# Announcements – Oct 22, 2013

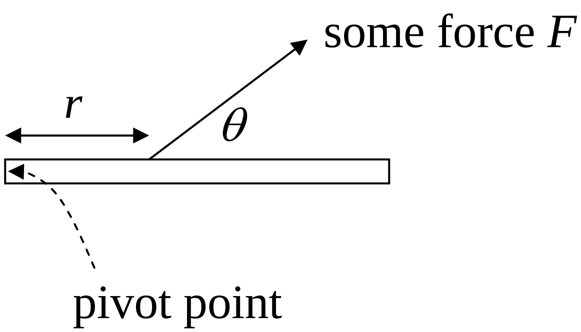
1. **Exam 3** starts one week from today
   a. Next Tuesday: in-class review
   b. Evening TA review → Tuesday night. survey sent out.
   c. Exam covers HW 11-17. (HW 17 doesn't really exist)

2. **Goal:** complete the connection between linear and angular quantities
   a. Distance $x \to \theta$
   b. Velocity $v \to \omega$
   c. Acceleration $a \to \alpha$
   d. Force $F \to \tau$
   e. Mass $m \to$ ?? (today)
   f. KE $\frac{1}{2} mv^2 \to$ ?? (today)
   g. Momentum $mv \to$ ?? (next time)

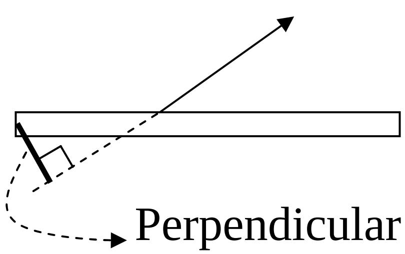
# Review of Torques

Definition of torque: (about a point)

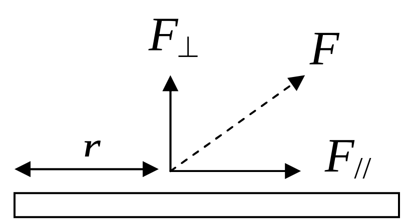
$$\boxed{\tau_p = r_\perp F = rF_\perp = rF\sin\theta}$$




Positive/negative:
- Produces a **clockwise** rotation = **negative**
- Produces a **counter-clockwise** rotation = **positive**



Perpendicular distance $r_\perp$,
the "lever arm" of the force:
$r_\perp = r\sin\theta$



# Equilibrium

$$\sum F = 0$$

$$\sum \tau_p = 0$$

Translation:

- if an object is not speeding up or slowing down, there is no net force on it
- if an object is not speeding up or slowing down its *rotation*, there is no net *torque* on it.

# From warmup (last time)

Ralph noticed that both torque and work are obtained by multiplying a force times a distance. He wants to know: how are they different? Do they have the same units? What can you tell Ralph to help him out?

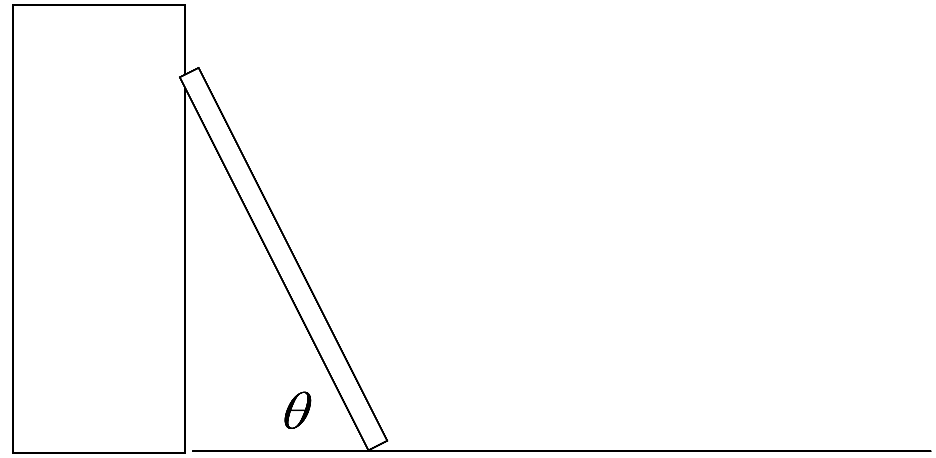
**“Pair share”**–I am now ready to share my neighbor’s answer if called on.

a. Yes

$W = F_{\parallel} d$ $\qquad$ $N \cdot m =$ “joules”

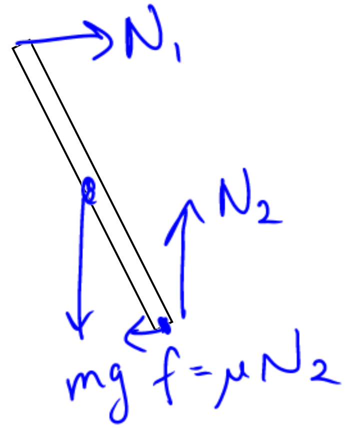
$\tau = F_{\perp} r$ $\qquad$ $N \cdot m$

