Announcements – 12 Nov 2013

→ Happy 11/12/13!

1. Exam 4 not that far away…
   a. Starts a week from Thursday! Yikes!
   b. Evening TA exam review voting soon

Concerts:

- Thurs 11/14 Acappellooza 7pm, 251 Tanner
                          $2, or 2 for $3

- Sun 11/17 Utah Baroque Ensemble, Free
                          7³⁰ pm, 600 N 800 E Orem

- Thurs 11/21 Acappella Jam 8 pm, JSB auditorium
                          $5 at door, $4 in advance
                                      (from Wilk Info Desk)

Review: Ideal Gas Law

molecules

Boltzmann's

$$PV = Nk_BT$$

$$PV = nRT$$

$R = 8.31 \dfrac{J}{mol \cdot K}$

moles

Worked Problem

How much volume will 1 liter of liquid nitrogen fill when it becomes gas?

Density of LN = 0.807 g/cm^3

Molar mass of N_2 = 28 g/mol

Temperature in this room = about 70° F   (=294.3 K)

Atmospheric pressure in Provo = 0.85 atm

Answer: 821 L

Worked Problem

What is the mass of all the air in this room? The average molar mass of the molecules in air (mainly nitrogen and oxygen) is 29.0 g/mol.

$$V = 22\,m \times 15.6\,m \times 30\,m$$

$$PV = nRT$$

$$\uparrow$$

$$\frac{m}{MM}$$

$$m = \frac{P \cdot V \cdot MM}{RT} = \frac{(.85 \cdot 1.01 \cdot 10^5\,Pa)(22 \cdot 15.6 \cdot 30\,m^3)(.029\,\frac{kg}{mol})}{(8.31\,\frac{J}{mol \cdot K})(294\,K)}$$

$$= \boxed{10492\,kg}$$

Answer: more than you'd expect!

Worked Problem

Use the ideal gas law to determine the density of air at 1 atm and 80° F (300K). (MM$_{air}$ = 29 g/mol)
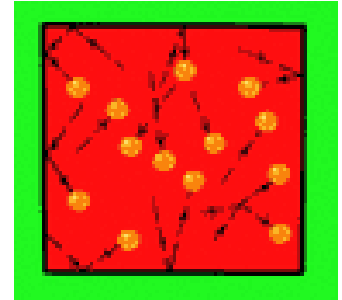
$$PV = nRT$$

$$n \uparrow = \frac{m}{MM}$$

$$\frac{m}{V} = \frac{P \cdot MM}{RT}$$

$$= \frac{(1.01 \cdot 10^5 \, Pa)(.029 \, kg/mol)}{(8.31 \, \frac{J}{mol \cdot k})(300k)}$$

$$= \boxed{1.175 \, kg/m^3}$$

Answer: 1.175 kg/m³

Molecular View of Pressure

Pressure: Comes from collision forces of molecules hitting wall

Related problem: You throw baseballs (mass 145 g) at a wall (area 9 m^2), at a speed of 85 mph (38 m/s). The collisions are elastic, and last for 0.05 seconds. (This is the time the ball is in contact with the wall.) A baseball hits the wall every 0.5 seconds. How much average pressure is generated by the baseballs? (Do this in three steps)

(a) How much force is generated by each hit? (Use impulse)

$$F \Delta t = \Delta p \quad \rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{mv - (-mv)}{t_{collision}}$$

$$= \frac{2(.145 kg)(38 m/s)}{.05 \, sec}$$

$$= \boxed{220 \, N}$$
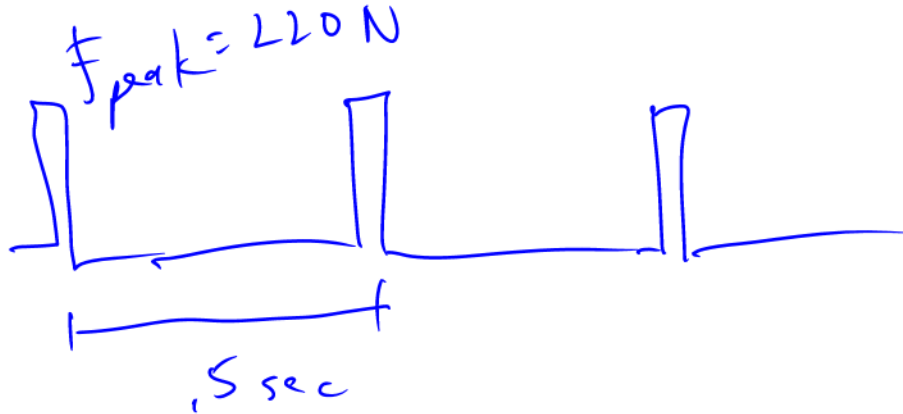
Answer: 220.4 N

.05 sec

(b) How much force is there, on average? $\boxed{22N}$

$F_{peak} = 220\,N$

.5 sec

$$F_{ave} = F_{peak} \cdot \dfrac{t_{collision}}{t_{between}}$$

(c) How much overall pressure is generated by the balls?

