Announcements – 15 Oct 2013

1. While you're waiting for class to start, see how many of these blanks you can fill out.

a_{tan} m/s^2

Tangential Accel.: Direction: tangent to the circle

Causes speed to increase Causes angular speed to also increase

Therefore, causes: angular acceleration

Definitions: θ = angle (rad) $\omega = \frac{\Delta\theta}{\Delta t}$ $\alpha = \frac{\Delta\omega}{\Delta t}$ α, $\frac{rad}{s^2}$

Connecting eqns: arc length s = $r\theta$ tan. v = $r\omega$ tan. a = $r\alpha$

Angular Kinematic Equations: $x \rightarrow \theta$ $v \rightarrow \omega$ $a \rightarrow \alpha$

1. $\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$
2. $\omega = \omega_0 + \alpha t$
3. $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

Centripetal Accel.:

Causes change in direction Direction: inward

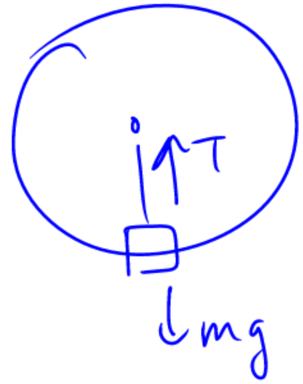
but not change in speed Magnitude: a_c = v^2/r

How to use with N2: Always include on the right hand side

Worked Problem

You swing a ball (mass *m*) in a vertical circle with a string; its speed is constant (*v*) through the whole circle. (a) What is the tension at the lowest point? (b) At the highest point?

(a) Picture:



v^2/r

Equation: $\sum F = ma$

$$T - mg = \frac{mv^2}{r} \longrightarrow T = \boxed{mg + \frac{mv^2}{r}}$$

(b) Picture:

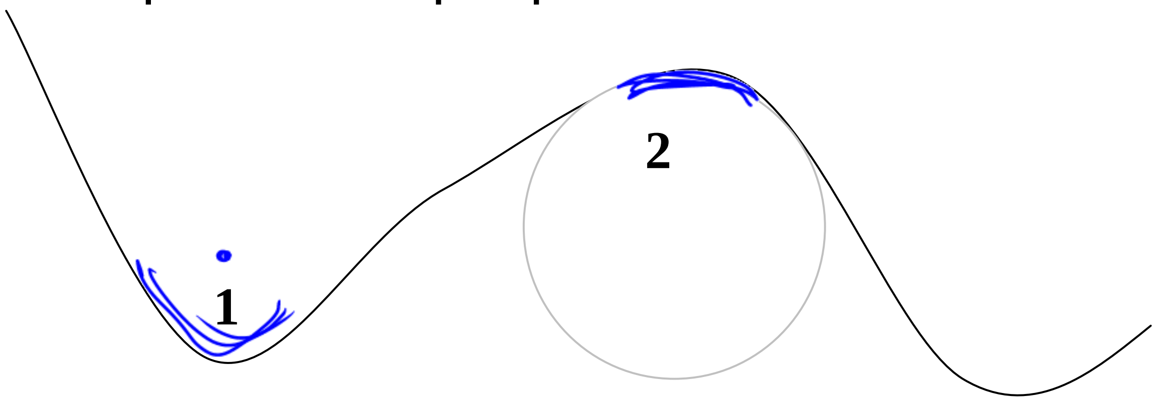
Equation: $\sum F = ma$

$$mg + T = \frac{mv^2}{r} \longrightarrow T = \boxed{\frac{mv^2}{r} - mg}$$

Answers: $mg + mv^2/r$, $mg \cancel{-} mv^2/r$

Unsafe roller coasters (no seatbelts)

For the top of an *outside* curve (pt 2), radius of curvature = 8 m, what is the maximum speed if the people are **not to fall out**?

