{v^2}{r}$$

Direction of centripetal acceleration?

→ let's call that positive

Bottom line:

If the object is in circular motion, then there is an acceleration.

This acceleration (v^2/r) goes on the **right hand side** of N2

From warmup

A ladybug sits on the outer edge of a merry-go-round that is turning around counter-clockwise without speeding up or slowing down. In what direction is the friction force that sticks the ladybug to the merry-go-round?

a. clockwise
b. counter-clockwise
c. inward
d. outward

From warmup

Ralph is confused about centripetal and centrifugal forces. When he is in a car which is turning to the left, he feels a force pushing him to the right. But the textbook says that the actual force is pushing him to the left. Can you explain this to him? What is he feeling during the turn?

“Pair share”–I am now ready to share my neighbor’s answer if called on.

a. Yes

Worked Problem

You swing a ball (mass *m*) in a vertical circle with a string; its speed is constant (*v*) through the whole circle. (a) What is the tension at the lowest point? (b) At the highest point?

(a) Picture:

Equation:

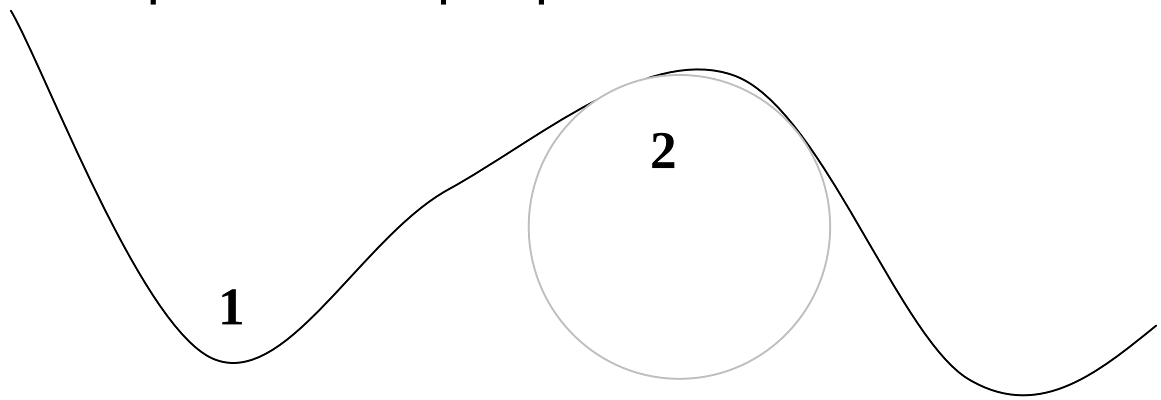
(b) Picture:

Equation:

Answers: $mg + mv^2/r$, $mg - mv^2/r$

Unsafe roller coasters (no seatbelts)

For the top of an *outside* curve (pt 2), radius of curvature = 8 m, what is the maximum speed if the people are **not to fall out**?



What's the difference between pt 1 and pt 2?

Free-body diagrams:

What happens to Normal force at pt 2 as speed increases?
→ Just as people fall out, the normal force is _______________.

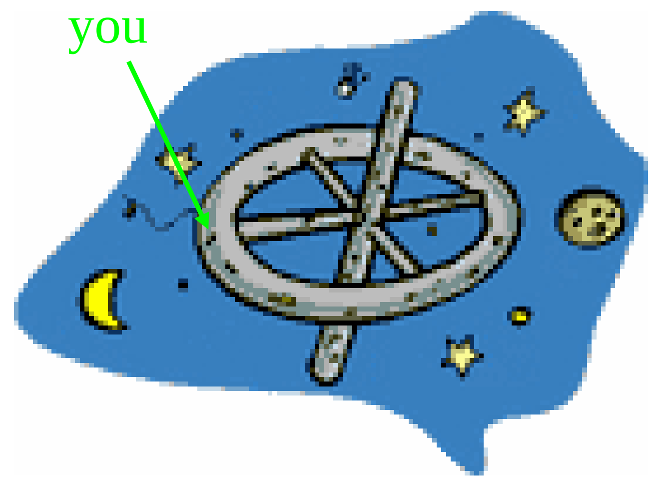
Solution to the problem (8 m radius of curvature):

Answer: 8.85 m/s

Question

Angular velocity of **earth** (1 rev/24 hours, convert to rad/s) gives speed at Provo = 792 mph! Why don't we fly off?

Space stations and “artificial gravity”



You are standing on a 50 m radius space station that rotates at just the right speed so that the **normal force** is

$N = m \times 9.8\ m/s^2$.

Your usual normal force = mg, so this feels “normal” to you! ☺

What direction do your feet face?

How fast must the space station rotate (rpm) in order to cause this?

Answer: 4.23 rpm