Announcements – 7 Oct 2014

Exam curve – very probably at least 5 points

1. Prayer

2. Energy review…

$E_{before} + W = E_{after}$

"Conservation of energy"

W includes work by all *nonconservative* forces

E includes both kinetic and potential energies of all objects

$W = F_{//}d$

$KE = \frac{1}{2}mv^2$

$PE_{gravity}$ (near surface of Earth) $= mgy$

y is measured relative to where you define y=0

$PE_{gravity}$ (otherwise) $= -GmM/r$

r is measured from center to center

$PE_{spring} = \frac{1}{2}kx^2$

x is measured from equilibrium (relaxed) position of spring

Also remember $F_{spring} = kx$, if needed in a N2 problem

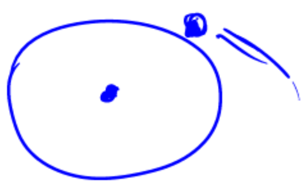
“Which of the problems from last night's HW assignment would you most like me to discuss in class today?”

Worked problem

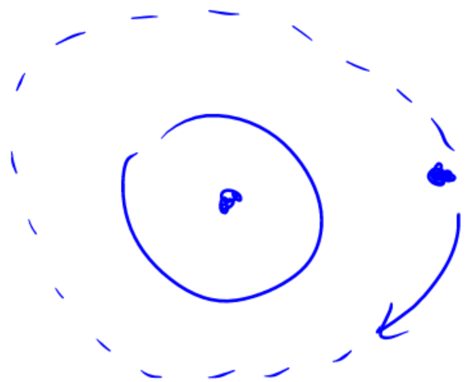
$$PE_g = -\frac{GMm}{r}$$

How much energy would you have to provide in order to “shoot” a 100 kg satellite into a near orbit like the ISS, 6712 km from center of earth? (E.g. via initial KE)

bef



after



$$E_{bef} + \cancel{W} = E_{aft}$$

$$KE_i + PE_i = KE_f + PE_f$$

$$KE_i - \frac{GMm}{r_E} = \frac{1}{2} m v_{orb}^2 - \frac{GMm}{r_{orb}}$$

orb. velocity?