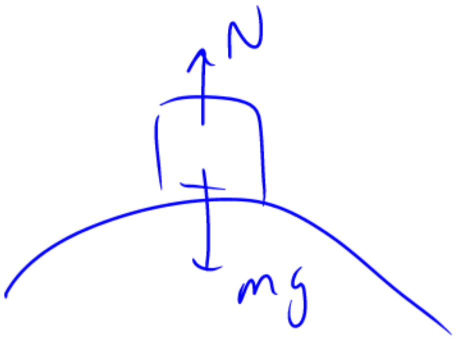


What's the difference between pt 1 and pt 2?

Free-body diagrams:

What happens to Normal force at pt 2 as speed increases?
→ Just as people fall out, the normal force is zero.

Solution to the problem (8 m radius of curvature):



$$\Sigma F = ma$$

$$mg - \cancel{N} = \frac{mv^2}{r}$$

N = 0 at critical speed

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{rg} = \sqrt{(8\,\text{m})(9.8\,\tfrac{\text{m}}{\text{s}^2})}$$

$$= 8.85\ \text{m/s}$$

Answer: 8.85 m/s

Question

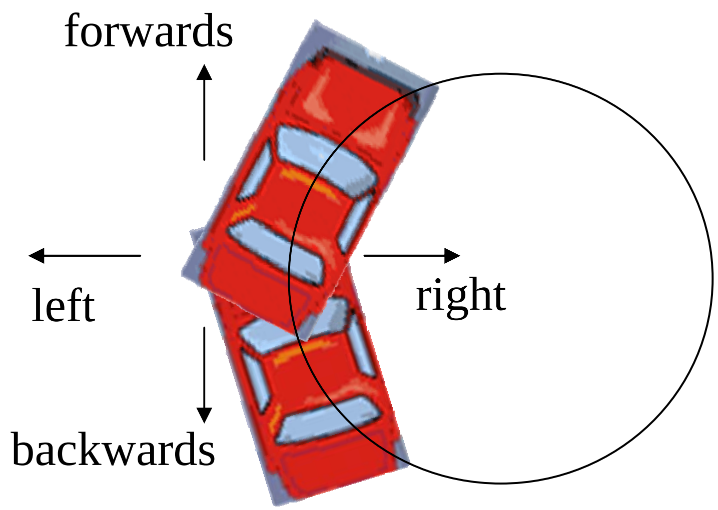
Angular velocity of **earth** (1 rev/24 hours, convert to rad/s) gives speed at Provo = 792 mph (354 m/s)! (Using 6.371×10^6 m as radius of earth and 40.24° as latitude of Provo.)

$v = r\omega$

Why don't we fly off?

$m\frac{v^2}{r}$

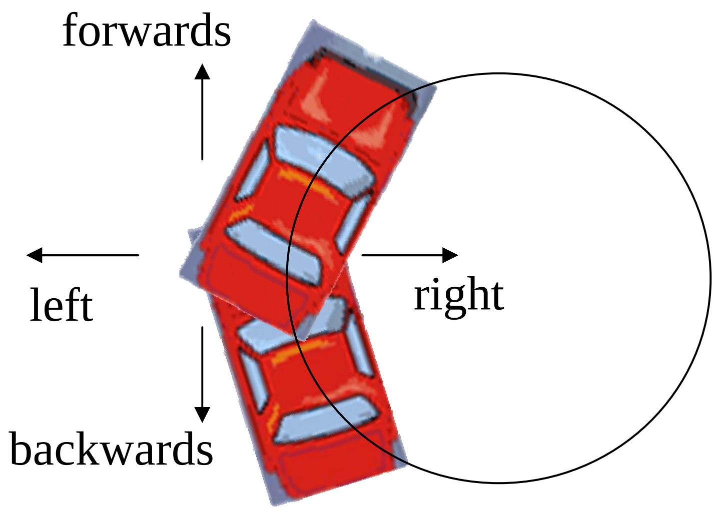
Scenario: Back Seat of Car



You are in the middle of the back seat of a car. The car turns right at constant speed, moving in a circle.

Question: What happens to you if no friction from seat?

