

Announcements – Thurs, 24 Oct 2013

1. **Tuesday evening, TA-led Exam Review session—results of doodle.com voting**

a. Time: _____ 6:30 - 8 pm

b. Place: to be scheduled

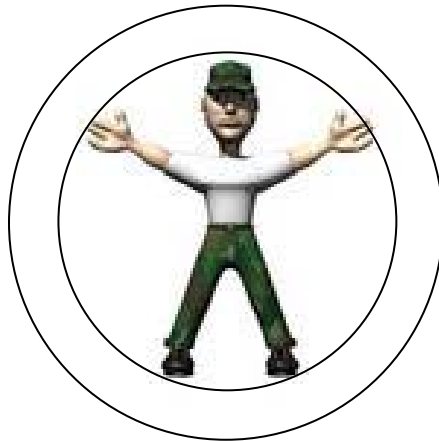
2. Physics of real world—<http://vimeo.com/76847836>
(Especially 4:45)

3. Photo contest!

Clicker quiz (from last time)

Mary and Fred are rolling a large tire down a hill. Mary says it will go faster if Fred gets inside the tire as shown and rolls down with it. Fred's not sure. What do you think?

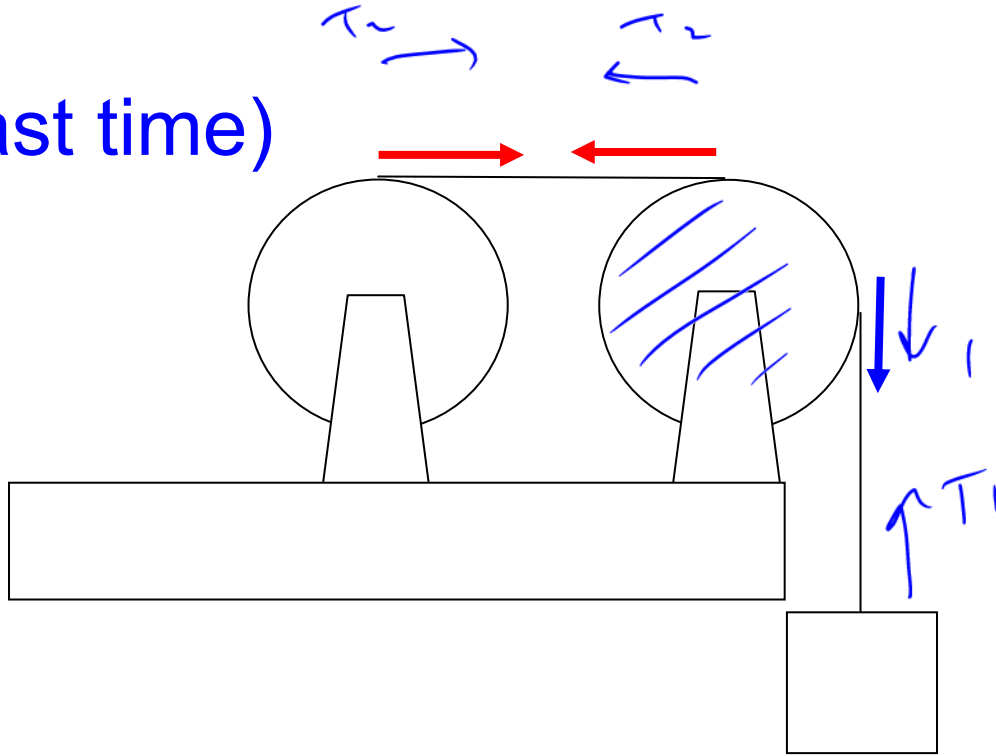
- a. It will go faster
- b. It will go slower
- c. It will take the same time



Clicker quiz (from last time)

$$\sum \tau = I \alpha$$

$(T_{1r}) - (T_{2r})$ \uparrow \uparrow



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is no slipping of the rope on either wheel; both wheels accelerate at the same rate. The tension in the rope is