$$\Sigma F = ma_c$$

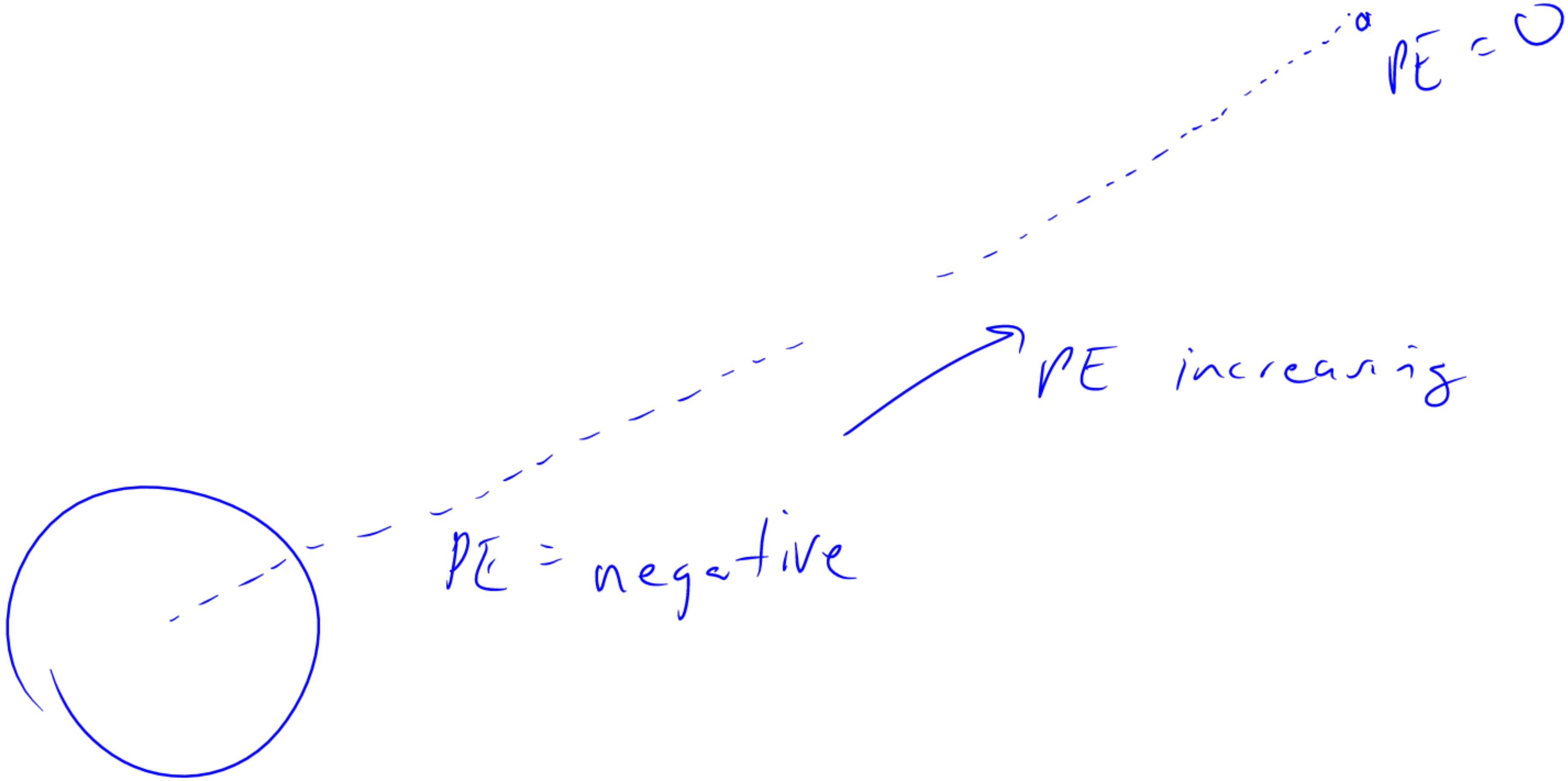
$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$v_{orb} = \sqrt{\frac{GM_{earth}}{r_{orb}}}$$

$$KE_i = \frac{GMm}{r_E} - \frac{GMm}{r_{orb}} + \frac{1}{2} m \underbrace{\left(\frac{GM_e}{r_{orb}}\right)}_{v_{orb}^2}$$

$$= \boxed{3.29 \cdot 10^9 \text{ J}}$$

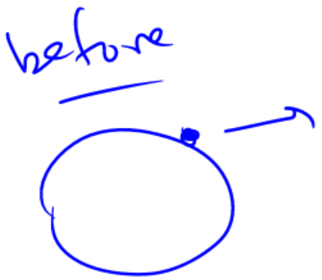
Answer: 3.29E9 J



$$PE = -\frac{GMm}{r}$$

Escape velocity

How fast do you have to shoot an object to get it infinitely far away form the Earth? R_{earth} = 6371 km; M_{earth} = 5.974 × 10^{24} kg (ignore the sun's gravitational pull)



$$E_{bef} + \cancel{W} = E_{aft}$$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_{earth}} = 0 + 0$$

$$\frac{1}{2} \cancel{m} v_0^2 = \frac{GM\cancel{m}}{r_{earth}}$$

$$\boxed{v_0 = \sqrt{\frac{2GM}{r_{earth}}}}$$

$$= \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{6371 \cdot 10^{3}}}$$

$$= \boxed{11{,}200\ m/s}$$

Compare to orbital velocity

$$v_{orb} = \sqrt{\frac{GM}{r_{earth}}}$$

Robert Heinlein:

"If you can get into orbit, then you're halfway to anywhere"

Answers: 11.2 km/s; 7.9 km/s

Power!