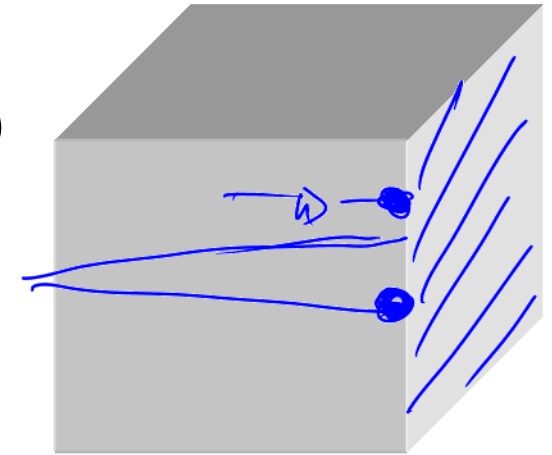$$P = \dfrac{F_{ave}}{area} = \dfrac{22N}{9\,m^2} = \boxed{2.449\,Pa}$$

Answers: 22.04 N; 2.449 Pa

$$v_x = \frac{x}{t} \rightarrow t = \frac{x}{v_x} = \frac{2L}{v_x}$$

The actual problem

A cube filled with gas (focus on x-direction for now)

Molecules (mass *m*) hit the right wall, at a speed of v_x. Elastic collisions. How much pressure is generated by the molecules? (Do this in four steps)

$$L$$

(a) How much force is generated by each hit?

$$F \, \Delta t = \Delta p \rightarrow F = \frac{\Delta p}{\Delta t} = \frac{mv_x - (-mv_x)}{t_{collision}} = \boxed{\frac{2mv_x}{t_{collision}}}$$

(b) How much force is there from one molecule, on average?

$$F_{ave} = F_{peak} \cdot \frac{t_{collision}}{t_{between}} = \left(\frac{2mv_x}{t_{collision}} \right) \left(\frac{t_{collision}}{2L/v_x} \right) = \boxed{\frac{mv_x^2}{L}}$$

Answers: $\dfrac{2mv_x}{time\ of\ collision}$; $\dfrac{mv_x^2}{L}$

(c) How much pressure is generated by the ~~molecules~~ one molecule?

$$P = \frac{F_{ave}}{area} = \frac{mv_x^2/L}{L^2} = \frac{mv_x^2}{L^3} = \boxed{\frac{mv_x^2}{Vol.}}$$

(d) Expand to N molecules, and 3 dimensions ($v_x = v_y = v_z$). P = ?

$$P_{tot} = N \cdot P_{one\ molecule}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v^2 = 3 v_x^2$$

$$v_x^2 = \boxed{\frac{1}{3} v^2}$$

$$\boxed{P_{tot} = \frac{N \cdot m \cdot \frac{1}{3} v^2}{Vol}}$$

Answers: $\frac{mv_x^2}{V}$; $\frac{Nm\left(\frac{1}{3}v^2\right)}{V}$

$$PV = Nm\left(\tfrac{1}{3}v^2\right)$$

→ Does this look familiar?

Compare to: $\underline{PV = N\,k_B T}$

$$\tfrac{3}{2} \cdot \tfrac{1}{3}mv^2 = k_B T \,\tfrac{3}{2}$$

$$\tfrac{1}{2}mv^2 = \tfrac{3}{2}k_B T$$

$$T = \frac{m}{k_B} \frac{1}{3} v^2$$

$$\rightarrow \frac{1}{2} m v^2 = ? \quad \frac{3}{2} k_B T$$

I will give you this equation on exam. Use it to solve for v!

Kinetic Energy

one molecule

$KE_{tot} = \dfrac{3}{2} k_B T$

$KE_x = \dfrac{1}{2} k_B T$

$KE_y = \dfrac{1}{2} k_B T$

$KE_z = \dfrac{1}{2} k_B T$

"Degrees of freedom"

total for all molecules

$= \dfrac{3}{2} N k_B T$

$= \dfrac{3}{2} n R T$

Equipartition Theorem

Only briefly mentioned in your book! And not by name!
(see page 390, Section 12.2 in 8th edition)

"The total kinetic energy of a system is shared equally among all of its <u>independent parts</u>, on the average, once the system has reached thermal equilibrium."