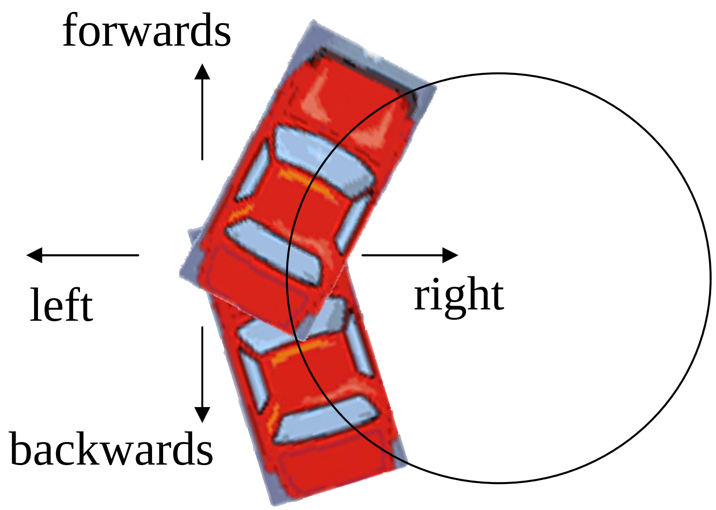
Clicker Quiz



The net horizontal force on you *after you are pressed up against the door* is:

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

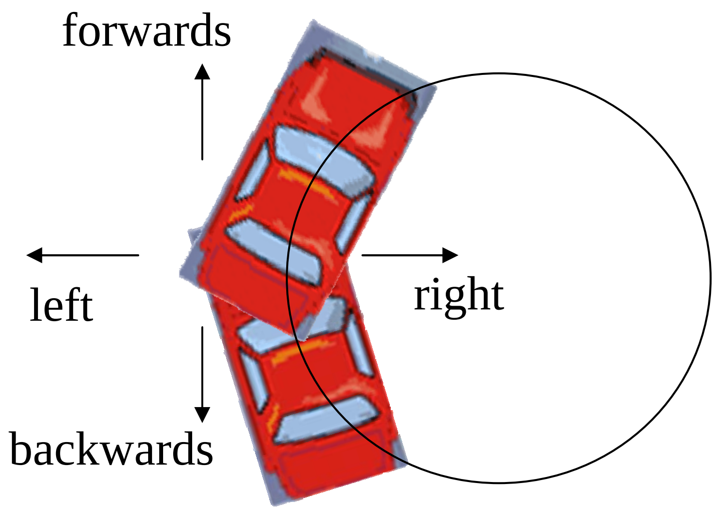
Clicker Quiz

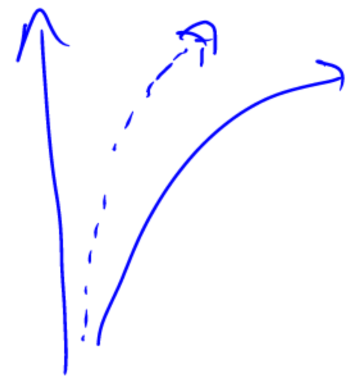


What if there's enough static friction so that you do not slide? In what direction is the static friction?

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

Clicker Quiz

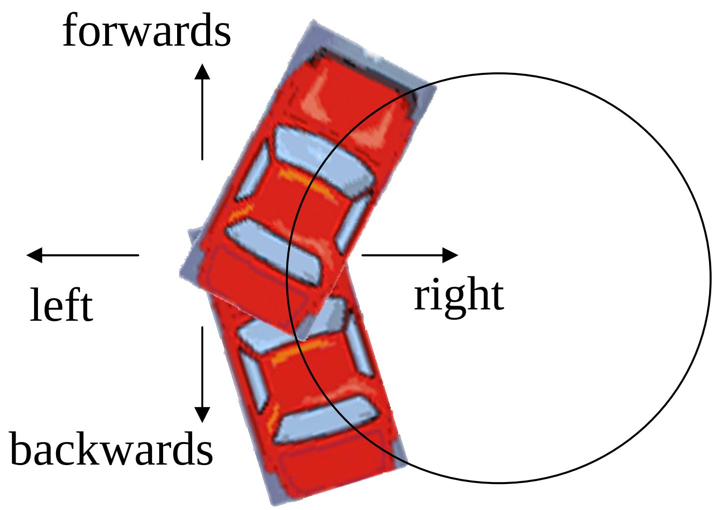




What if there's only a little bit of friction, so that you are sliding, but not as much as if no friction. In what direction is this kinetic friction?