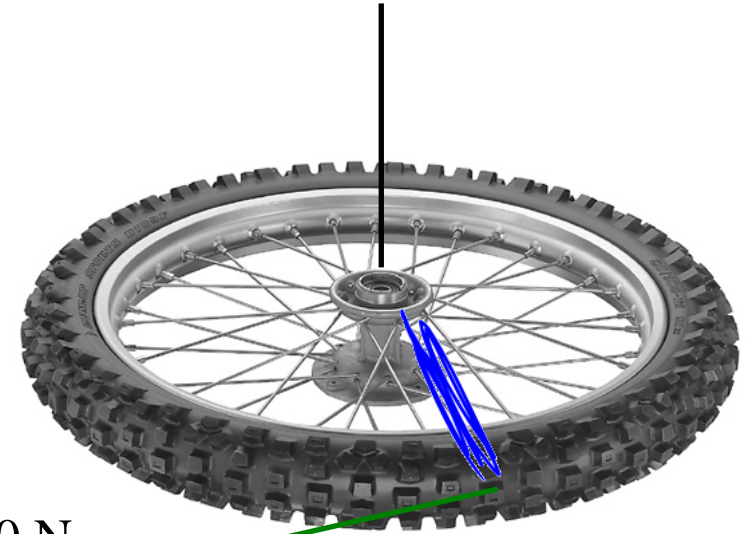
- Largest between the disks
- Largest above the mass
- The same in both places.

(What's the difference with our old "massless pulleys"?)

Worked Problem

A bicycle tire ($r = 0.4 \text{ m}$, $I = 0.8 \text{ kg}\cdot\text{m}^2$) is hanging from a string from the ceiling, not moving. You push tangentially on the edge with a 30 N force for 0.3 seconds . What is ω_f ? (*Hint: because time is given, might be simplest to do it with N2, not energy.*)

$F = 30 \text{ N}$



$$\tau = I \alpha$$

$$(30)(.4) = (.8) \alpha$$

$$\alpha = \frac{12}{.8} \text{ rad/s}^2$$

$$v_f = v_o + at$$

$$\omega_f = \omega_o + \alpha t$$

$$\omega_f = 0 + \left(\frac{12}{.8}\right)(.3)$$

$$= 4.5 \text{ rad/s}$$

Answer: 4.5 rad/s

Angular Correspondences Review

Kinematics

Distance: x

Velocity: v

Acceleration: a

$$\begin{cases} x = x_0 + v_0 t + \frac{1}{2} a t^2 \\ v = v_0 + a t \\ v^2 = v_0^2 + 2a(x - x_0) \end{cases}$$

Angle: θ

Angular velocity: ω

Angular acceleration: α

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Mass

→ Mass: m

Moment of inertia: I

Force/Newton's 2nd Law

→ Force: F

$$\rightarrow \sum \vec{F} = m\vec{a}$$

Torque: τ

$$\sum \tau = I\alpha$$

Energy

$$\rightarrow KE_{trans} = \frac{1}{2} m v^2$$

$$KE_{rot} = \frac{1}{2} I \omega^2$$

Momentum...

$$\vec{p} = m\vec{v}$$

Angular momentum??

$I \omega$

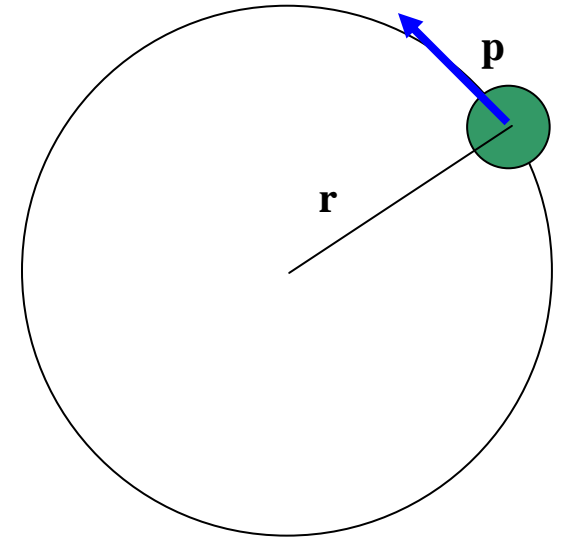
Angular momentum

Imagine a mass m on a thin rod moving in a circle, with constant speed v . It has linear momentum

$$\vec{p} = \underline{m\vec{v}}.$$

Is \vec{p} constant? No

Is $|\vec{p}|$ constant? Yes
(magnitude)