Specifically, each "degree of freedom",
of each molecule, has "thermal energy" of: $\underline{\quad \frac{1}{2} k_B T \quad}$ $per\ molecule$

<u>"independent parts"</u>: larger for molecules that can
- rotate
- vibrate

(requires more than one atom)
→ **such molecules have more "internal energy"**

Translational kinetic energy

Three independent directions

transl. KE   3 translational deg. of freedom

Result:

$$KE_{ave} = \frac{1}{2}mv_{ave}^{2} = \frac{3}{2}k_{B}T$$

Rotational kinetic energy

We'll revisit this later

From warmup

An ideal gas has a mixture of heavy and light molecules at the same temperature. The molecules with the most [translational] KE are…

    a. heavy
    b. light
    c. same

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

KE

Demos

Kinetic theory machine

Molecular speed

Worked Problem

How fast are the oxygen molecules traveling in this room? (300 K)

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT$$

$$v = \sqrt{\frac{3k_BT}{m}}$$

$$m = \frac{\text{Mass of 1 mole}}{\text{\# molecules in a mole}}$$
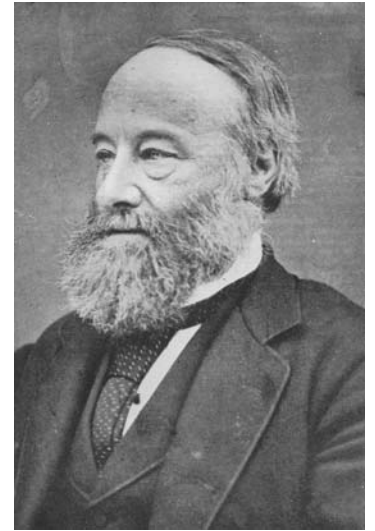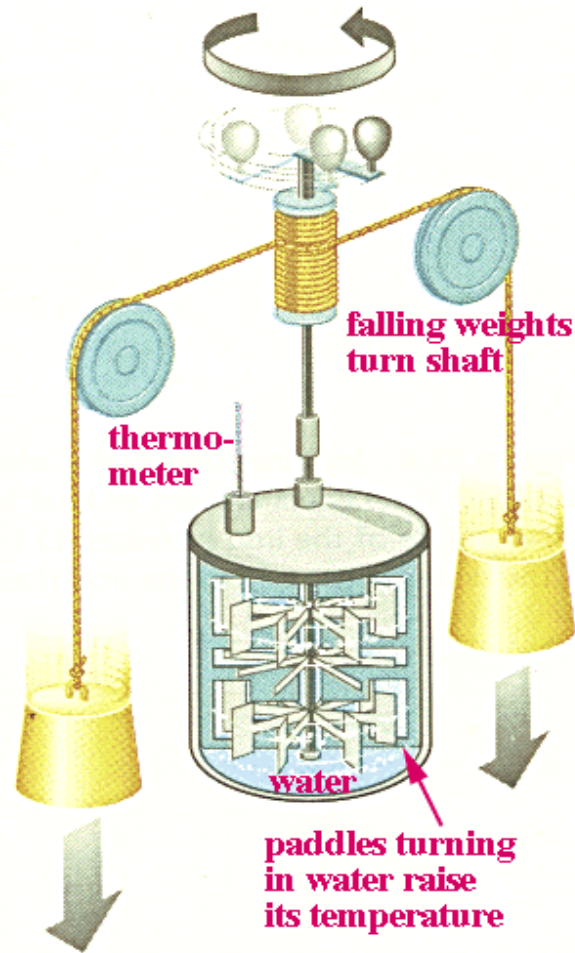
$$= \frac{.032 \text{ kg}}{6.022 \cdot 10^{23}}$$

$$= \sqrt{\frac{3 \cdot \left(1.38 \cdot 10^{23} \frac{J}{K}\right)(300 K)}{\left(.032 \text{ kg} / 6.022 \cdot 10^{23}\right)}}$$

$$= \boxed{483 \text{ m/s}}$$

Answer: 483.46 m/s (= 1081 mph!)

Heat

Heat is <u>random kinetic energy</u>!

Symbol: Q
Units: Joules

"Mechanical equivalent of heat":
James Joule 1849

falling weights turn shaft

thermo-meter

water

paddles turning in water raise its temperature
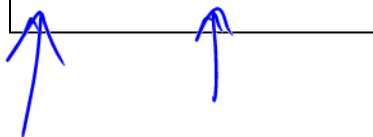
calories vs. Calories

1 calorie = 4.186 J

Food calorie: 1 Cal
                = a *kilocalorie*

"Specific heat"

How much does T rise when heat energy is added?

