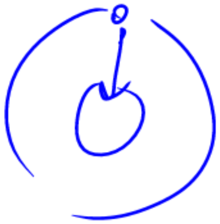
Announcements – Oct 17, 2013

1. No announcements!

Real satellites:

http://science.nasa.gov/RealTime/JTrack/3d/JTrack3d.html



International space station, 340.5 km above surface of Earth (R_e = 6,371 km)	7.707 km/s
Geostationary orbit, 35,786 km above surface	3.075 km/s
Moon, r = 381,715 km	1.022 km/s

$$\sum F = m a_c$$

$$\boxed{\frac{GM\cancel{m}}{r^{\cancel{2}}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{GM_{earth}}{r_{orbit}}}$$

Worked Problem: How long does it take ISS to orbit?

$$v = \sqrt{\frac{6.67\cdot10^{-11}\cdot5.98\cdot10^{24}}{(6371000+340500)}} = 7709 \text{ m/s}$$

$$v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(6371000+340500)}{7709}$$

$$= \boxed{91 \text{ min}}$$

Answer: 91.2 min

Satellite game

http://www.science-animations.com/support-files/satellite.swf

From warmup (last time)

If the Earth attracts the moon with gravitational force, why doesn't the moon fall into the Earth? Give an explanation that a friend in junior high school could follow.

“Pair share”–I am now ready to share my neighbor’s answer if called on.

a. Yes

Clicker quiz

You are on planet Xarthon, which has a mass of 2× that of the earth and a radius 2× as big. If you throw a ball at the surface, and you will find that $g_{Xarthon}$ is ____ g_{earth}

a. larger than
[✓ b.] smaller than
c. the same as

$$\sum F = ma$$

$$\boxed{\frac{GM\cancel{m}}{r^2}} = \cancel{m}a$$

Clicker quiz

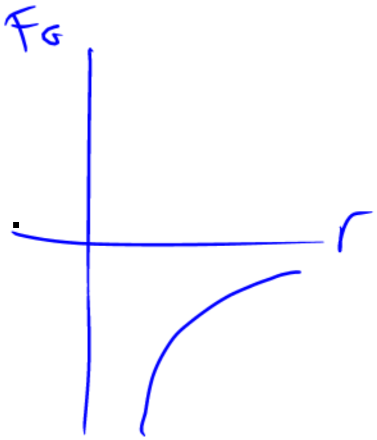
Satellites in higher orbits are travelling slower, so to “shoot” a satellite from the surface of the earth into a high orbit (i.e. with a cannon), you would provide it with _________ initial kinetic energy than for a satellite in a low orbit

[✓ a. more]
b. less
c. same

Gravitational PE

Need new **PE$_{\text{gravity}}$**

PE = mgy just won't work…
Force isn't "mg" any more!



Using calculus to calculate work done against (non-constant) gravitational force…

$$\boxed{PE_G = -\frac{GMm}{r}}$$

$$\frac{GMm}{r^2}$$

Here the negative sign is critically important!!
(not a vector direction)

Before: PE = 0 when y=0, wherever you want!