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

Clicker Quiz



In what direction is the friction force from the road acting on the car's tires?

a. Towards the left
b. Towards the right
c. Forwards
d. Backwards

Worked Problem: Floor-dropping ride

If the coefficient of friction is μ, what minimum speed *v* must you be going before the floor is removed?

$$\sum F_{inward} = m a_{inward}$$

$$N = m\frac{v^2}{r}$$

$$\sum F_y = m a_y$$

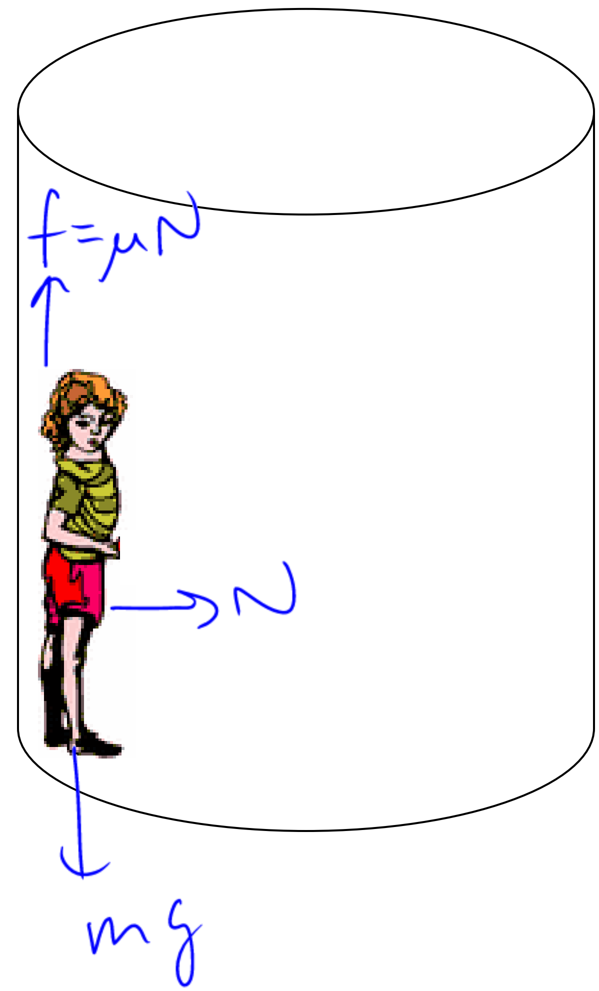
$$f - mg = 0$$

$$\mu N = mg$$

$$N = \frac{mg}{\mu}$$

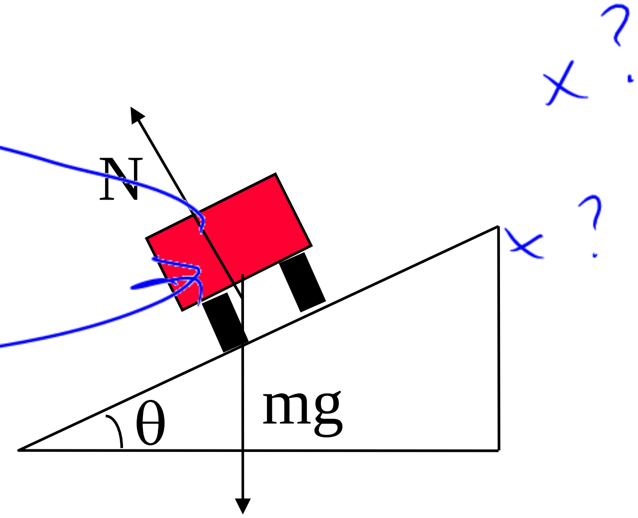
$$\frac{\cancel{m}g}{\mu} = \frac{\cancel{m}v^2}{r}$$

$$\boxed{v = \sqrt{\frac{rg}{\mu}}}$$



Answer: $\sqrt{\frac{rg}{\mu}}$

Banked roadways



Consider turn with no friction…
What direction will car go if slight banking?
What direction if steep banking?