The rate at which energy is produced or consumed

$P = \frac{E}{t}$

Or… (equivalently)
Power is the rate at which work is being done

$\frac{W}{t}$ $\frac{J}{sec}$

$= \text{Watt}$

SI units: 1 Watt = 1 J/s = 1 N·m/s = 1 kg·m^2/s^3 (yuck!)

For reference: 1 horsepower (hp) = 745.7 W

$W = F_{\parallel} d$

$P = \frac{W}{t}$

$= F_{\parallel} \cdot \frac{x}{t}$

v

$= \boxed{F_{\parallel} v}$

Another useful formula, if constant velocity:

$$\boxed{P = F_{//} v}$$

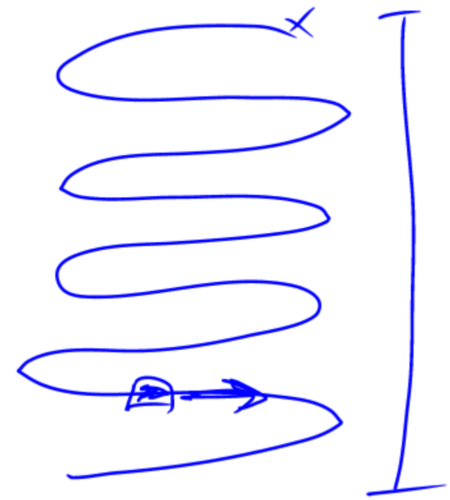


Image credit: Wikipedia. (Northern end of Stelvio Pass, Italy)

From warmup: Switchbacks on mountain roads (consider only work done against gravity):

a. increase the work needed to go up a mountain

b. decrease the work needed to go up a mountain

c. keep the work needed the same

Power: decreased

$\cancel{E_{bef}} + W = E_{aft}$

$W = PE_f$

Experimental Problem

How much "horsepower" can a person generate?

Experiment: jumping from a stand-still $\rightarrow$ Volunteer needed!

Parameters:

mass (kg) = 82 kg

measured height jumped (m) = .61 m

measured "impulse time" (s) (time while legs are exerting force on ground)

= .2 s

What was the work done by his/her body during the impulse time?

$W = mgy = (82\,\text{kg})(9.8\,\text{m/s}^2)(.61\,\text{m}) = 490.2\,\text{J}$

How much horsepower?

$P = \frac{490.2\,\text{J}}{.2\,\text{s}} = 2451\,\text{W} \times \frac{1\,\text{hp}}{745.7\,\text{W}}$

$= \boxed{3.3\,\text{hp}}$

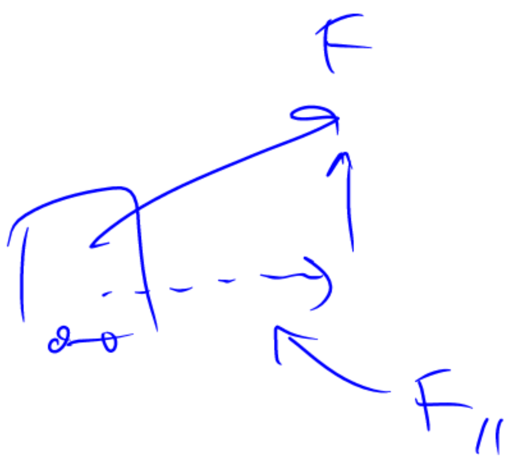
Clicker quiz

A car weighing 3000 N moves at a speed of 30 m/s on level ground. To do this, it pushes backwards on the road with a 5000 N force. What is the power output of the car engine?

a. 0 kW
b. 60 kW
c. 90 kW
d. 150 kW
e. 240 kW

$$P = F_{\parallel} \cdot v$$

$$= (5000\,N)(30\,m/s)$$



Where are we now?