- temperature rise is proportional to heat added
- the more mass… the less the temperature rises
- material dependent

$$Q = mc\Delta T$$

c = "specific heat" (closely related to "heat capacity")

$m\,c$ sometimes called "thermal mass"

TABLE 11.1

Specific Heats of Some Materials at Atmospheric Pressure

Substance	J/kg·°C	cal/g·°C
Aluminum	900	0.215
Beryllium	1 820	0.436
Cadmium	230	0.055
Copper	387	0.0924
Germanium	322	0.077
Glass	837	0.200
Gold	129	0.0308
Ice	2 090	0.500
Iron	448	0.107
Lead	128	0.0305
Mercury	138	0.033
Silicon	703	0.168
Silver	234	0.056
Steam	2 010	0.480
Water	4 186	1.00

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Question:

If you add 5 J of heat to a mass of water, and 5 J of heat to the same mass of copper, which one increases the most in temperature?

   a. Water

   b. Copper

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Lead	128	0.0305
Mercury	138	0.033
Silicon	703	0.168
Silver	234	0.056
Steam	2 010	0.480
Water	4 186	1.00

From warmup

The fact that desert sand is very hot in the day and very cold at night is evidence that sand has a:
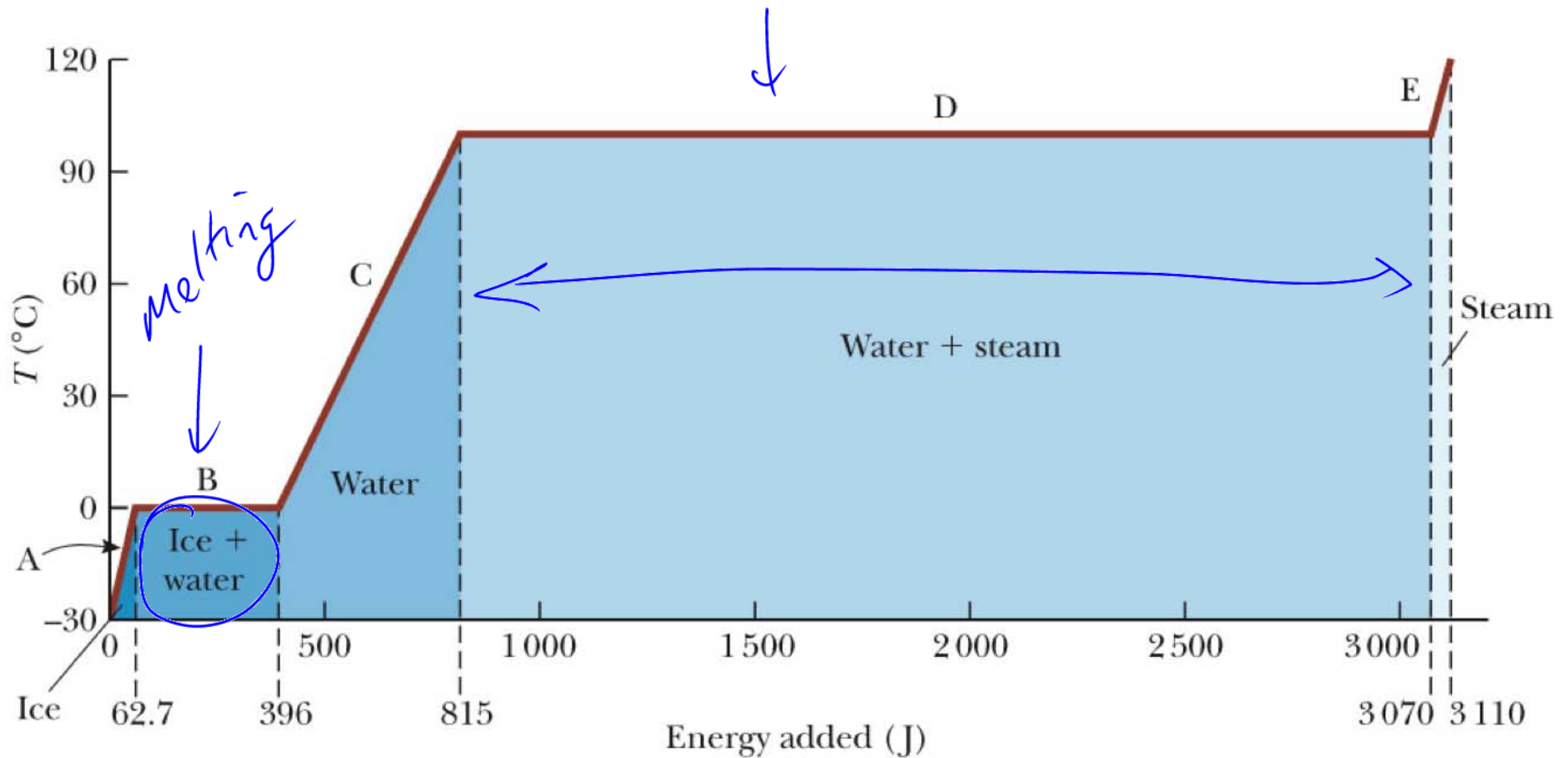
    a. low specific heat
    b. high specific heat