In between?

So, why do they bank turns?

HW Problem, 13-1 (due Thurs): what should the banking angle be so that there is no sideways friction force needed? (given overall turn radius and speed)

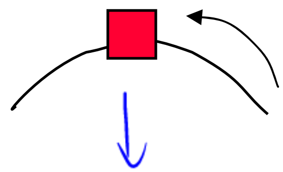
Hardest part: which way to draw the axes?? Conflicting advice:

- Colton: "Make the positive x-axis be along the inclined plane"
- ✓ Colton: "Make the positive x-axis be towards the center of the circle"

Conflict resolved:

Combined Centripetal and Tangential

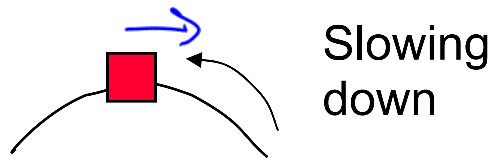
Example: Going around a corner while <u>slowing down</u>



Clicker quiz: The centripetal acceleration at this instant is

a. up
b. down
c. left
d. right
e. zero

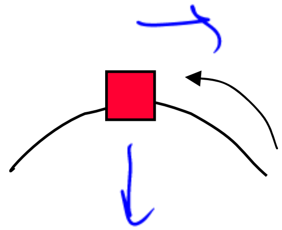
Clicker quiz



The tangential acceleration at that instant is