With new equation: PE = 0 when r = ∞

$E_{bef} + W = E_{aftr}$

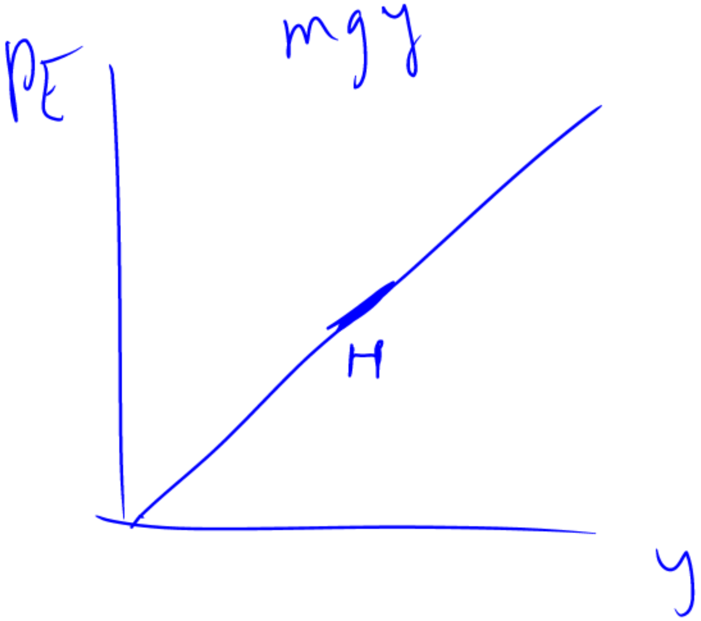
PE_i PE_f

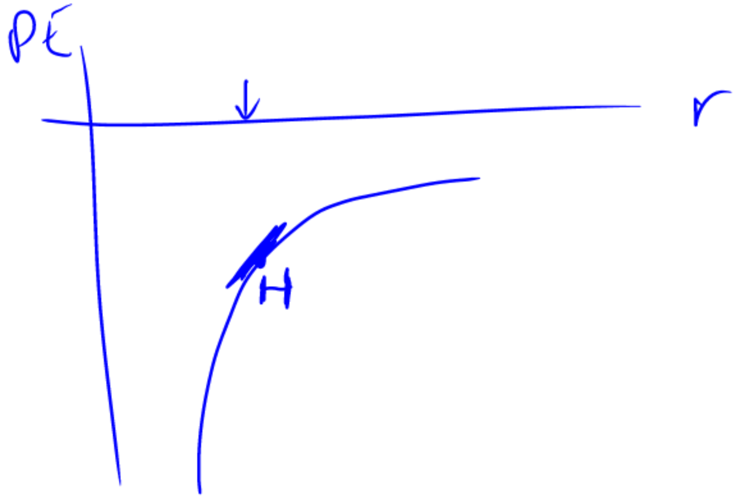
From warmup

Ralph noticed the negative sign in the general equation for gravitation potential energy, PE=−GMm/r, and he read the book's statement that "this expression reduces to PE=mgh close to the surface of Earth". He is very confused, because among other things one equation has a negative sign and the other one doesn't! How can they possibly be equivalent? What can you tell Ralph to help him out?

"Pair share"–I am now ready to share my neighbor's answer if called on.

a. Yes





Worked problem

How much energy would you have to provide in order to “shoot” a 100 kg satellite into a near orbit like the ISS, 6712 km from center of earth? (E.g. via initial KE)

$$E_{bef} + \cancel{W} = E_{aft}$$

↙

$$KE_i - \frac{GMm}{r_e} = \frac{1}{2}mv_f^2 - \frac{GMm}{r} + \frac{GM_{earth}m}{r_e}$$

$$KE_i = \frac{1}{2}m\left(7709\,\frac{m}{s}\right)^2 - \frac{GM_{earth}\times m}{6712000}$$

$$\boxed{KE_i = 3.29\cdot10^9\ J}$$

…into a much farther geostationary orbit? (42,157 km)

$$KE_i = \frac{1}{2}m\underset{\substack{\uparrow\\ \text{new}\\ \text{orb. speed}}}{v_f^2} - \frac{GM_e m}{42157000} + \frac{GM_e m}{r_e}$$

$$= \boxed{5.79\cdot10^9\ J}$$

…to an orbit at the moon’s distance (381,715 km)

$$KE_i = \text{etc}$$

$$= \boxed{6.21\cdot10^9\ J}$$

Answers: 3.29E9 J, 5.79E9 J, 6.21E9 J

Escape velocity

$$\boxed{v_{orb} = \sqrt{\frac{G M_{earth}}{r_{orb}}}}$$

Same question: …into an orbit very, very far away from the Earth?? (ignore the sun’s gravitational pull)

Hints: What is its orbital velocity? = 0 when you are very far away

What is its final kinetic energy?

Final potential energy?

$$KE_i = \cancel{\frac{1}{2} m v_f^2} - \cancel{\frac{G M m}{r_{orb}}} + \frac{G M m}{r_e}$$

$$\frac{1}{2} \cancel{m} v_0^2 = \frac{G M \cancel{m}}{r_e} \longrightarrow \boxed{v_{esc} = \sqrt{\frac{2GM}{r_e}}}$$

Robert Heinlein:

> **“If you can get into orbit, then you're halfway to anywhere”**

Answers: 6.26E9 J

From warmup

The "escape velocity" of a planet is the speed needed for a rocket to go from the surface of the planet into orbit.

a. true
[✓ b. false]

Worked Problem

What is the escape velocity of the earth?
(R_{earth} = 6371 km; M_{earth} = 5.974×10^{24} kg)

$$V_{esc} = \sqrt{\frac{2GM_e}{r_e}}$$

$$= \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 5.974 \cdot 10^{24}}{6371000}}$$

$$= \boxed{11200 \ m/s}$$

Answer: 11.2 km/s

Torque and equilibrium

A force supplies a ***torque*** on an object when it is applied in such a way that could cause the object to rotate

Definition: $\boxed{\tau = r_\perp F}$

Note: where do you measure the distance *r* from?

If the object is rotating: axis of rotation
If the object is standing still: any pivot point

Above all, be consistent

Positive vs. negative torques:

Yes!

CCW = +

CW = −

Is torque a vector?

Yes!