## Problem:
(Like HW 14-4)



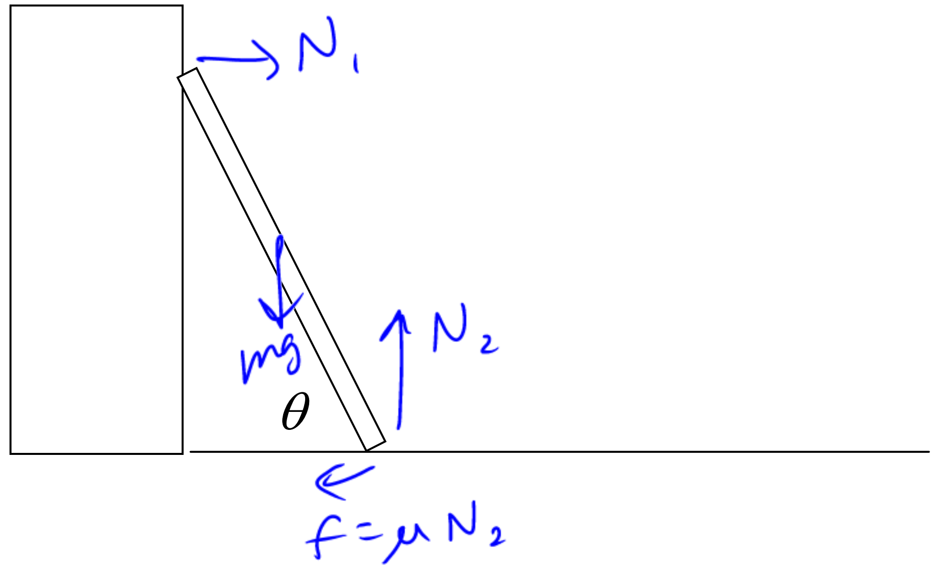

A ladder leans against a **frictionless** wall. The ground has static coefficient of friction $\mu$. What's the smallest angle $\theta$ such that the ladder doesn't slip? Length of ladder is *d*, mass of ladder is *m*.

Draw a FBD of ladder:




**Clicker quiz**: I have done so
a. yes

# Clicker quiz



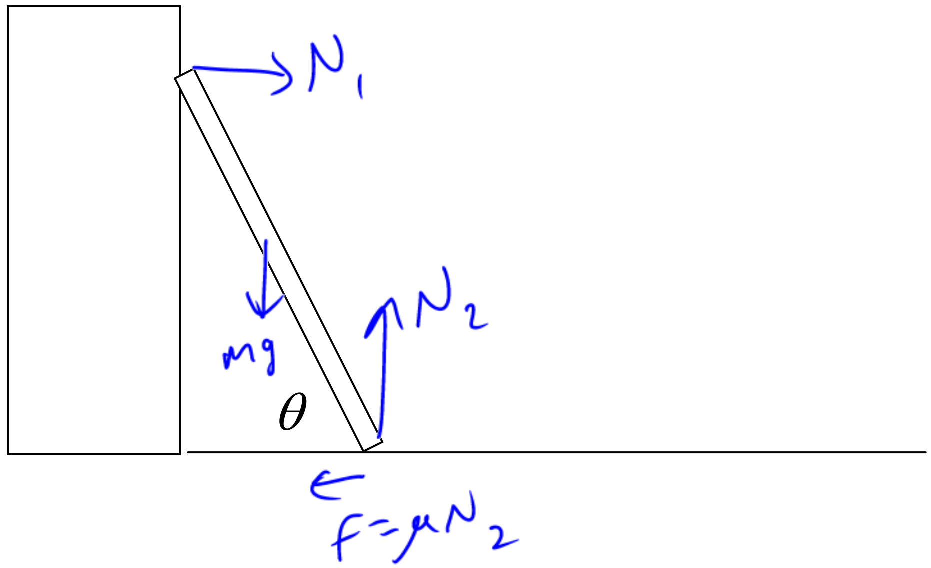

The ground's frictional *force* is _________ compared to the wall's normal force.

a. more than
b. less than
c. the same
d. can't tell

$$\sum F_x = 0$$

$$N_1 = \mu N_2$$

# Clicker quiz



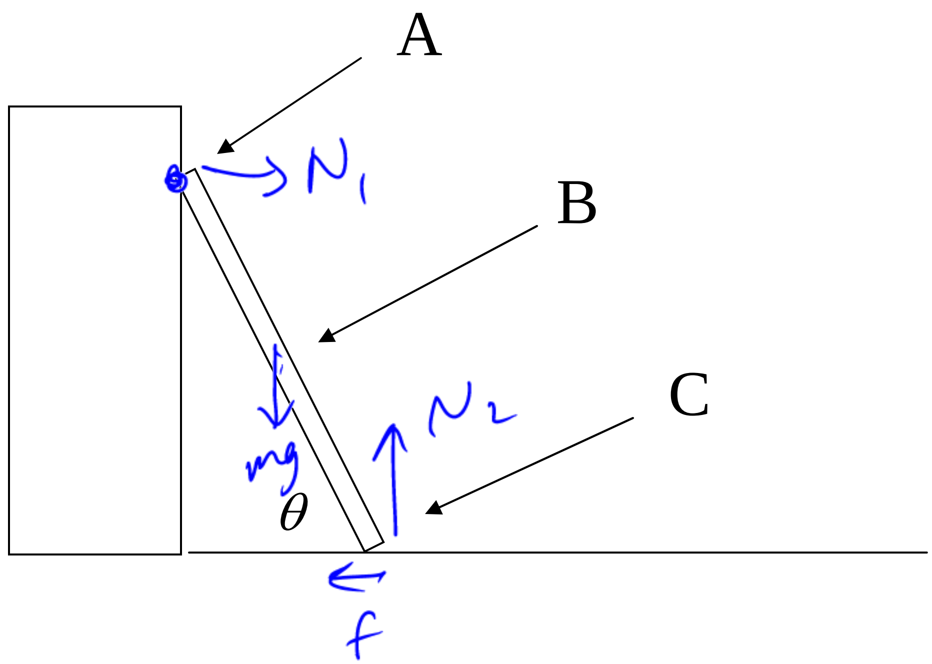

The ground's normal *force* pushing upward is _________ compared to the weight.

