

Announcements – 15 Oct 2013

1. While you're waiting for class to start, see how many of these blanks you can fill out.

Tangential Accel.: a_{tan} m/s^2 Direction: tangent to the circle
 Causes speed to increase Causes angular speed to also increase
 Therefore, causes: a angular a acceleration

Definitions: $\theta = \text{angle (rad)}$ $\omega = \frac{\Delta\theta}{\Delta t}$

Connecting eqns: arc length $s = r\theta$

$\alpha = \frac{\Delta\omega}{\Delta t}$ α , $\frac{rad}{s^2}$
 tan. $v = \frac{r\omega}{r}$ tan. $a = \frac{r\alpha}{r}$

Angular Kinematic Equations: $x \rightarrow \theta$

1. $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
2. $\omega = \omega_0 + \alpha t$
3. $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

$v \rightarrow \omega$ $a \rightarrow \alpha$



Centripetal Accel.:

Causes change in direction
 but not change in speed

Direction: inward
 Magnitude: $a_c = \frac{v^2}{r}$

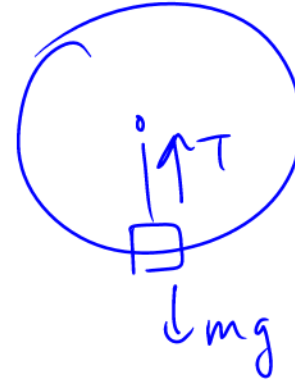
How to use with N2: Always include on the right hand side

Worked Problem



You swing a ball (mass m) in a vertical circle with a string; its speed is constant (v) through the whole circle. (a) What is the tension at the lowest point? (b) At the highest point?

(a) Picture:



Equation:

$$\sum F = ma \quad \downarrow \quad v^2/r$$
$$T - mg = \frac{mv^2}{r} \quad \rightarrow \quad T = mg + \frac{mv^2}{r}$$

(b) Picture:



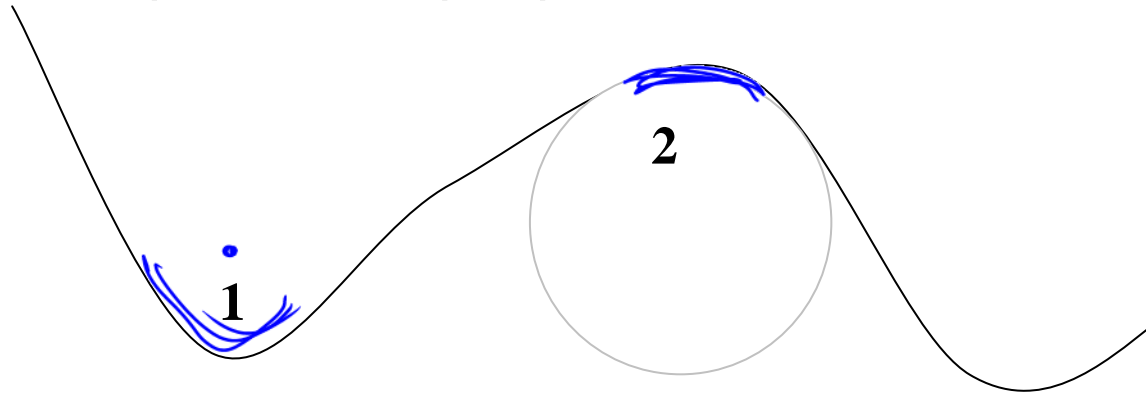
Equation:

$$\sum F = ma$$
$$mg + T = \frac{mv^2}{r} \quad \rightarrow \quad T = \frac{mv^2}{r} - mg$$

Answers: $mg + mv^2/r$, $mg - mv^2/r$

Unsafe roller coasters (no seatbelts)

For the top of an *outside* curve (pt 2), radius of curvature = 8 m, what is the maximum speed if the people are **not to fall out**?



What's the difference between pt 1 and pt 2?

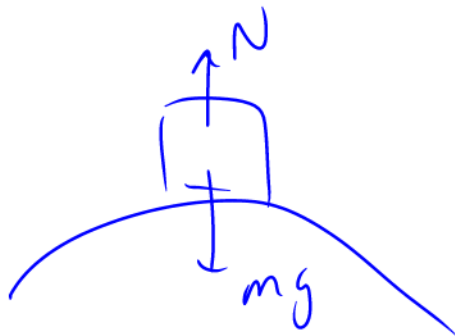
Free-body diagrams:



What happens to Normal force at pt 2 as speed increases?