From warmup

In order to apply the most torque to a bolt, you should:

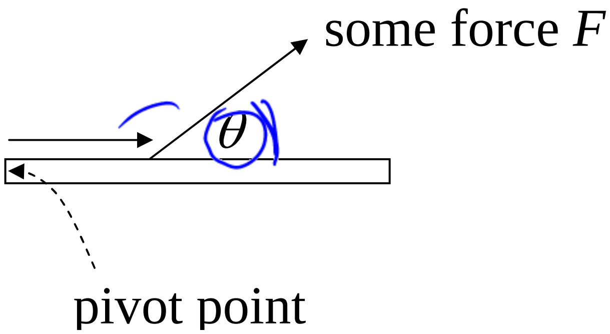
[✓ a.] use a wrench with a long handle
b. use a wrench with a short handle
c. there would be no difference

Clicker quiz

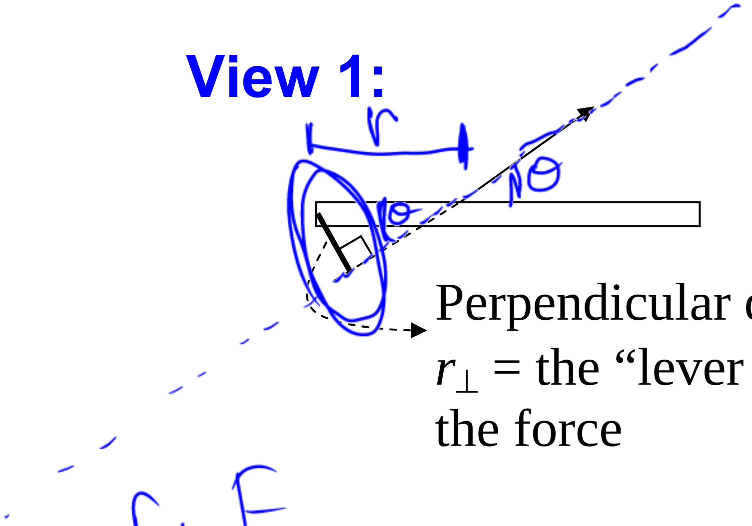
In order to apply the most torque, you should:

[✓ a.] apply the force perpendicular to ***r***
b. apply the force at a 45° angle from ***r***
c. no difference

"Lever Arm"



View 1:

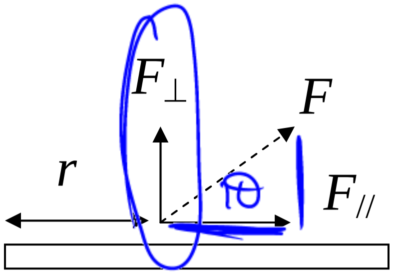


Perpendicular distance $r_\perp$ = the "lever arm" of the force

$r_\perp F$

$(r \sin\theta) F$

View 2:



$r F_\perp = r F \sin\theta$

$$\boxed{\tau = r_\perp F = rF_\perp = rF\sin\theta}$$

Demo: T-handle torque

Torque tug-of-war

Torque Summary

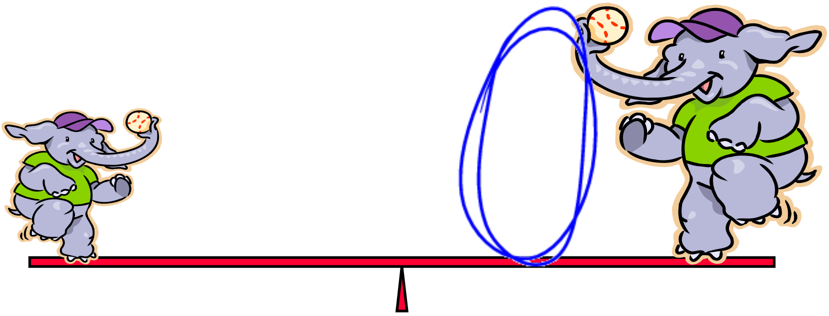
$$\boxed{\tau = r_\perp F = rF_\perp = rF\sin\theta}$$

→ **but be careful about which angle you call θ!**

Note: If you are familiar with vector cross products,
you can write it like this: $\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$

(direction of τ?)

From warmup



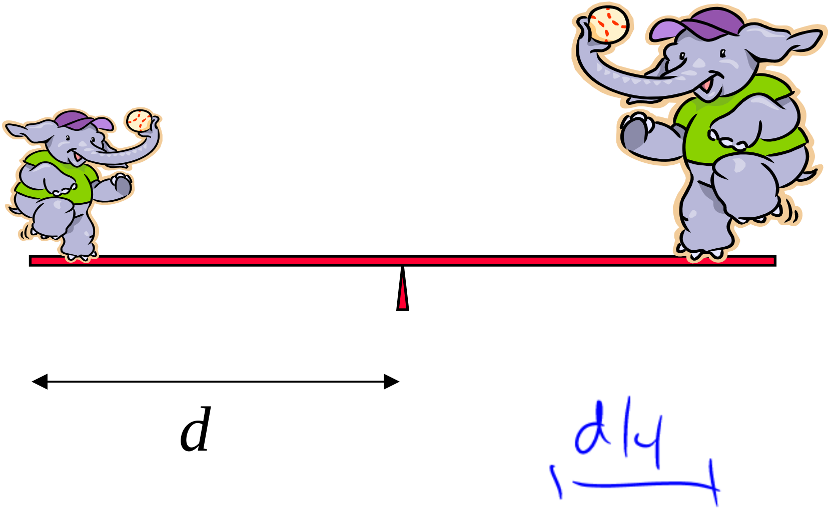
Two people sit on a seesaw. They sit in positions such that the seesaw is balanced in a horizontal position. The two people must weigh the same amount.