a. more than
b. less than
c. the same
d. can't tell

$$\Sigma F_y = 0$$

$$N_2 = mg$$

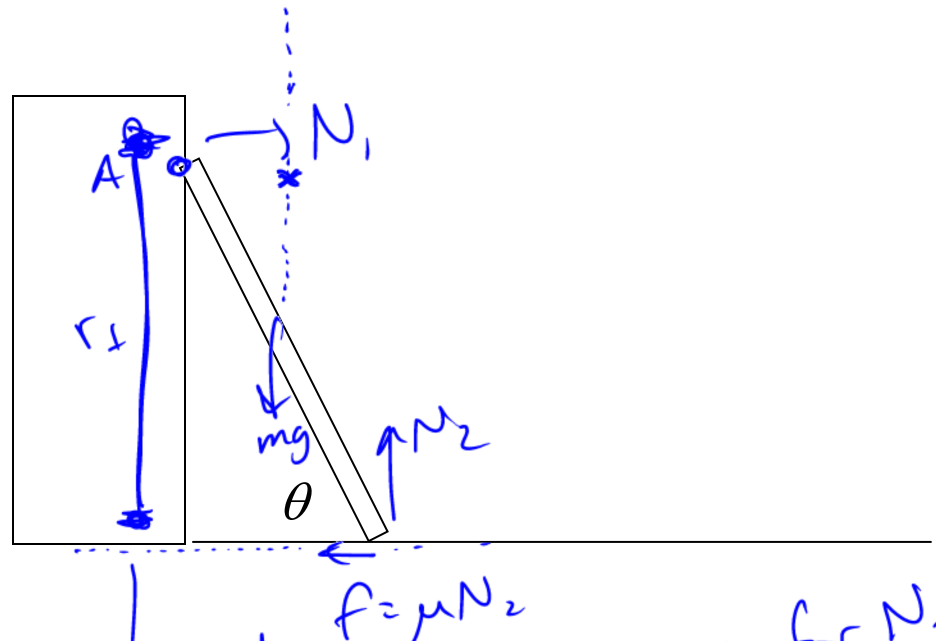
# Clicker quiz




To solve the problem, we need to use $\Sigma\tau = 0$… but about which "pivot point" should we compute the torques?

a. A
b. B
c. C

# Solved problem



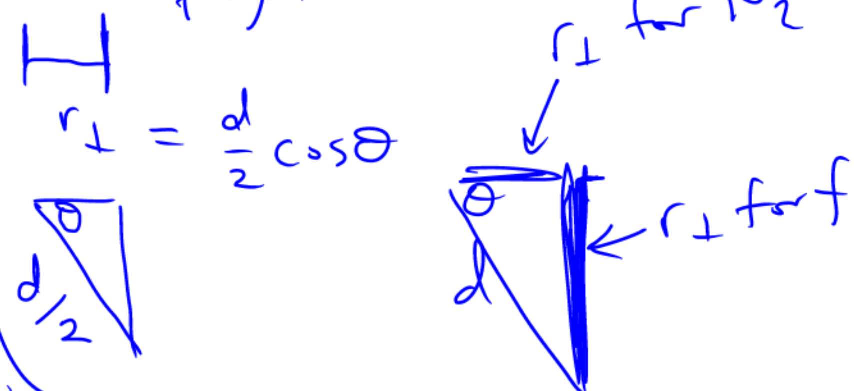

$\Sigma F_x = 0 \rightarrow N_1 = \mu N_2$

$\Sigma F_y = 0 \rightarrow N_2 = mg$

$\Sigma \tau_A = 0$

$\rightarrow -(mg)\left(\frac{d}{2}\cos\theta\right) + N_2(d\cos\theta) - (\mu N_2)(d\sin\theta) = 0$

$N_2 = mg$ $\quad$ $N_2 = mg$




$-\cancel{mg}\frac{\cancel{d}}{2}\cos\theta + \cancel{mg}\cancel{d}\cos\theta - \mu \cancel{mg}\cancel{d}\sin\theta = 0$

$-\frac{1}{2}\cos\theta + \cos\theta = \mu\sin\theta \rightarrow \frac{1}{2}\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}} = \mu\frac{\sin\theta}{\cos\theta}$

$\frac{1}{2} = \mu\tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2\mu}\right)}$