What do we need in order to affect $|\vec{p}|$? Torque

Derivation of Angular Momentum

Force-momentum relationship

Start with Newton 2:

$$\sum \vec{F} = m\vec{a}$$

$$= \frac{m \Delta v}{\Delta t}$$

$$= \frac{\Delta p}{\Delta t}$$

Torque-ang. mom. relationship

$$\sum \tau = I\alpha$$

$$= I \frac{\Delta \omega}{\Delta t}$$

$$= \frac{\Delta (I\omega)}{\Delta t}$$

Define $L = I\omega$

$$L = I\omega$$

If no net external force,
no change in momentum

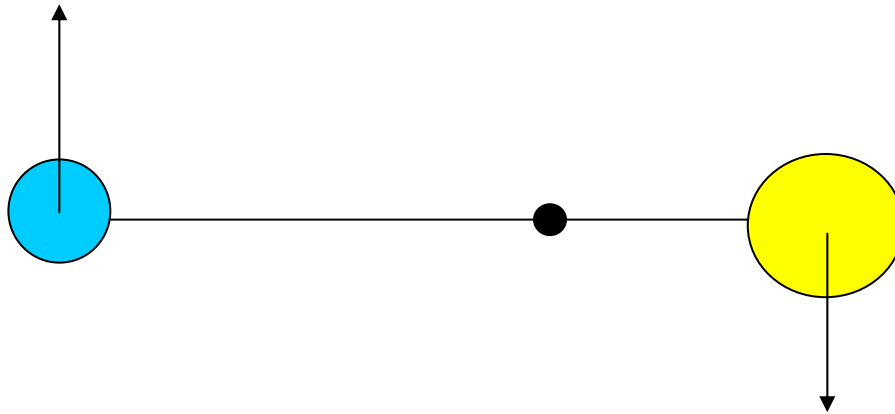
If no net external torque, no
change in angular momentum

Conservation of Angular momentum

$$\boxed{\Sigma L_{bef} = \Sigma L_{aft}} \rightarrow \text{if and only if } \textit{no net external torque}$$

$$I_i \omega_i = I_f \omega_f$$

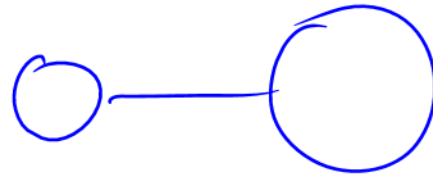

Problem



$$I_i \omega_i = I_f \omega_f$$

dec. \nearrow inc.

Two space stations are connected by a cable. They are rotating about their center of mass. Someone in the blue station pulls the cable in so they are each closer to the center of rotation. What happens?



Demo: Hoberman sphere

Clicker quiz

Is rotational kinetic energy conserved in the Hoberman sphere? The final KE is _____ as the initial KE:

- a. more
- b. less
- c. the same

Hint: is there any non-conservative work done?

From warmup

Rotating stool, student with weights. What happens to her moment of inertia as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same

What happens to her rotational speed as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same

What happens to her rotational kinetic energy as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same

Application to skaters? (frictionless ice)

Demo: spinning chair



Worked Problem

A skater has an initial ω of 2 rad/s and $I = 30 \text{ kg}\cdot\text{m}^2$. When she brings in her arms, $I = 10 \text{ kg}\cdot\text{m}^2$. What is her final ω ?