→ Just as people fall out, the normal force is zero.

Solution to the problem (8 m radius of curvature):



$$\sum F = ma$$
$$mg - \cancel{N} = m \frac{v^2}{r}$$

↑
= 0
at critical
speed

$$\cancel{mg} = \cancel{m} \frac{v^2}{r}$$

$$r = \sqrt{rg}$$

$$= \sqrt{(8\text{ m})(9.8 \frac{\text{m}}{\text{s}^2})}$$

$$= \underline{8.85 \text{ m/s}}$$

Answer: 8.85 m/s

Question

Angular velocity of **earth** (1 rev/24 hours, convert to rad/s) gives speed at Provo = 792 mph (354 m/s)! (Using 6.371×10^6 m as radius of earth and 40.24° as latitude of Provo.)

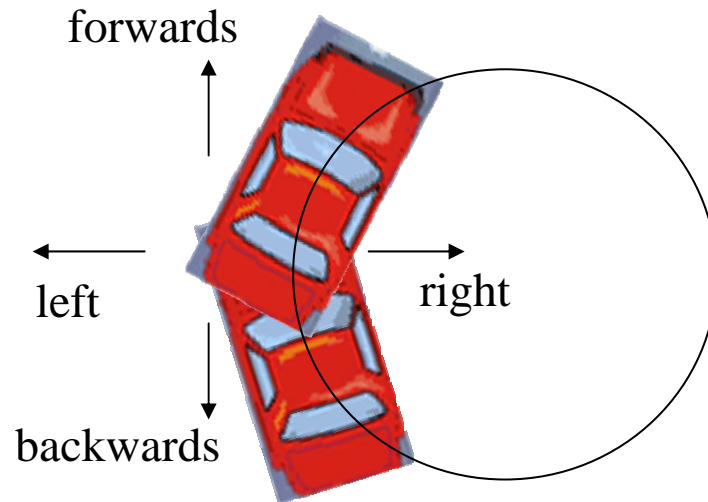
$$v = r\omega$$

Why don't we fly off?



$$m \frac{v^2}{r}$$

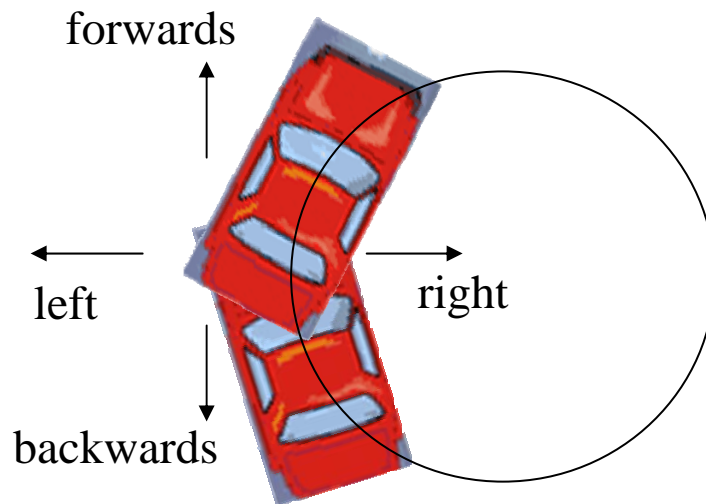
Scenario: Back Seat of Car



You are in the middle of the back seat of a car. The car turns right at constant speed, moving in a circle.

Question: What happens to you if no friction from seat?

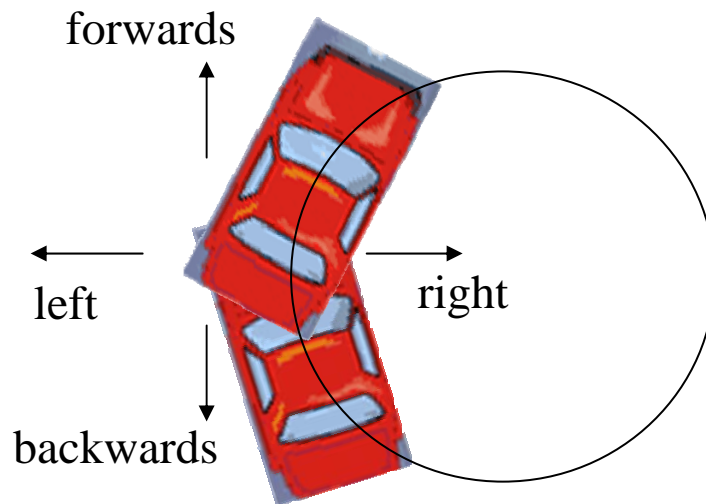
Clicker Quiz



The net horizontal force on you *after you are pressed up against the door* is:

- a. Towards the left
- b. Towards the right
- c. Forwards
- d. Backwards

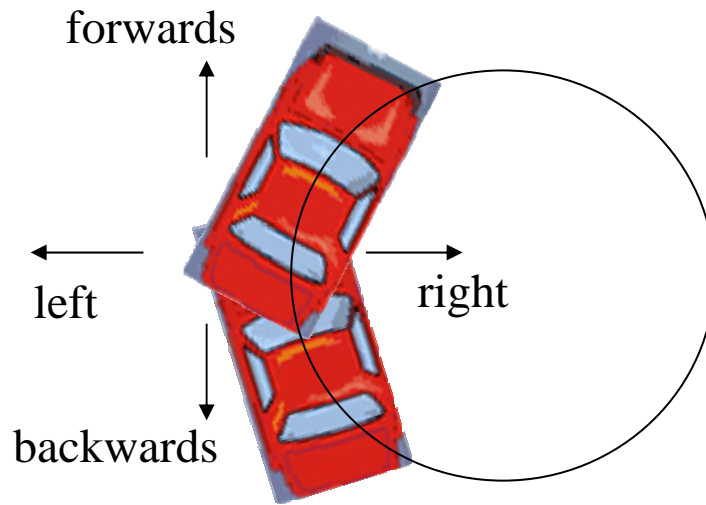
Clicker Quiz



What if there's enough static friction so that you do not slide? In what direction is the static friction?

- a. Towards the left
- b. Towards the right
- c. Forwards
- d. Backwards

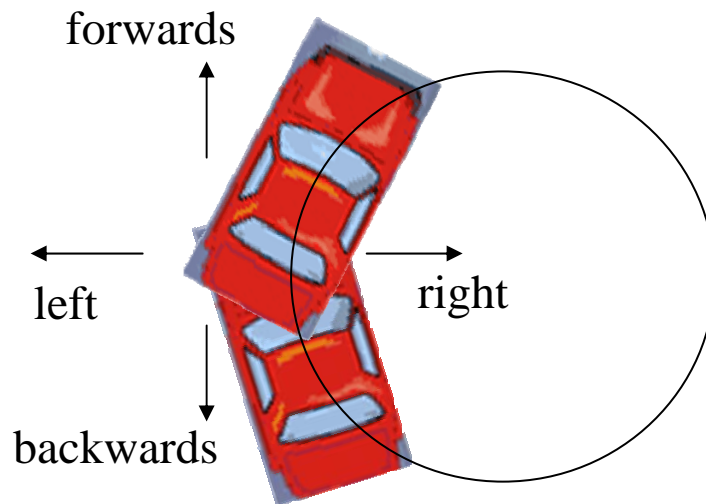
Clicker Quiz



What if there's only a little bit of friction, so that you are sliding, but not as much as if no friction. In what direction is this kinetic friction?

- a. Towards the left
- b. Towards the right
- c. Forwards
- d. Backwards

Clicker Quiz



In what direction is the friction force from the road acting on the car's tires?

- a. Towards the left
- b. Towards the right
- c. Forwards
- d. Backwards

Worked Problem: Floor-dropping ride

If the coefficient of friction is μ , what minimum speed v must you be going before the floor is removed?