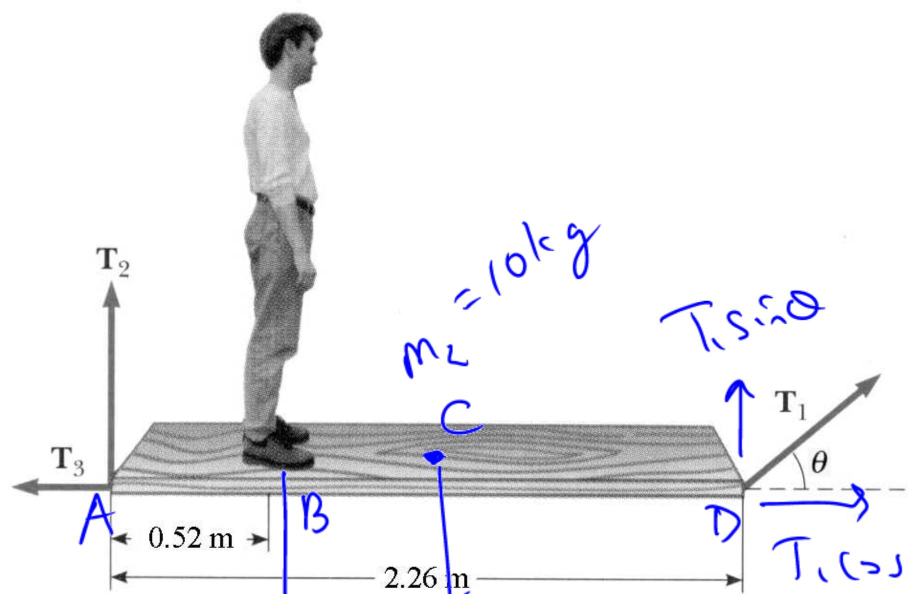
If $\mu = 0.5 \rightarrow \theta = 45°$; $\quad \mu = 0.7 \rightarrow \theta = 35.5°$; $\quad \mu = 0.9 \rightarrow \theta = 29.1°$

Answer: $\theta = \tan^{-1}\left(1/(2\mu)\right)$

# One more equilibrium problem:

A uniform plank of length 2.26 m and mass 10 kg is balanced by three ropes as indicated in the figure, with $\theta = 35°$. A 75 kg person is standing 0.52 m from the left end. Find the tensions in all three ropes.

$m_1 = 75\,kg$




$m_1 g$ $\quad$ $m_2 g$

$\sum F_x = 0 \qquad T_3 = T_1 \cos\theta$

$\sum F_y = 0 \qquad m_1 g + m_2 g = T_2 + T_1 \sin\theta$

$\sum \tau_A = 0 \qquad -(m_1 g)(.52) - m_2 g(1.13) + T_1 \sin 35° (2.26) = 0$

?

Solve for $T_1$

then use Eqn 1 to find $T_3$
Eqn 2 to find $T_2$

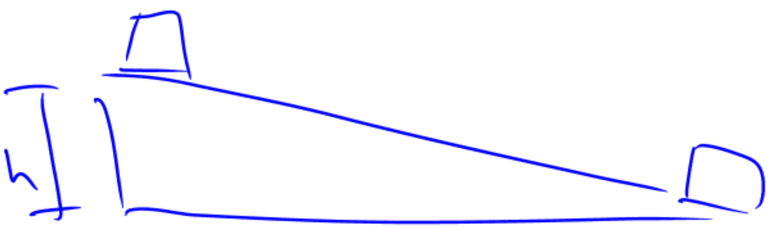
Answers: 380.3 N, 311.5 N, 614.9 N

# Rotational kinetic energy

**Demo…** a cart races a ball (video from warmup). Who wins? Why?

$+ KE_{rot}$

**Review:** How fast is **cart** going at bottom? (Energy)




$$PE_i = KE_f$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gh}$$

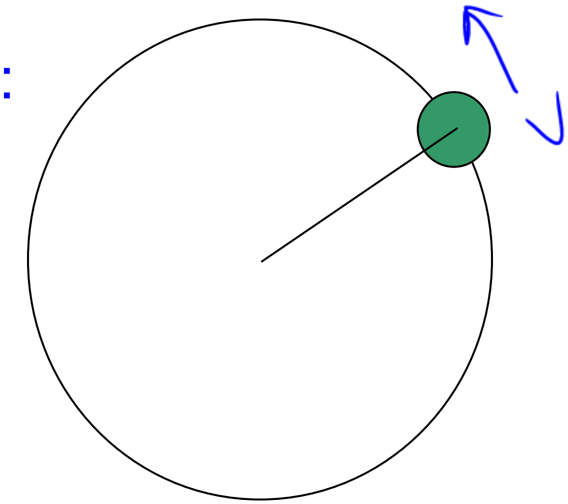
How long did it take to get there? (Kinematics)

We could do this

→ **What's different about the ball?**

Kinetic energy of a **"point mass"** rotating in a circle:

$KE = \frac{1}{2} m v_{tan}^2$




$v_{tan} = \omega \cdot r$

Write in terms of ω:

$\frac{1}{2} m \omega^2 r^2$

$\frac{1}{2} m v^2$

$\frac{1}{2} (m r^2) \omega^2 \leftarrow KE = \frac{1}{2} I \omega^2$

$$KE_{rot} = \frac{1}{2}(\text{something})\omega^2$$

→ what's the something?