$$\sum L_{\text{bef}} = \sum L_{\text{aft}}$$

$$I_i \omega_i = I_f \omega_f$$

$$(30 \text{ kg}\cdot\text{m}^2)(2 \frac{\text{rad}}{\text{s}}) = (10) \text{ kg}\cdot\text{m}^2 \cdot \omega_f$$

How much work did it take to do this?

$$\begin{aligned} K E_i &= \frac{1}{2} I_i \omega_i^2 \\ &= \frac{1}{2} (30)(2^2) \\ &= \underline{\underline{60 \text{ J}}} \end{aligned}$$

$$K E_f = \frac{1}{2} (10)(6^2) = \underline{\underline{180 \text{ J}}}$$

$$\begin{aligned} \omega_f &= \frac{30}{10} \cdot 2 \\ &= \boxed{6 \text{ rad/s}} \end{aligned}$$

$$W = 120 \text{ J}$$

Answers: 6 rad/s, 120 J

Is L conserved in these cases?



“Teacups”: central post is connected to the platform floor

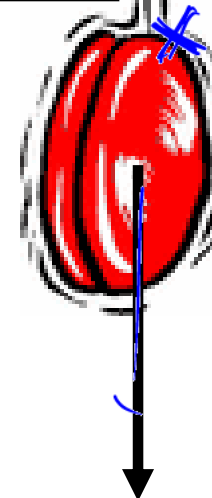
A = Yes
B = No

Train on circular track



Pocket watch with internal spring

Yo-yo



Videos

- train on circular track
- pocket watch

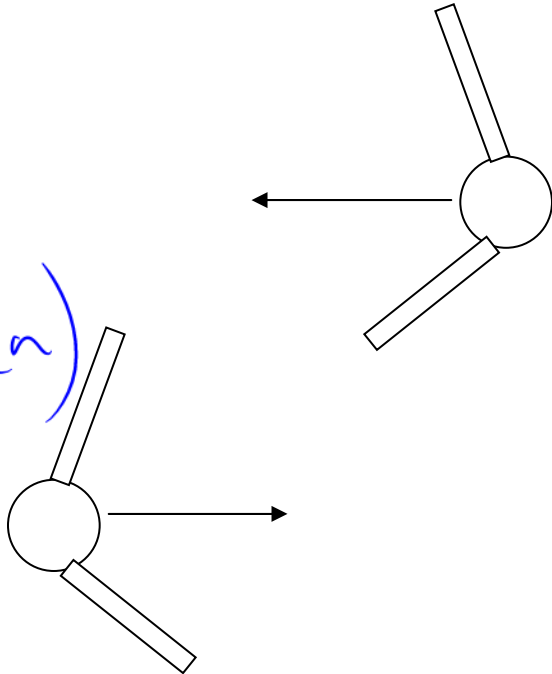
$$L = I\omega$$

Food for thought: two skaters joining hands

Before

Any L?

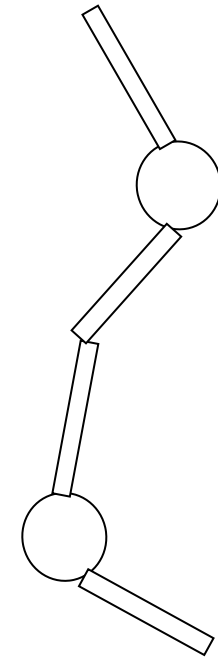
Yes
(hidden)



After

Any L?

Yes



Angular momentum conserved \Leftrightarrow No external torque
(system=both skaters)

No external torque

Clicker quiz: Is there an external torque here? I.e. was angular momentum conserved?

- a. Yes external torque/ang. mom. not conserved
- b. No external torque/ang. mom. is conserved

Another expression for L...