a. true

[✓ b. false]

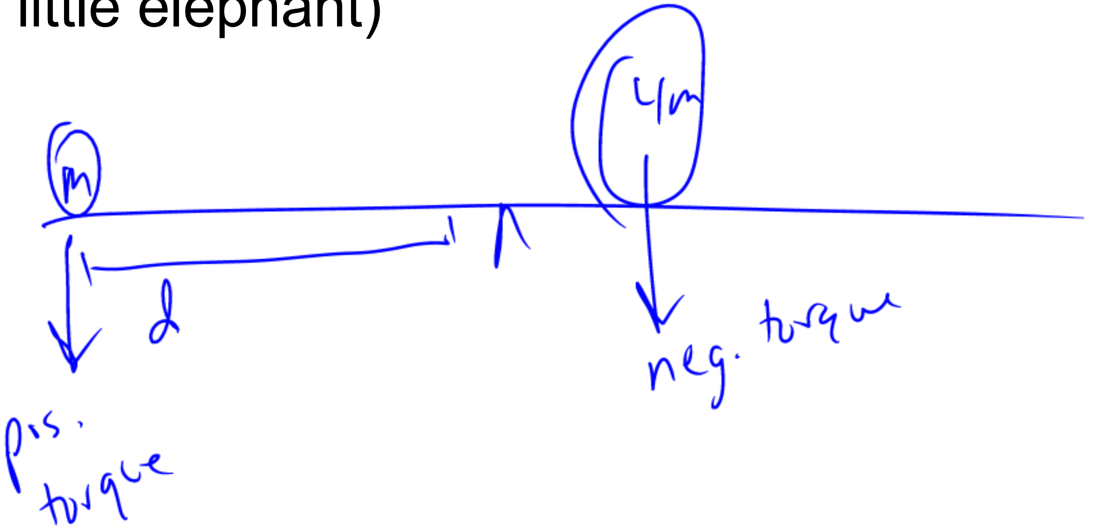
Clicker quiz

$\tau = rF$

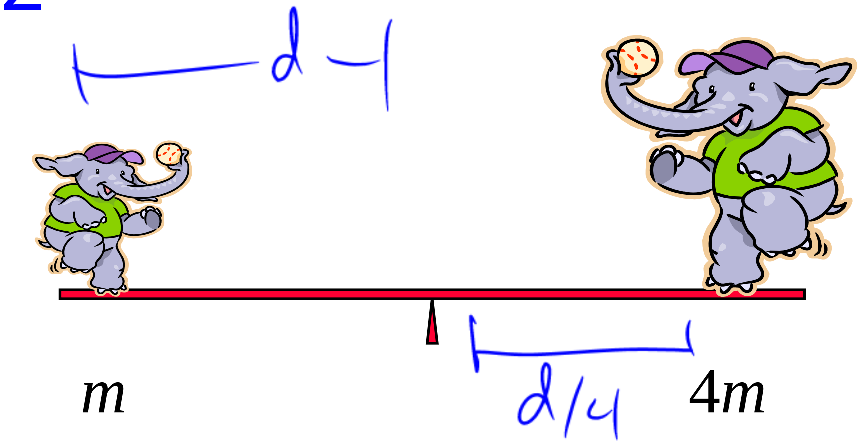


Where should the large elephant stand in order to balance the seesaw? (mass = 4 times the little elephant)

a. *d*
b. *d*/2
[✓ c.] *d*/4
d. *d*/6
e. *d*/8



Clicker quiz



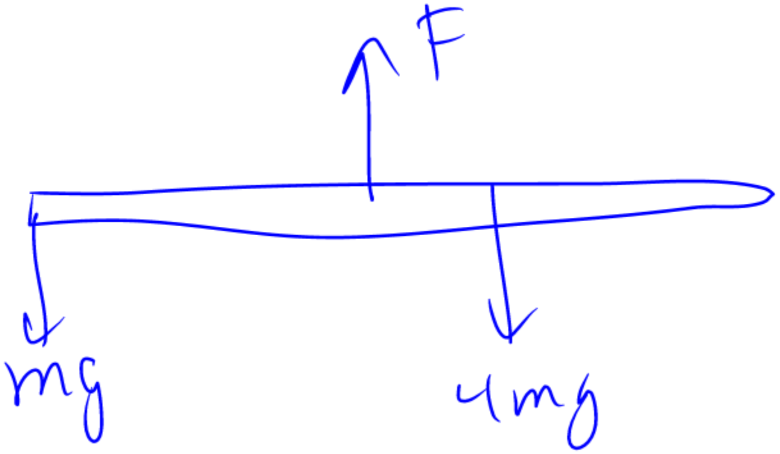
Not rotating: $\boxed{\Sigma \tau = 0}$

Not moving up/down $\boxed{\Sigma F = 0}$

"Equilibrium"