$mr^2$

"moment of inertia"

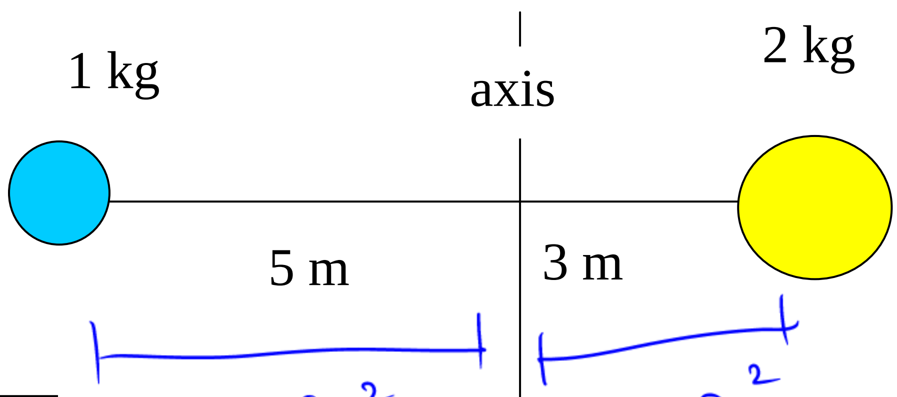
$I$

# "Moment of inertia"

$\boxed{I_{pt\,mass} = mr^2}$ (rotating in a circle; *r* = radius of circle)

Kinetic energy in terms of *I* and ω: $\boxed{KE_{rot} = \frac{1}{2} I \omega^2}$

Moment of inertia for two masses? (connected with a rod)

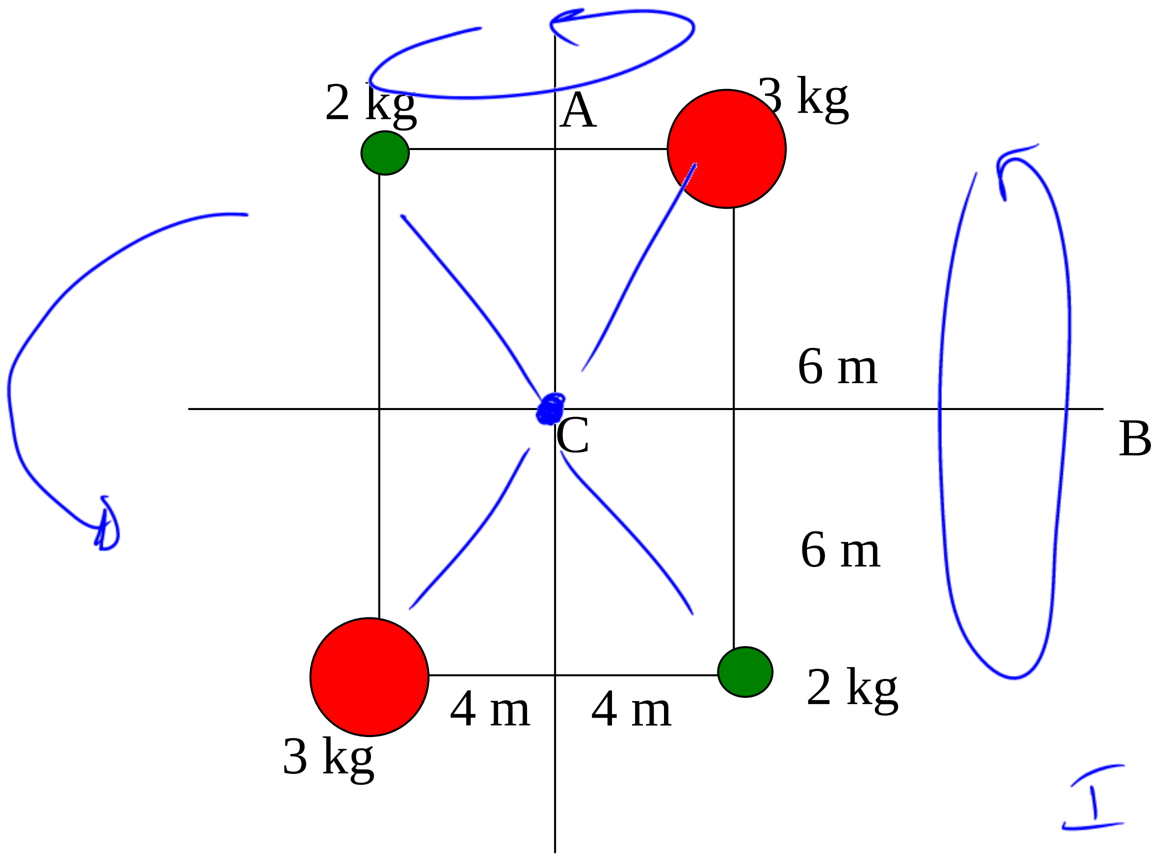



$m_2 r_2^2$ $m_1 r_1^2$

$\boxed{I = I_1 + I_2 + \ldots}$

$I_{tot} = m_1 r_1^2 + m_2 r_2^2$

# Clicker quiz




**Tip**: If size of object is much smaller than rotation radius, treat it as a "point mass"

$I = m_1 r_1^2 + m_2 r_2^2 + \ldots$

Does *I* change when you rotate about axis A vs. axis B?

a. About axis A has larger *I*

b. About axis B has larger *I*

c. They have the same *I*

$I_A = m_{tot} \cdot (4m)^2$

$I_B = m_{tot} \cdot (6m)^2$

# Worked problem

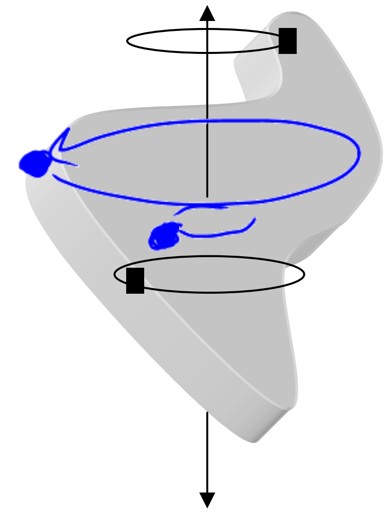
What's the total moment of inertia about axis C? (C is into the page)