$$\sum F_{\text{inward}} = m a_{\text{inward}}$$

$$N = m \frac{v^2}{r}$$

$$\sum F_y = m a_y$$

$$f - mg = 0$$

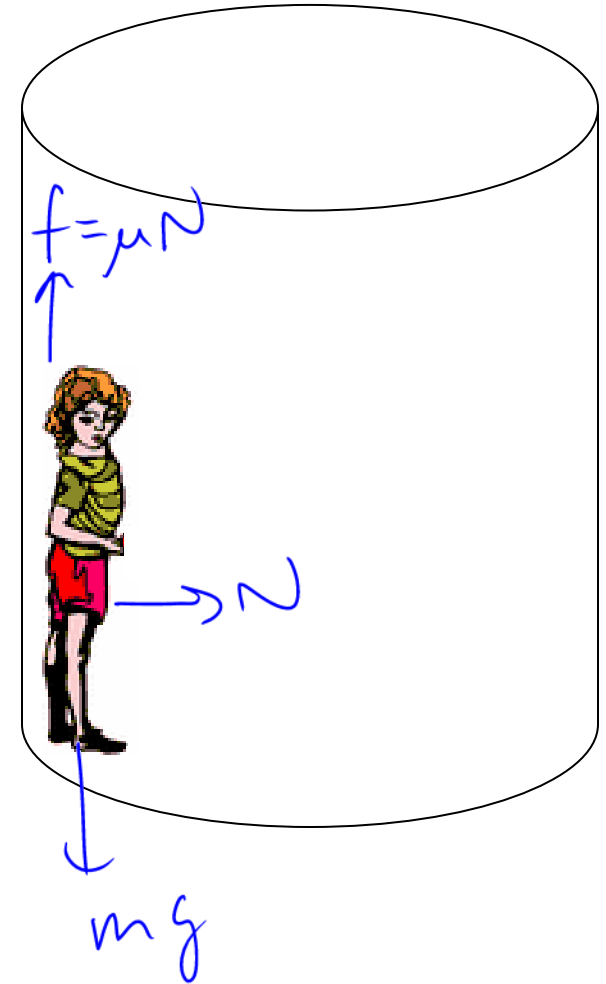
$$\mu N = mg$$

$$N = \frac{mg}{\mu}$$

$$\frac{mg}{\mu} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{rg}{\mu}}$$

Answer: $\sqrt{\frac{rg}{\mu}}$



Banked roadways

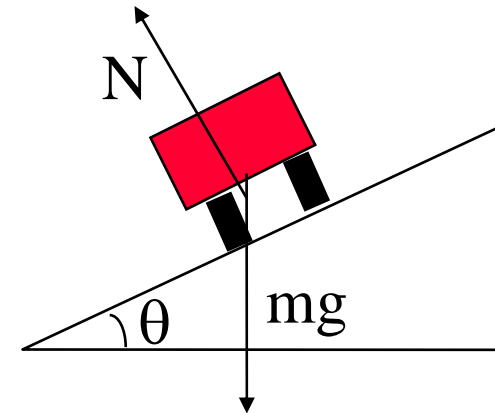
Consider turn with no friction...

What direction will car go if slight banking?

What direction if steep banking?

In between?

So, why do they bank turns?



HW Problem, 13-1 (due Thurs): what should the banking angle be so that there is no sideways friction force needed? (given overall turn radius and speed)

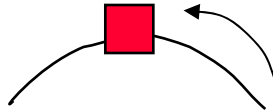
Hardest part: which way to draw the axes?? Conflicting advice:

- Colton: “Make the positive x-axis be along the inclined plane”
- Colton: “Make the positive x-axis be towards the center of the circle”

Conflict resolved:

Combined Centripetal and Tangential

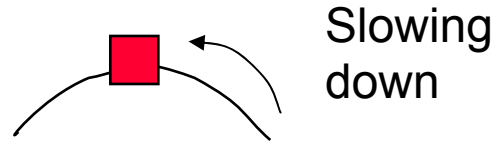
Example: Going around a corner while slowing down



Clicker quiz: The centripetal acceleration at this instant is

- a. up
- b. down
- c. left
- d. right
- e. zero

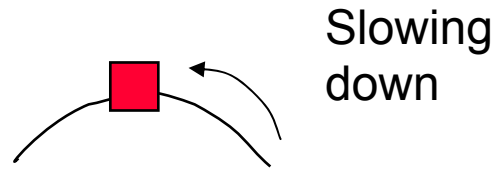
Clicker quiz



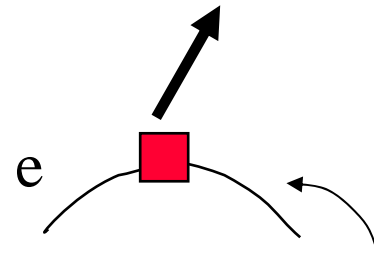
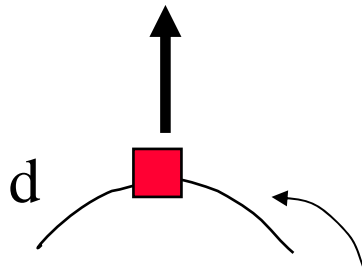
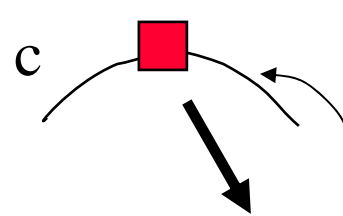
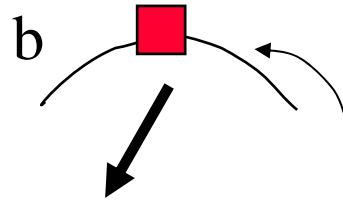
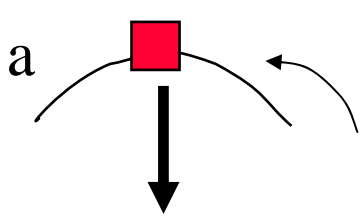
The tangential acceleration at that instant is