When the see-saw is balanced, what is the upwards force from the pivot point? (Or, equivalently, the downward force *on* the pivot point.)

a. *mg*
b. 4*mg*
c. 5*mg*
d. 6*mg*
e. 8*mg*

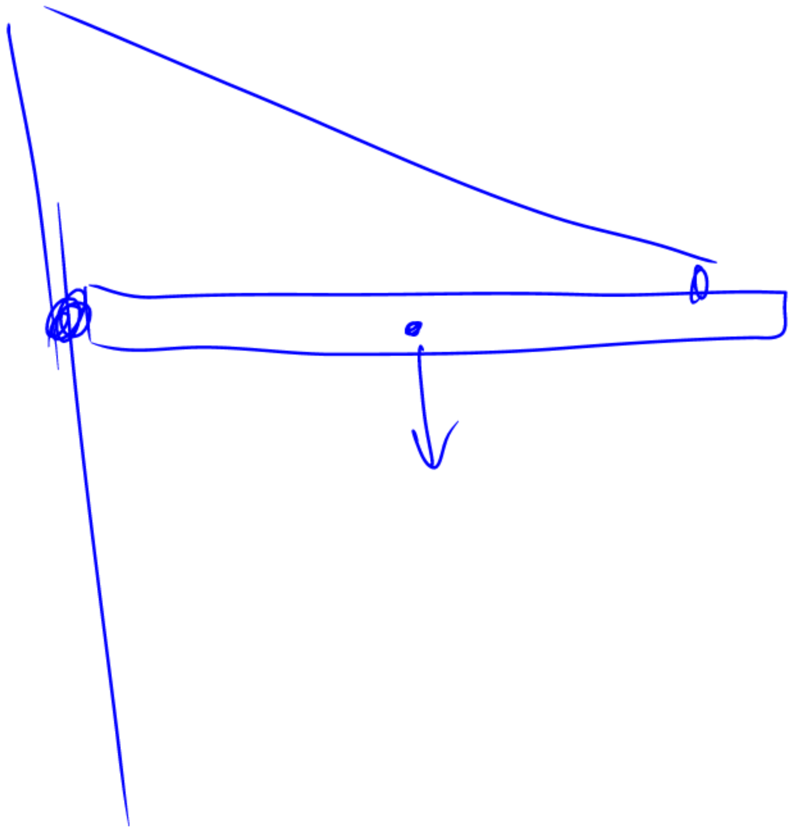


$\Sigma F = 0$

$F - mg - 4mg = 0$

$\boxed{F = 5mg}$

Demos: Center of mass



Equilibrium

What concepts are involved?

1. If an object is not moving ("translational equilibrium"), then…

$$\boxed{\Sigma F = 0}$$

2. If an object is not ***rotating*** ("rotational equilibrium"), then…

$$\boxed{\Sigma \tau = 0}$$

A new blueprint equation!

From Warmup

If an object is in equilibrium:

- a. the net force on it must be zero
- b. the net torque on it must be zero
- [✓ c.] both of the above
- d. neither of the above

General advice

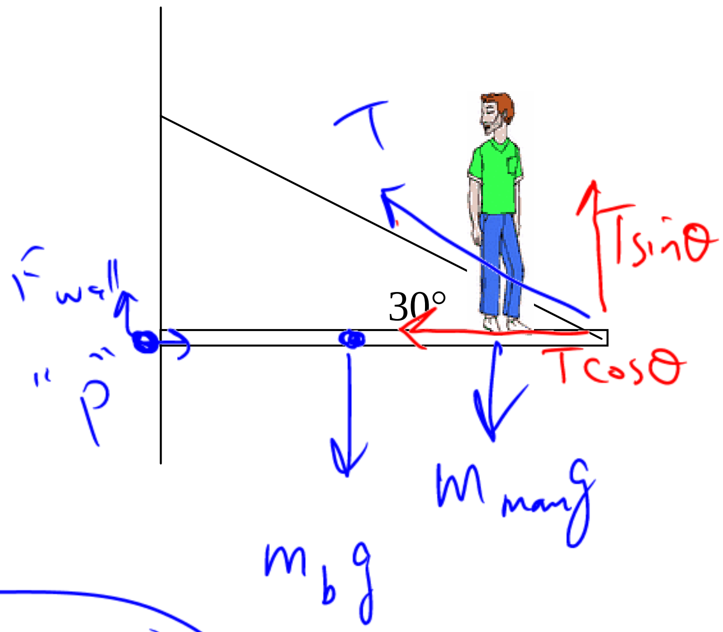
Think carefully about the pivot point

and the sign of the torques

$\tau = r F_\perp$

Worked problem

A 1500 N man, 1 meter from the right end, is standing on a board supported by a wall and a rope. The board weighs 800 N and is 4 meters long. What is the tension in the rope?