<u>Topics</u>

Kinematics (velocity, acceleration)
Vectors & 2D Motion
Forces & Newton's Laws
Work & Energy
Momentum
Rotations, Torque, and Angular Momentum
Pressure
Fluids & Solids
Temperature, Heat, and Heat Flow
Laws of Thermodynamics
Vibrations & Waves

Mechanics

Thermodynamics

Part Mechanics,
Part Sound
Part Optics

Conserved quantities

Energy

→ When no non-conservative work done, $E_{bef} = E_{aft}$

Mass

→ If not converted to/from energy ($E=mc^2$),

(total mass)$_{bef}$ = (total mass)$_{aft}$

Charge

→ (total charge)$_{bef}$ = (total charge)$_{aft}$

I.e., if some positive charge flows out of a neutral object, it will leave the object with negatively charged

<u>Often conserved (used to e.g. balance chemical reactions)</u>

Number of each type of atom

Number of electrons

Etc.

A new conserved quantity… **momentum**

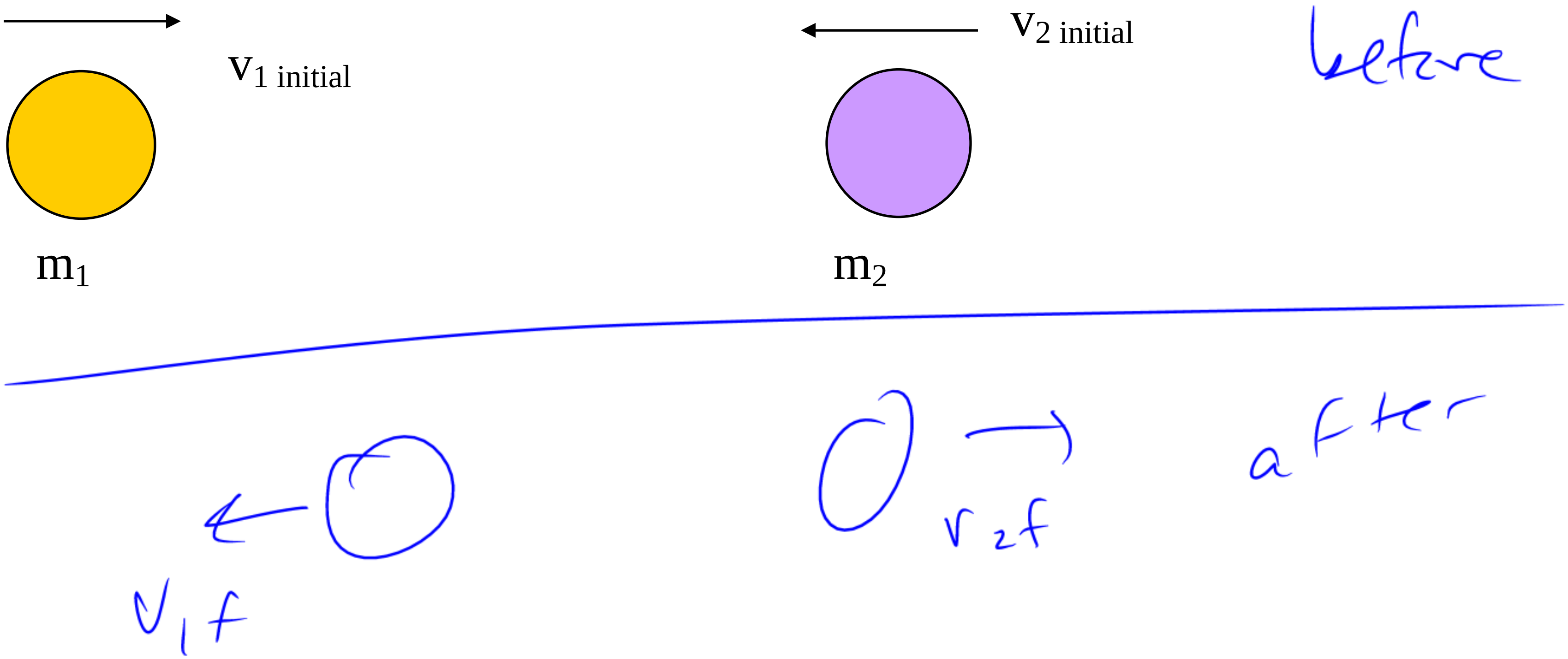
Define $\boxed{\vec{p} = m\vec{v}}$ for each object, then

$$\boxed{\sum \vec{p}_{before} = \sum \vec{p}_{after}}$$ (if no external forces)

Another blueprint equation!