Answer: $I_{tot}$ = 520 kg·m$^2$

# Demo

Variable "I-rotator"

# “Extended” objects



Must add up $mr^2$ *for each bit of mass* in the object

Which bits of mass contribute the most to *I*?

far from axis

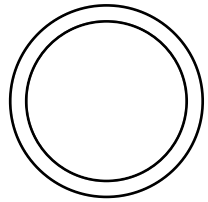
**From warmup.** Moment of inertia is biggest for:

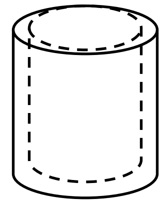
a. compact objects
b. objects that are spread out
c. neither; doesn’t depend on shape

**Demo:** Long “I-bars”

# Which of these objects will have the largest I?

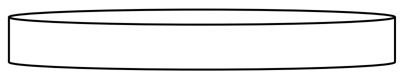
Hoop/cylindrical shell

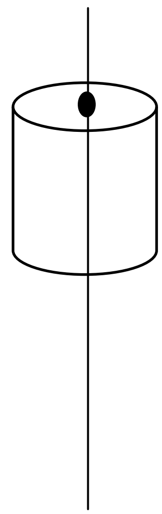




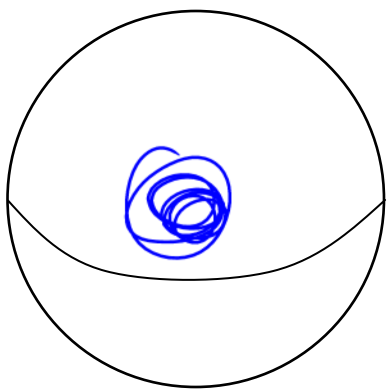
largest I

Solid disk/cylinder



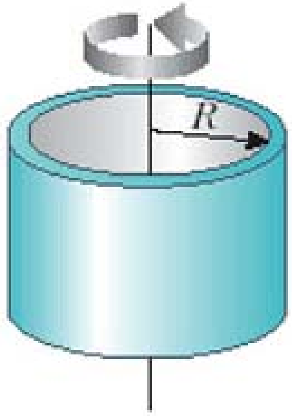


Solid sphere

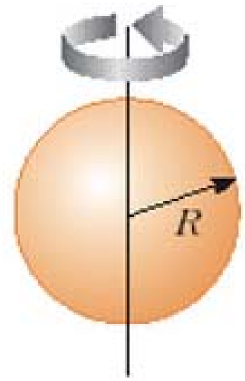


smallest I

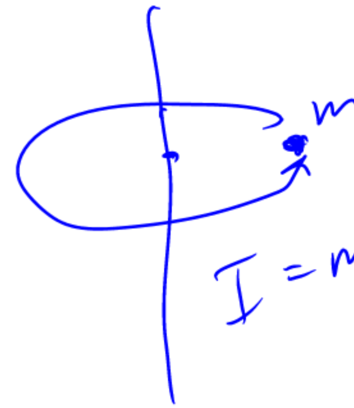
Hoop or thin cylindrical shell
$I = MR^2$




Solid sphere
$I = \frac{2}{5} MR^2$

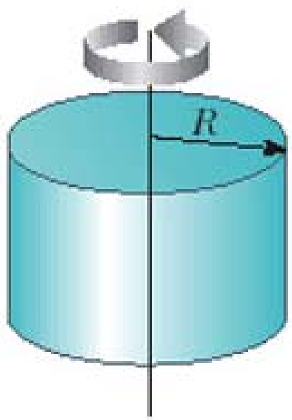



I pt mass in a circle

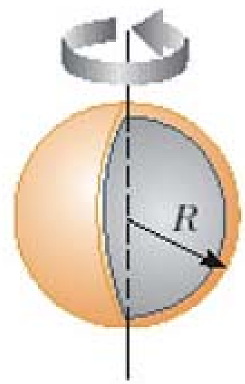



$I = mr^2$

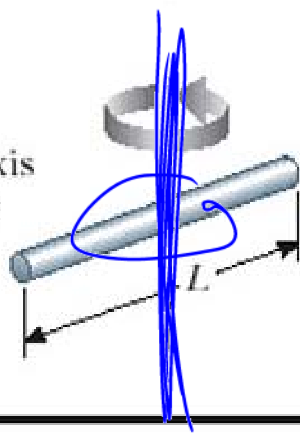
Solid cylinder or disk
$I = \frac{1}{2} MR^2$



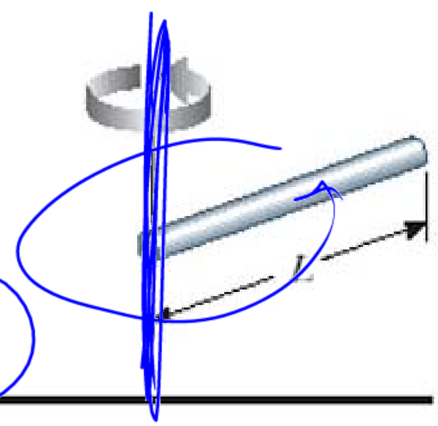

Thin spherical shell
$I = \frac{2}{3} MR^2$




Long thin rod with rotation axis through center
$I = \frac{1}{12} ML^2$




Long thin rod with rotation axis through end
$I = \frac{1}{3} ML^2$



# Clicker quiz

Which kind of rolling object will be moving the <u>fastest</u> at the bottom of an incline?

a. Hoop → $I = mr^2$
b. Solid disk → $I = \frac{1}{2} mr^2$
c. Sphere → $I = \frac{2}{5} mr^2$
d. They will all tie
e. Depends on size and/or mass

Additional question: Which object will get to the bottom first?