Start with

$$\tau = r_{\perp} F$$

$$= r_{\perp} \left(\frac{\Delta p}{\Delta t} \right)$$

$$\tau = \frac{\Delta(r_{\perp} p)}{\Delta t}$$

Remember

$$F_{net} = \Delta p / \Delta t$$

$$\tau_{net} = \Delta L / \Delta t$$

$$L = r_{\perp} p$$

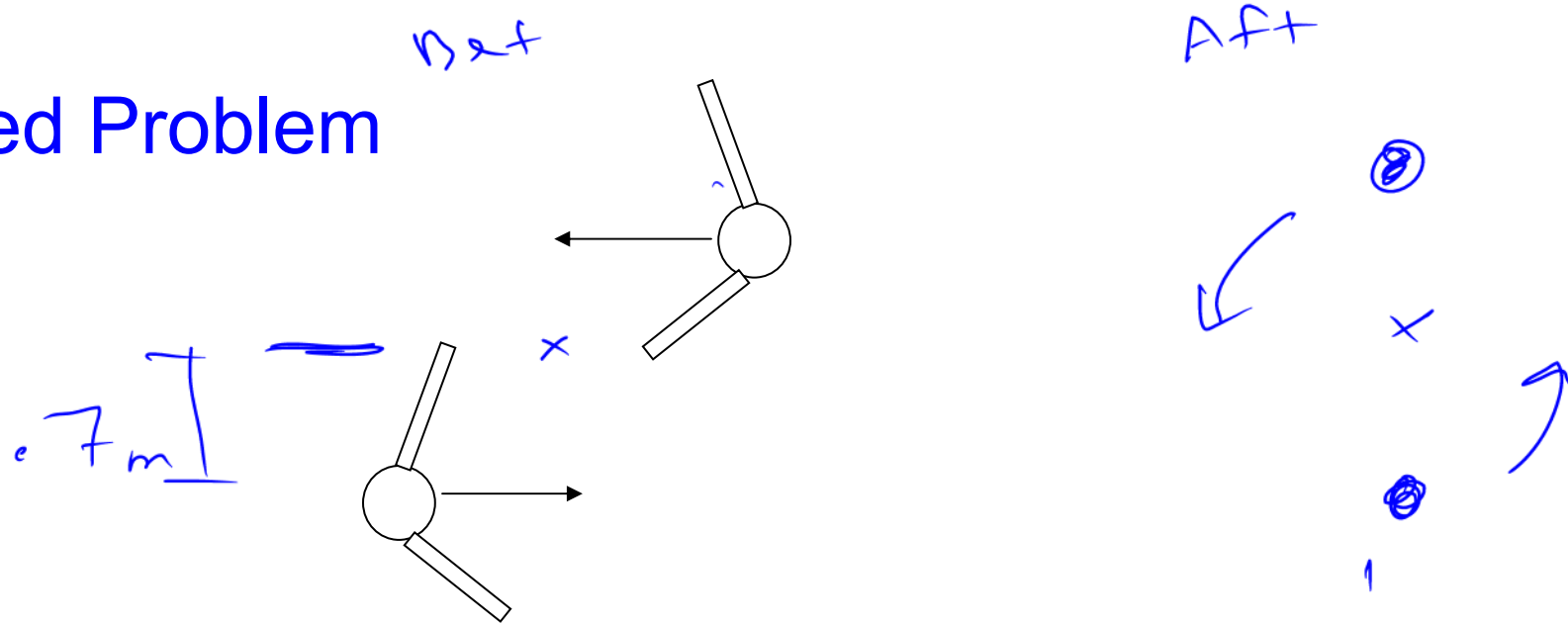
Result:

$$L = r_{\perp} p$$

$$(= r p_{\perp} = r p \sin \theta)$$

$$L = I \omega$$

Worked Problem



The skaters have 0.7 m arms and are each 62 kg. They come together at 3.5 m/s. How fast (rad/s) are they turning afterwards?

$$\sum L_{\text{before}} = \sum L_{\text{after}} \quad \begin{matrix} \swarrow m_1 r_1^2 \\ \swarrow m_2 r_2^2 \end{matrix}$$

$$r_{\perp 1} p_1 + r_{\perp 2} p_2 = I_1 \omega_1 + I_2 \omega_2$$

$$+ (0.7 \text{ m})(62 \text{ kg})(3.5 \text{ m/s}) + (0.7)(62)(3.5) = (62)(0.7)^2 \omega + (62)(0.7)^2 \omega$$

$$\omega = 5 \text{ rad/s}$$

Answer: 5 rad/s

Comment on vectors... (aka L has a direction!)