1. Draw all of the forces present.
Note: gravity acts at the *center of mass*

1b. Divide forces into components

2. Use Σ**F** blueprint equation(s)

$$\Sigma F_x = 0 \longrightarrow \boxed{F_{wall\,x} - T\cos\theta = 0}$$

$$\Sigma F_y = 0 \longrightarrow \boxed{F_{wall\,y} - m_b g - m_m g + T\sin\theta = 0}$$

3. Use Στ blueprint equation
→ which pivot point to use?

$$\Sigma \tau_p = 0$$

$$\boxed{-(m_b g)(2) - (m_m g)(3) + T\sin(30^\circ)(4) = 0}$$

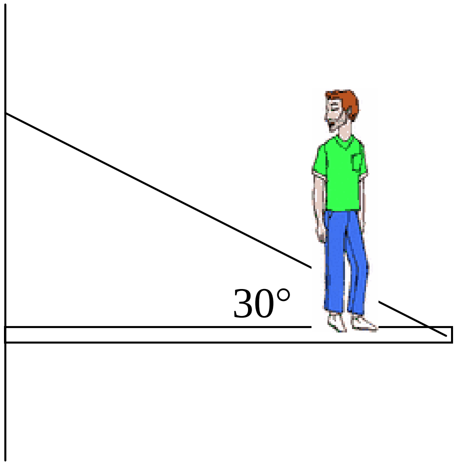
$$T = \frac{m_b g\, 2 + m_m g\, 3}{\sin 30 \cdot 4}$$

4. Use the filled-in blueprints to solve for what you’re looking for.

Answer: T = 3050 N

Additional question

What are the horizontal and vertical forces of the wall on the board?



Answers: F_x = 2641 N to right, F_y = 775 N up

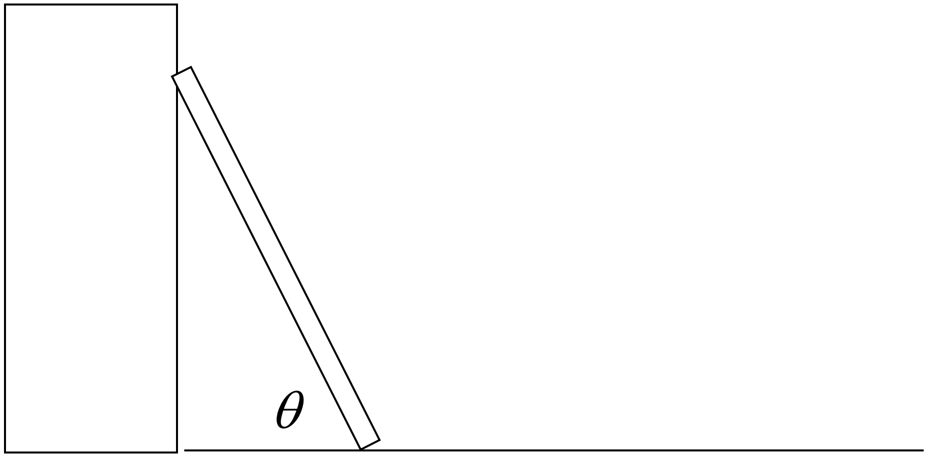
From warmup

Ralph noticed that both torque and work are obtained by multiplying a force times a distance. He wants to know: how are they different? Do they have the same units? What can you tell Ralph to help him out?

“Pair share”–I am now ready to share my neighbor’s answer if called on.

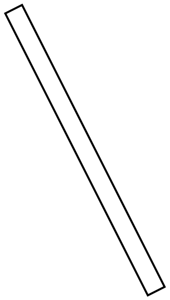
b. Yes

Problem:
(Like HW 14-4)



A ladder leans against a **frictionless** wall. The ground has static coefficient of friction μ. What's the smallest angle θ such that the ladder doesn't slip? Length of ladder is *d*, mass of ladder is *m*.

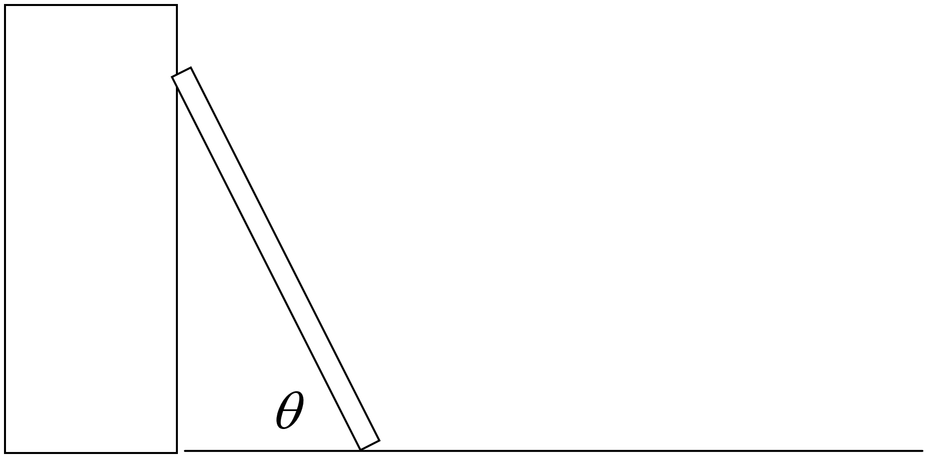
Draw a FBD of ladder:



Clicker quiz: I have done so

a. yes

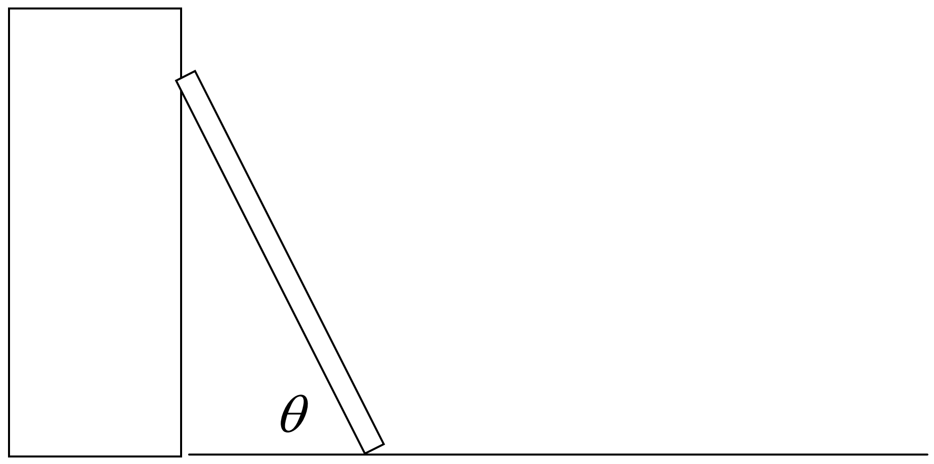
Clicker quiz



The ground's frictional *force* is ________ compared to the wall's normal force.

a. more than
b. less than
c. the same
d. can't tell

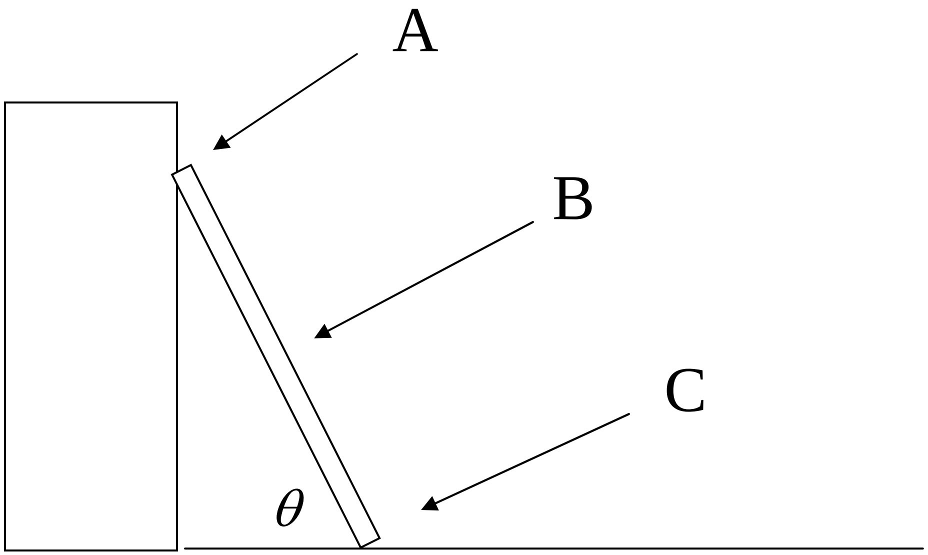
Clicker quiz



The ground's normal *force* pushing upward is _________ compared to the weight.

a. more than
b. less than
c. the same
d. can't tell

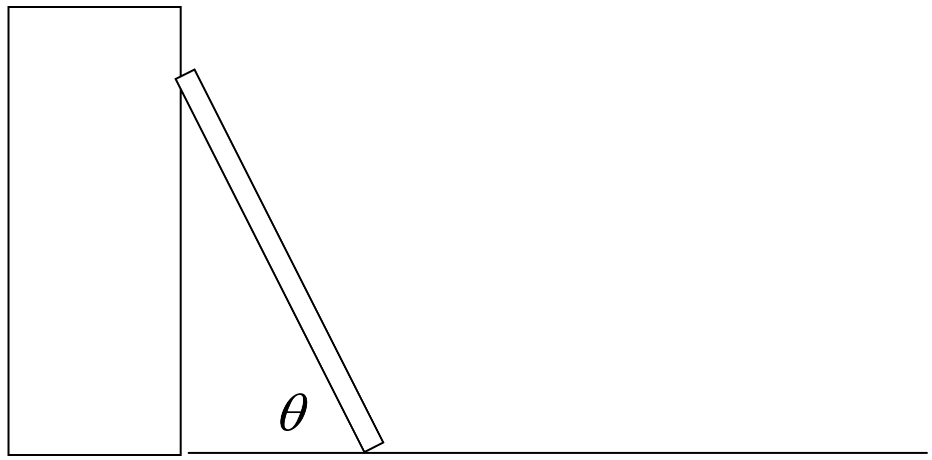
Clicker quiz



To solve the problem, we need to use $\Sigma\tau = 0$… but about which point should we compute the torques?