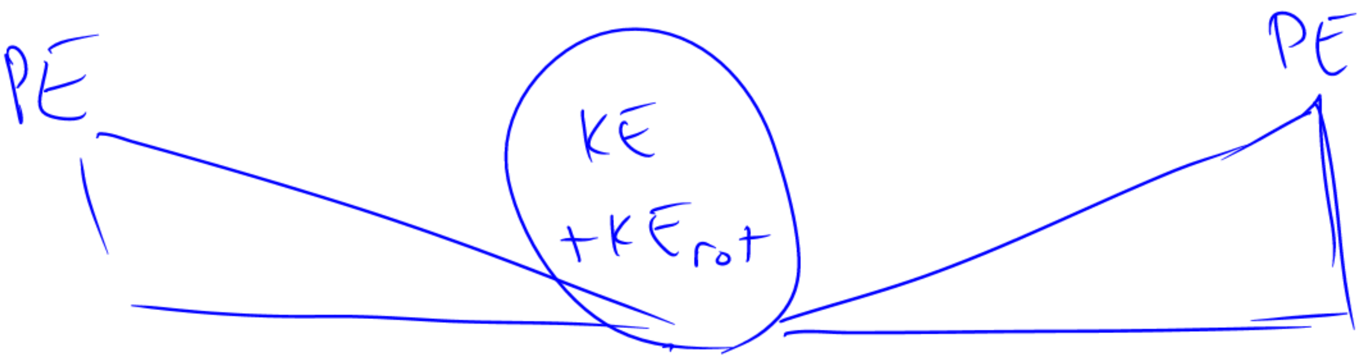
# Demo: Moment of inertia races

Hoop vs. sphere
Hoop vs. disk
Big disk vs. little disk
Big hoop vs. little hoop
Big sphere vs. little sphere

# Clicker quiz

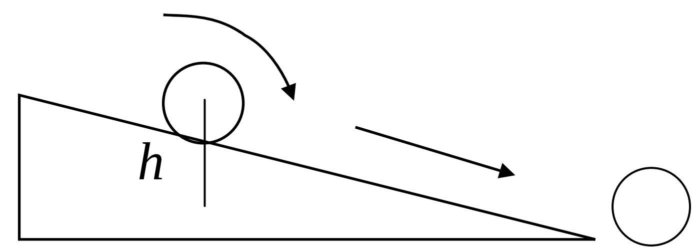
If they continued on, which would go the farthest up a hill on the other side?

a. Hoop
b. Solid disk
c. Sphere
d. All would end at the same height



# Worked Problem

An object with moment of inertia *I* rolls down a height *h* without slipping. Find the speed at bottom.




$$E_{bef} = E_{aft}$$

$$PE = KE_{trans.} + KE_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$mgh = \left(\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2}\right)v^2$$

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2}}} \quad \cdot\frac{1/m}{1/m}\ \frac{2}{2} = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

Answer: $v = \sqrt{\dfrac{2gh}{1 + I/mR^2}}$

$$I_{sphere} = \frac{2}{5}mr^2$$

# Newton's second law for rotation

$$\boxed{\sum \tau_p = I\alpha}$$

$\tau \quad I \quad \alpha$

still also have $\sum \vec{F} = m\vec{a}$ … but acceleration of what?

**From warmup.** Angular acceleration will definitely increase if:

a. torque is decreased and moment~~um~~ of inertia is decreased
b. torque is decreased and moment~~um~~ of inertia is increased
c. torque is increased and moment~~um~~ of inertia is decreased
d. torque is increased and moment~~um~~ of inertia is increased

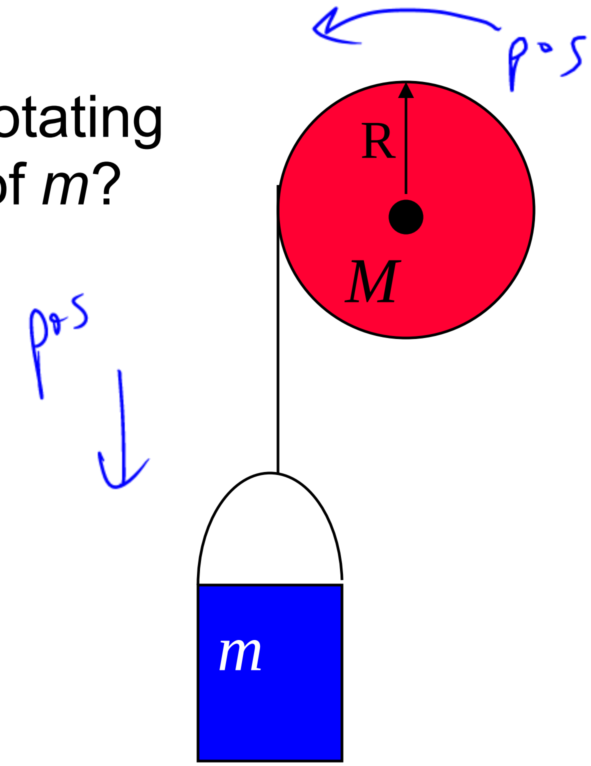
# From warmup

Ralph heard his instructor say **"Moment of inertia plays the same role in rotational motion that mass does in linear motion."** This confuses him. What does it mean?