Does ω have a direction? yes

Therefore L has direction

$$L \approx I\omega$$

Thus with **no external torques...**

...both magn. and direct. of L stay the same

Demo: gyroscope

From warmup: <http://science.howstuffworks.com/gyroscope1.htm>

Ralph watched the video with the bicycle wheel, but became very confused. He had learned that angular momentum is conserved, but in this case isn't the angular momentum of the wheel constantly changing in direction as the wheel spins around. What's up?



“Pair share”—I am now ready to share my neighbor's answer if called on.

a. Yes

Demo: bicycle wheel

Demo: Angular momentum with external torque

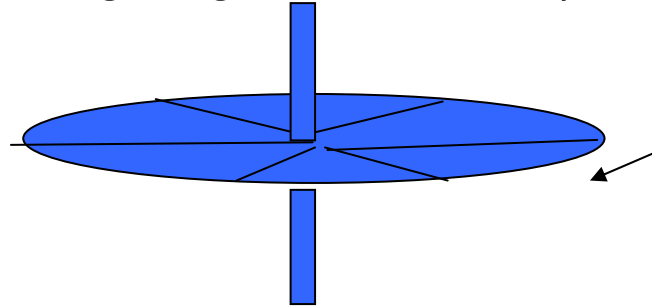
(wacky briefcase)

Demo: briefcase

To fully describe what happens to angular momentum with external torque takes more math than we have... just understand that strange things can happen. 😊

Clicker quiz

José sits still on frictionless ice, holding a bicycle wheel that's already spinning. Viewed from above it is going **clockwise** (CW).

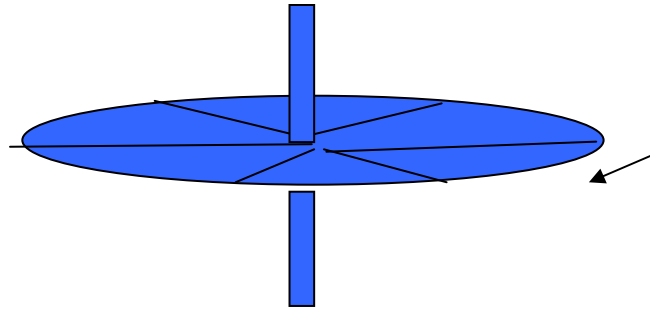


If he grabs on to the wheel edge firmly and stops it from spinning he will:

- a. Start to turn CW (viewed from the top)
- b. Start to turn CCW
- c. Remain sitting without turning

$$\cancel{(L_J + L_w)}_{\text{before}} = \cancel{(L_J + L_w)}_{\text{after}}$$

Clicker quiz



José still on frictionless ice holding this spinning wheel. Viewed from above it is going **clockwise** (CW).

If, instead of stopping the wheel, he carefully turns it over so it is going CCW (viewed from the top), he will start to:

- a. Turn CW, but slower than in the previous problem
- b. Turn CCW, but slower than in the previous problem
- c. Turn CW, but faster than in the previous problem
- d. Turn CCW, but faster than in the previous problem
- e. Remain sitting without turning

$$(\cancel{L_J} + L_W)_{\text{bef}} = (L_J + \underbrace{(L_W)}_{\text{negative}})_{\text{aft}}$$
$$L_J_{\text{aft}} = 2L_W$$

Demos: rotating platform, bicycle wheel

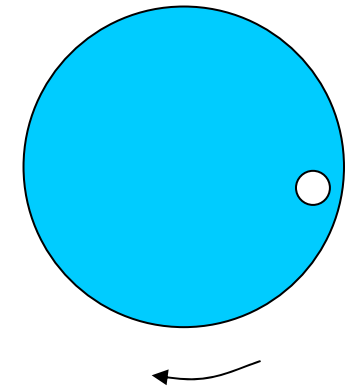
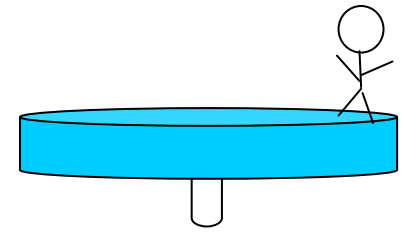
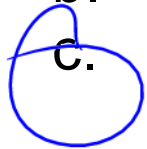
Demo: double bicycle wheels

Clicker quiz

What will happen to the **rotational speed ω** of the merry-go-round if the girl...