a. A
b. B
c. C

Solved problem



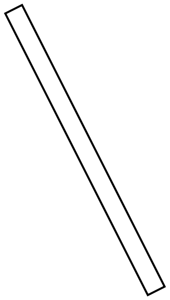
If $\mu = 0.5 \rightarrow \theta = 45°$; $\mu = 0.7 \rightarrow \theta = 35.5°$; $\mu = 0.9 \rightarrow \theta = 29.1°$

Answer: $\theta = \tan^{-1}\left(1/(2\mu)\right)$

Modification

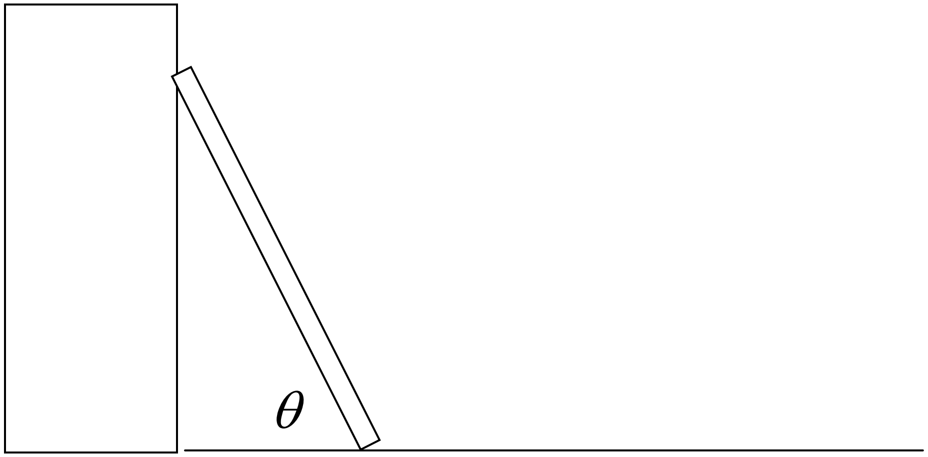
Suppose the wall ***also*** has friction, μ. What's the angle θ now? (Think: bigger or smaller?)

New FBD:



Equations:

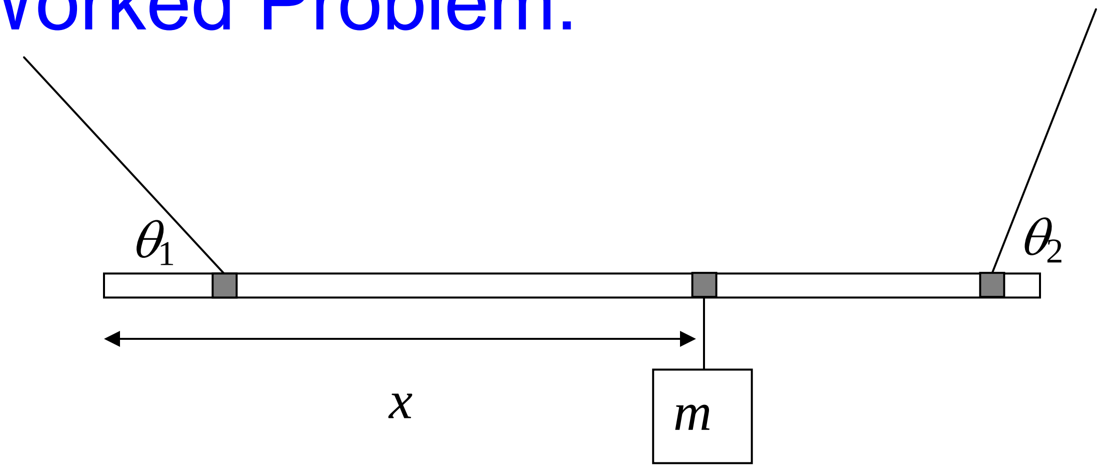
Solved problem



If $\mu = 0.5 \rightarrow \theta = 36.9°$; $\mu = 0.7 \rightarrow \theta = 20.0°$; $\mu = 0.9 \rightarrow \theta = 6.0°$

Answer: $\theta = \tan^{-1}\left(1/(2\mu) - \mu/2\right)$

Worked Problem:



A 0.4 kg meterstick is suspended from support pillars (not shown) via two strings at θ_1 =41.4° and θ_2 = 60°, with tensions of 2N and 3N. The strings are attached at 10 cm and 5 cm from the two ends of the meterstick. The stick is *not* in equilibrium until an additional mass is hung from a point in the middle. Find the unknown x and m.

Answers: 0.171 kg, 38.2 cm