**“Pair share”**–I am now ready to share my neighbor’s answer if called on.

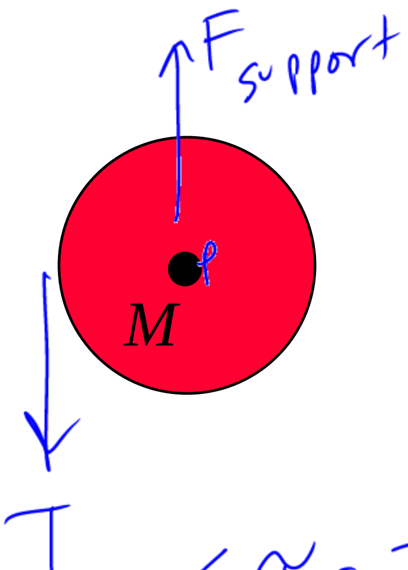
b. Yes

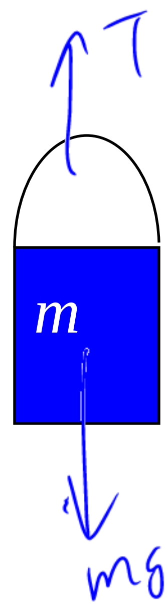
**Worked problem:** A falling mass starts a cylinder rotating (<u>not</u> a "massless pulley"). What is the acceleration of *m*?




$$a_{tan} = \alpha r$$

Start with FBDs:




$$\sum \tau_p = I\alpha$$

$$(T)\cancel{R} = \left(\tfrac{1}{2}MR^{\cancel{2}}\right)\left(\frac{a}{\cancel{R}}\right)$$

$$T = \tfrac{1}{2}Ma$$

$$\sum F = ma$$

$$mg - T = ma$$

plug in and solve for a

**Write equations…**

Cylinder

Pail

Make a connection between $\alpha$ and *a*:

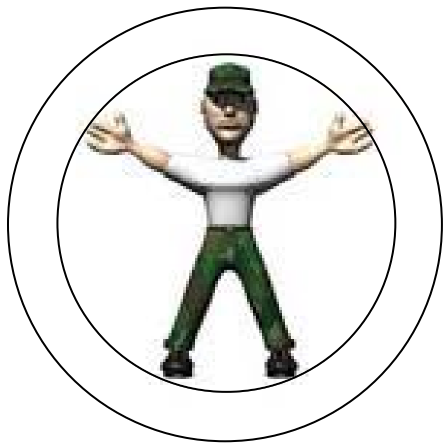
Answer: $a = \frac{m}{m + M/2} g$

# Alternate method:

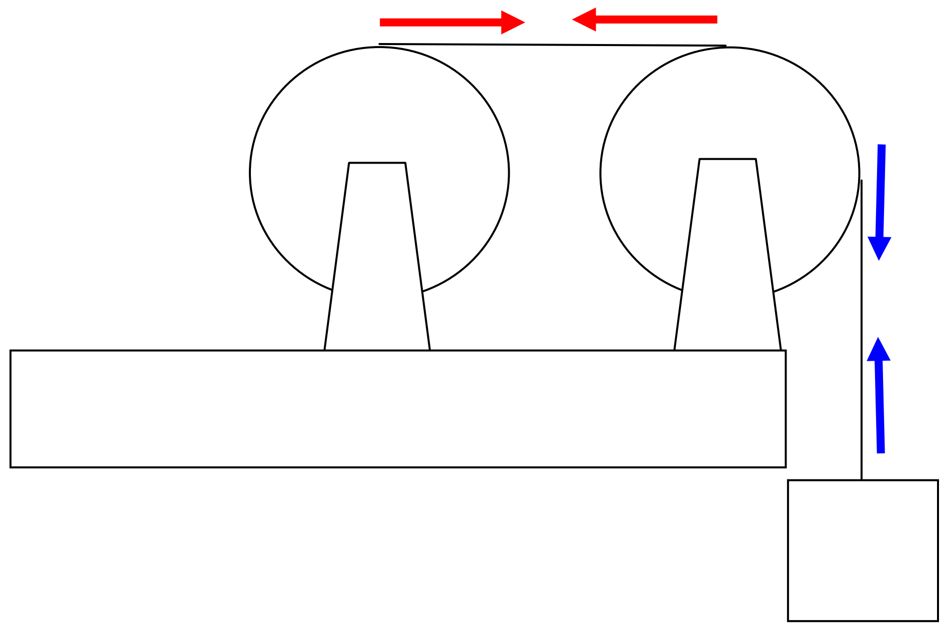
# Clicker quiz

Mary and Fred are rolling a large tire down a hill. Mary says it will go faster if Fred gets inside the tire as shown and rolls down with it. Fred's not sure. What do you think?

a. It will go faster
b. It will go slower
c. It will take the same time



# Clicker quiz



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is no slipping of the rope on either wheel; both wheels accelerate at the same rate. The tension in the rope is

a. Largest between the disks (red arrows)
b. Largest above the mass (blue arrows)
c. The same in both places.

(What's the difference with our old "massless pulleys"?)