- a. up
- b. down
- c. left
- d. right
- e. zero

Clicker quiz



Which figure represents the total \mathbf{a} vector?



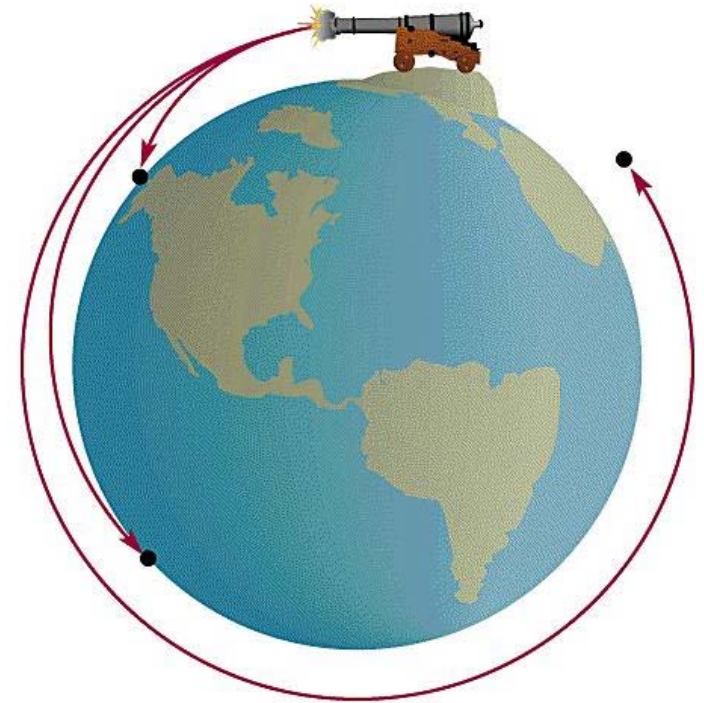
On to Gravity!!



Newton's thoughts about the moon's orbit and projectile motion, c. 1670:

Parabola of projectile turns into a **circle**.
The apple, the cannonball, and **the Moon**

→ all are in _____



Newton's Law of Gravity:

All masses attract all other masses!

$$F_G = G \frac{mM}{r^2}$$

r measured from _____

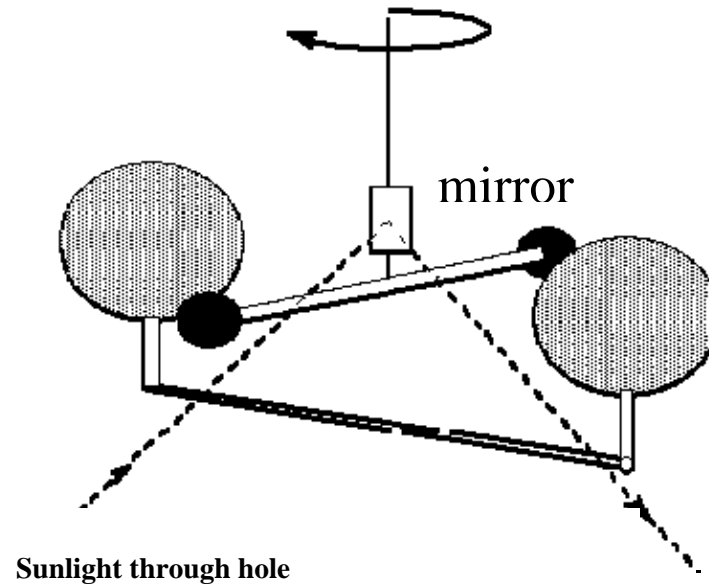
(sometimes written with negative sign)

Proportionality constant: $G = 6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

Near the surface of the earth:

$$R_{\text{Earth}} = 6.371 \times 10^6 \text{ m}$$
$$M_{\text{Earth}} = 5.974 \times 10^{24} \text{ kg}$$

Cavendish Experiment



1783: first measurement of forces between “regular” masses, by Cavendish.