Phase Changes

From warmup: Thermal energy that is used to melt or freeze something is called:

    a.   latent heat

    b.   specific heat

    c.   thermal mass

Phase Changes

boiling

During phase change, no *T* increase
→ but heat still needed to complete the phase change
→ both phases co-exist

Latent Heat Equation

$$Q = mL$$

L depends on
- Material
- Type of phase change (i.e. solid-liquid, liquid-gas, or other)

Water:

$L_{melting/freezing} = 3.33 \times 10^5$ J/kg   333000

$L_{boiling/condensing} = 2.26 \times 10^6$ J/kg

2,260,000

Clicker quiz

If you want to melt a cube of ice that's initially at -40°C, which part takes the most energy?

a. Raising the temperature to 0°C

b. Converting from solid to liquid phase

c. Same energy

$Q = mc\Delta T \leftarrow (4186)(40)$

$\approx 160{,}000$

$Q = mL$

Water:

$c = 4186 \text{ J/(kg °C)}$

$L_{melting/freezing} = 3.33 \times 10^5 \text{ J/kg}$

333000

From warmup

Ralph's professor stated "If you add an ice cube to a glass of water, the temperature of the water does not necessarily decrease." That seems bizarre to him, because ice is obviously used to cool down water! Can you help him understand what his professor may have been talking about?

"Pair share"–I am now ready to share my neighbor's answer if called on.

    a. Yes

Calorimetry
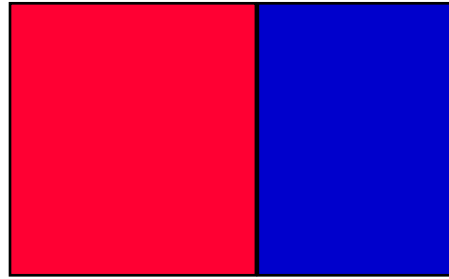
Conservation of energy

$$Q_{gained\,by\,cold\,objects} = Q_{lost\,by\,hot\,objects}$$

Blueprint!

(assuming no heat flow to outside)

→ On both sides of equation use only *positive* quantities

(absolute values)

→ Don't forget melting and boiling *mL* terms if needed
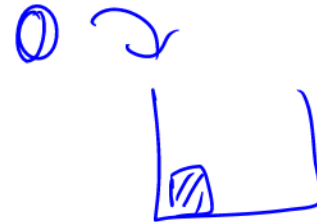
My method vs. book's method

Worked Problem

$L = 333000$ J/kg

$C_{ice} = 2090$ J/kg·°C

0.2 kg of iron at 100° C is added to an insulated container with 0.2 kg of ice at -10° C. How much ice melts if they come to equilibrium at 0° C?
(Ref: $c_{iron} = 448$ J/kg·°C)

Start with: $Q_{gained\ by\ ice} = Q_{lost\ by\ iron}$

$$\left(mc\Delta T\right)_{ice\ raising\ to\ 0°C} + m_{unknown} L = \left(mc\Delta T\right)_{iron\ cooling\ to\ 0°C}$$

$$(.2)(2090)(0 - -10) + m_{unknown}(333000) = (.2)(448)(100-0)$$

$$\uparrow$$
$$10$$

$$\uparrow$$
$$100$$

Solve for $m_{unknown}$

$$= .01435\ kg$$

Answer: 14.35 g

Worked Problem

5 g of hot iron at 300° C is added to 100 g of water at 30° C. What is the final temperature?

Answer: 31.44° C

Worked Problem

500 g of hot iron at 300° C is added to 100 g of water at 30° C. What is the final temperature?

Answers: 124.1 (not real answer), -395.3° C (not real answer), 100° C

Clicker quizzes (review)

1. Which molecules have the most kinetic energy?
   a. The heavy ones
   b. The light ones
   c. Same

2. Which molecules have the fastest speed?
   a. The heavy ones
   b. The light ones
   c. Same