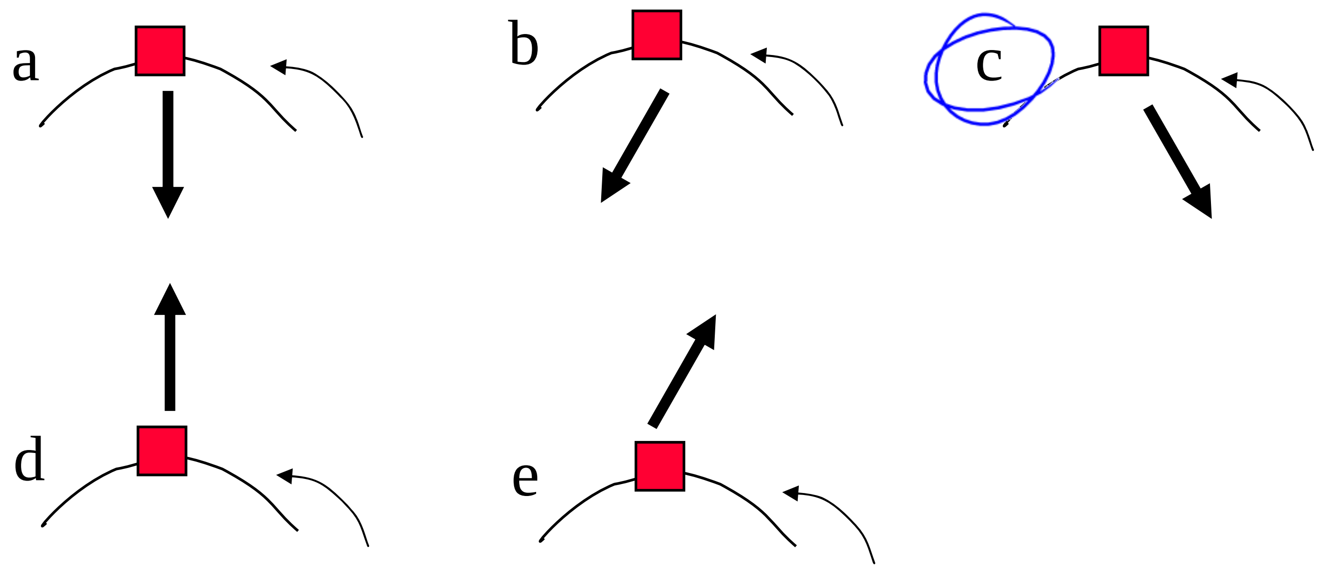
a. up
b. down
c. left
d. right
e. zero

Clicker quiz

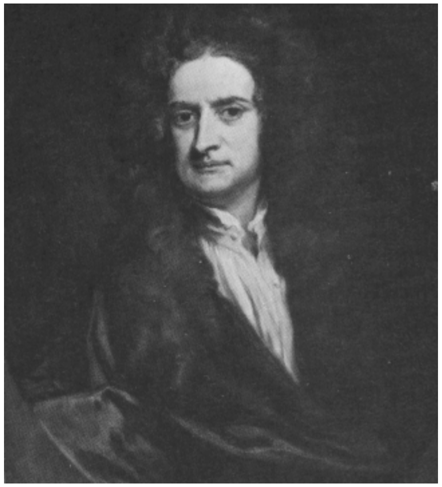


Slowing down

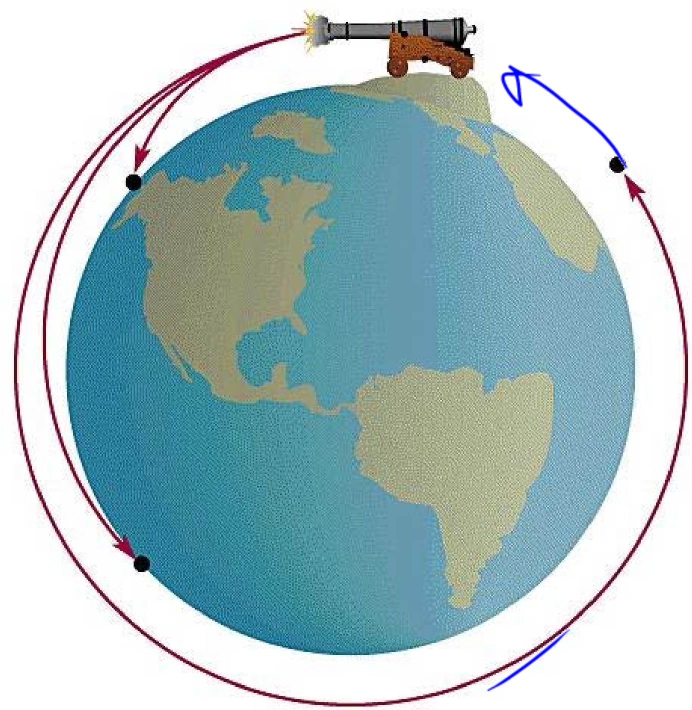
Which figure represents the total ***a*** vector?



On to Gravity!!



Newton's thoughts about the moon's orbit and projectile motion, c. 1670:



Parabola of projectile turns into a circle.
The apple, the cannonball, and the Moon

→ all are in free fall

Newton's Law of Gravity:

All masses attract all other masses!

$$\boxed{F_G = G\frac{mM}{r^2}}$$

r measured from center of mass. (for each object)

(sometimes written with negative sign)

$F_g = mg$

Proportionality constant: $G = 6.674 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

Near the surface of the earth:

$R_{Earth} = 6.371\times10^{6}$ m
$M_{Earth} = 5.974 \times 10^{24}$ kg

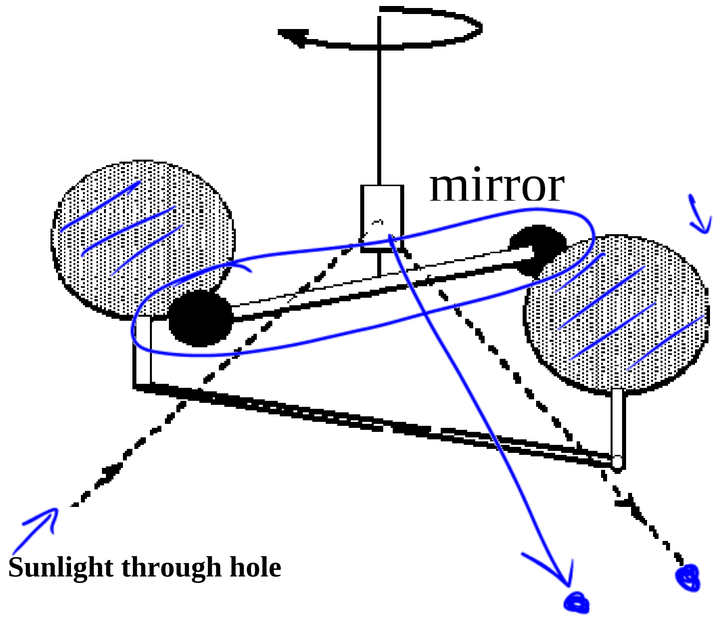
$\sum F = ma$

$G\frac{\cancel{m}M}{r^2} = \cancel{m}a$

$$a = \frac{(6.674\cdot10^{-11})(5.974\cdot10^{24})}{(6.371\cdot10^{6})^2}$$

$= 9.8\ \text{m/s}^2$

Cavendish Experiment



1783: first measurement of forces between “regular” masses, by Cavendish.

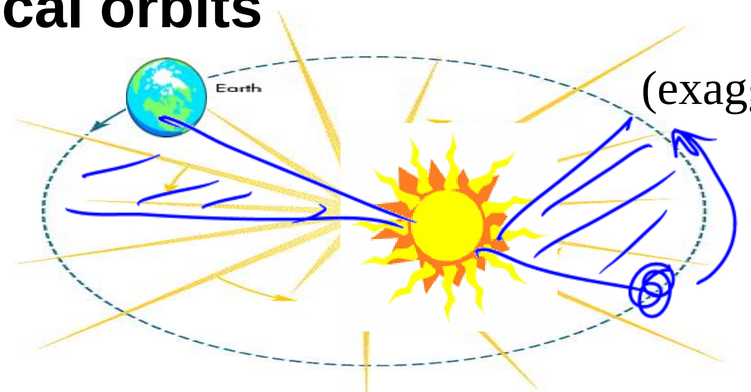
“Weighing the world” vs. determining *G*

→ Most accurate such measurement for 102 years!
(only 1% off of today’s value)

How did Newton know it was **inverse square**?

<u>Kepler's laws</u> (about 1600) came from observations of the planets in our solar system:

1. Elliptical orbits



(exaggerated ellipse)

2. Equal areas in equal times: fastest close to Sun

3. $T^2 \sim r^3$

$\approx$ (T = "orbital period" = 1 "year")

All three can be exactly predicted using Newton's Second Law together with Newton's Law of Gravity! (Done in Phys 321)

From warmup

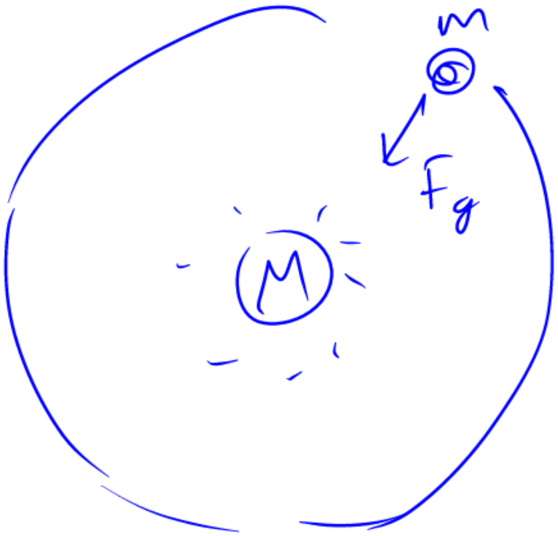
Which is not one of Kepler's laws?