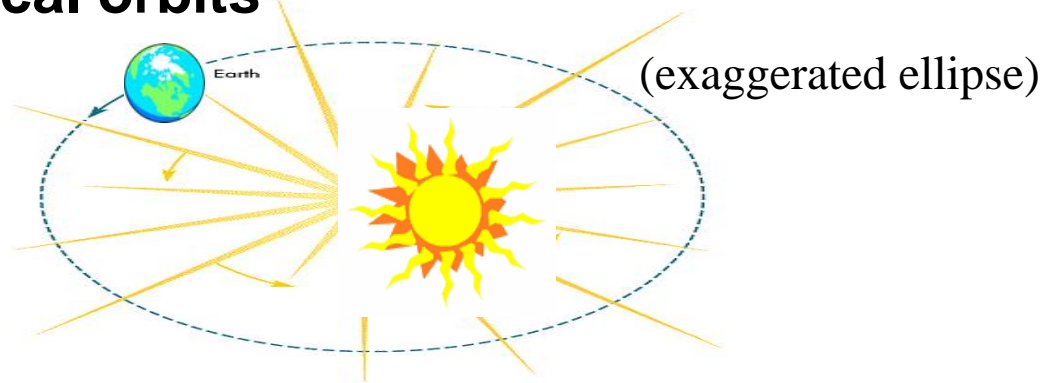
“Weighing the world” vs. determining G

→ Most accurate such measurement for 102 years!
(only 1% off of today’s value)

How did Newton know it was **inverse square**?

Kepler's laws (about 1600) came from observations of the planets in our solar system:

1. Elliptical orbits



2. Equal areas in equal times: fastest close to Sun

3. $T^2 \sim r^3$

(T = "orbital period" = _____)

All three can be exactly predicted using Newton's Second Law together with Newton's Law of Gravity! (Done in Phys 321)

From warmup

Which is not one of Kepler's laws?

- a. Planets all move in the same plane
- b. Planets move in elliptical orbits
- c. Equal areas swept out in equal time: faster closer to sun
- d. The period of orbit increases as r increases

Worked Problem

Figure out what the proportionality constant is in Kepler's Third Law, $T^2 \sim r^3$, in terms of G and the mass of the sun. Assume a circular planetary orbit.

Answer: $4\pi^2/(GM)$

Worked Problem

How long is Jupiter's year? ($r_{\text{Jupiter}} \approx 5.2 r_{\text{Earth}}$)

Answer: 11.86 years

Satellites

Question: What's the difference between the earth revolving around the sun, and a satellite revolving around the earth?

Orbital Velocity

On the moon (no air friction, mass M) someone really *could* get into orbit by being fired horizontally off the highest mountain (radius r).

How fast would you have to shoot that person?

How long would it take him to go around once?
“orbital period”

Answers: $v = \sqrt{GM/r}$, $2\pi r/v$

Circular orbits

For each v , only one r will work

For each r , only one v will work!

Clicker quiz: A satellite in a higher orbit will be going _____ than a satellite in a lower orbit.

a. faster

b. slower

Real satellites:

<http://science.nasa.gov/RealTime/JTrack/3d/JTrack3d.html>

International space station, 340.5 km above surface of Earth ($R_e = 6,371$ km)	7.707 km/s
Geostationary orbit, 35,786 km above surface	3.075 km/s
Moon, $r = 381,715$ km	1.022 km/s

Worked Problem: How long does it take ISS to orbit?

Answer: 91.2 min

From warmup

If the Earth attracts the moon with gravitational force, why doesn't the moon fall into the Earth? Give an explanation that a friend in junior high school could follow.

“Pair share”—I am now ready to share my neighbor’s answer if called on.

a. Yes

Clicker quiz

You are on planet Xarthon, which has a mass of $2\times$ that of the earth and a radius $2\times$ as big. If you throw a ball at the surface, and you will find that

g_{Xarthon} is _____ g_{earth}

- a. larger than
- b. smaller than
- c. the same as

Clicker quiz

Satellites in higher orbits are travelling slower, so to “shoot” a satellite from the surface of the earth into a high orbit (i.e. with a cannon), you would provide it with _____ initial kinetic energy than for a satellite in a low orbit

- a. more
- b. less
- c. same

Next time...

Gravitational PE

Need new $\mathbf{PE}_{\text{gravity}}$

$\mathbf{PE = mgy}$ just won't work...
Force isn't "mg" any more!