...walks towards the center?

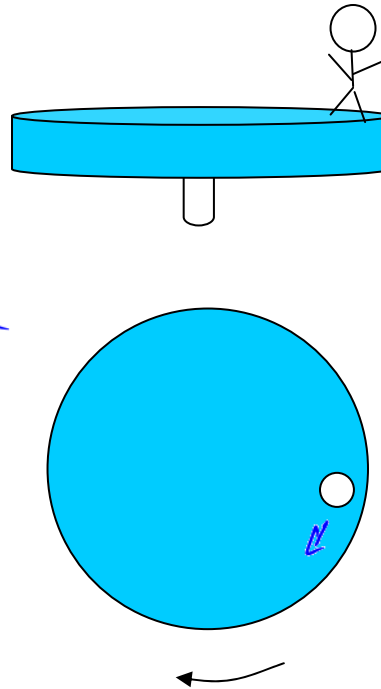
- a. it slows down
- b. it stays same speed
- c. it speeds up



Clicker quiz

$$L_G + L_{MGR} = \cancel{L_G} + L_{MGR}$$

↑

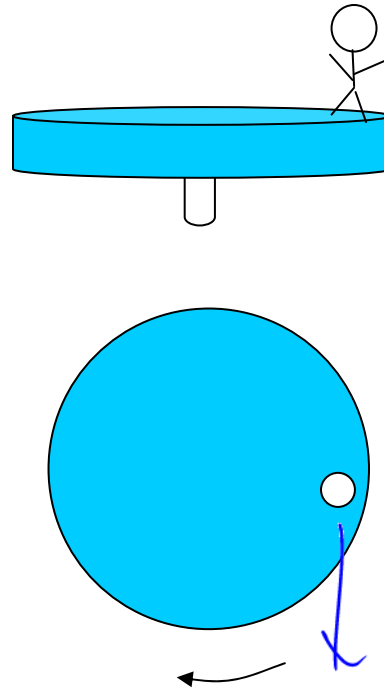


...starts running opposite to the spinning so she is at rest vs the ground?

- a. it slows down
- b. it stays same speed
- c. it speeds up

HINT: Sometimes it's easier to think of the **forces (torques)** she puts on the merry-go-round to change, rather than conservation of L.

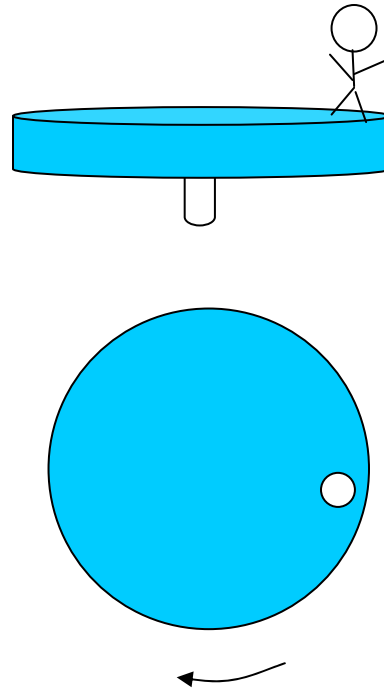
Clicker quiz



...slips off when she steps on a frictionless icy part?

- a. it slows down
- b. it stays same speed
- c. it speeds up

Clicker quiz



...throws her shoe off tangentially in the direction she's moving?

- a. it slows down
- b. it stays same speed
- c. it speeds up