a. Planets all move in the same plane
b. Planets move in elliptical orbits
c. Equal areas swept out in equal time: faster closer to sun
d. The period of orbit increases as r increases

$v = \frac{2\pi r}{T}$

Worked Problem

Figure out what the proportionality constant is in Kepler's Third Law, $T^2 \sim r^3$, in terms of *G* and the mass of the sun. Assume a circular planetary orbit.



$$\Sigma F_g = m a_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM_{sun}} \cdot r^3}$$

Answer: $4\pi^2/(GM)$

$\Sigma F = ma.$

Worked Problem

How long is Jupiter's year? ($r_{Jupiter} \approx 5.2\ r_{Earth}$)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\rightarrow \quad T^2 = C \cdot r^2$$

$$\frac{T_J^2 = \cancel{C}\, r_J^3}{T_E^2 = \cancel{C}\, r_E^3}$$

$$T_J^2 = T_E^2 \cdot \left(\frac{r_J^3}{r_E^3} \right)$$

Answer: 11.86 years

Satellites

Question: What's the difference between the earth revolving around the sun, and a satellite revolving around the earth?

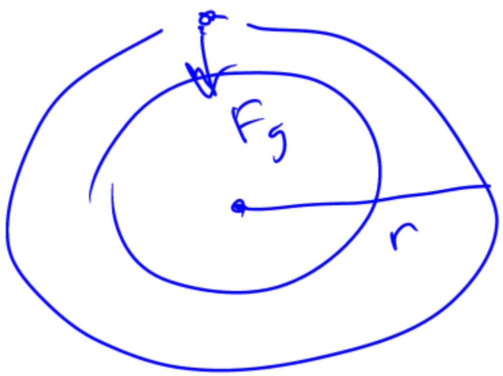
Not much → $M = M_{earth}$

$m = m_{sat.}$

Orbital Velocity

On the moon (no air friction, mass M) someone really *could* get into orbit by being fired horizontally off the highest mountain (radius r).

How fast would you have to shoot that person?



$$\Sigma F = ma$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

How long would it take him to go around once?

“orbital period”

$$v = \frac{2\pi r}{T}$$

$$\boxed{T = \frac{2\pi r}{v}}$$

Answers: v=sqrt(GM/r), 2πr/v

Circular orbits

For each *v*, only one *r* will work
For each *r*, only one *v* will work!

Clicker quiz: A satellite in a higher orbit will be going ________ than a satellite in a lower orbit.

a. faster
b. slower

Real satellites:

http://science.nasa.gov/RealTime/JTrack/3d/JTrack3d.html

International space station, 340.5 km above surface of Earth (R_e = 6,371 km)	7.707 km/s
Geostationary orbit, 35,786 km above surface	3.075 km/s
Moon, r = 381,715 km	1.022 km/s

Worked Problem: How long does it take ISS to orbit?

Answer: 91.2 min

From warmup

If the Earth attracts the moon with gravitational force, why doesn't the moon fall into the Earth? Give an explanation that a friend in junior high school could follow.

"Pair share"–I am now ready to share my neighbor's answer if called on.

a. Yes

Clicker quiz

You are on planet Xarthon, which has a mass of 2× that of the earth and a radius 2× as big. If you throw a ball at the surface, and you will find that $g_{Xarthon}$ is ____ g_{earth}

a. larger than
b. smaller than
c. the same as

Clicker quiz

Satellites in higher orbits are travelling slower, so to "shoot" a satellite from the surface of the earth into a high orbit (i.e. with a cannon), you would provide it with ________ initial kinetic energy than for a satellite in a low orbit

a. more
b. less
c. same

Next time…

Gravitational PE

Need new $\mathbf{PE_{gravity}}$

PE = mgy just won't work…
Force isn't "mg" any more!