Careful: Momentum is a vector!

Momentum: used for Collision Problems



$$\Sigma P_{bef} = \Sigma P_{after}$$

$$m_1 v_1 - m_2 v_2 = -m_1 v_{1f} + m_2 v_{2f}$$

Derivation of conservation law:

$$\Sigma\vec{F}_1 = m_1\vec{a}_1 \qquad\qquad \Sigma\vec{F}_2 = m_2\vec{a}_2$$

$-F_{21}$

$F_{21} = m_1 a_1 \quad = \quad F_{12} = m_2 a_2$

Newton's 3rd Law: the forces involved in the collision are equal and opposite

If no other forces, then…

$$\mathbf{F}_{2\text{-}1} + \mathbf{F}_{1\text{-}2} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2$$

$$0 = m_1\Delta\mathbf{v}_1/\Delta t + m_2\,\Delta\mathbf{v}_2/\Delta t$$

Multiply by Δt (which is the same for both)

$$m_1\Delta\mathbf{v}_1 + m_2\,\Delta\mathbf{v}_2 = 0$$

$$m_1(\mathbf{v}_{1\text{ final}} - \mathbf{v}_{1\text{ initial}}) + m_2(\mathbf{v}_{2\text{ final}} - \mathbf{v}_{2\text{ final}}) = 0$$

$$m_1\mathbf{v}_{1\text{ initial}} + m_2\,\mathbf{v}_{2\text{ initial}} = m_1\,\mathbf{v}_{1\text{ final}} + m_2\,\mathbf{v}_{2\text{ final}}$$

… and there you have it!

From warmup

no outside forces

The total momentum of an isolated system of objects is conserved

a. only if conservative forces act between the objects

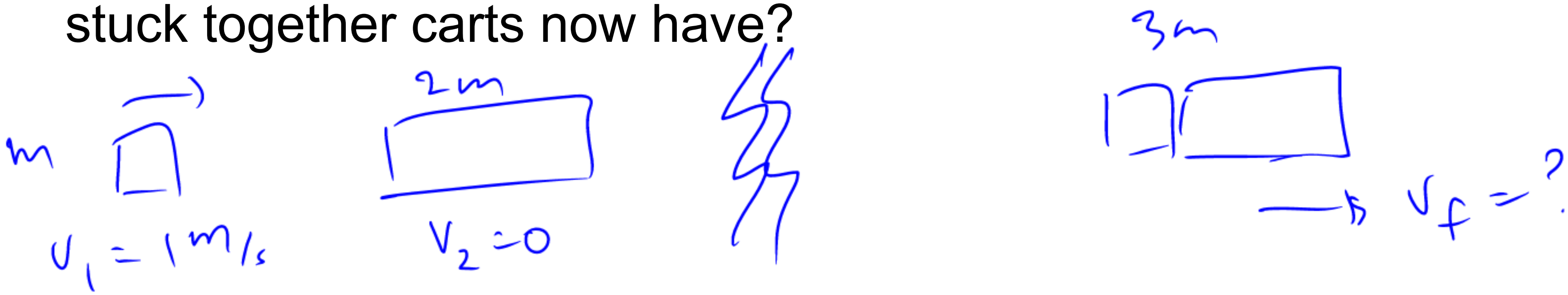
b. regardless of the nature of the forces between the objects.

Why use conservation of momentum?

Demo Problem: A cart moving at 1 m/s runs into a second cart (stationary) with the same mass and sticks to it. What velocity do the two stuck together carts now have?

Sorry

Demo Problem: A cart moving at 1 m/s runs into a second cart (stationary) with *twice* the mass and sticks to it. What velocity do the two stuck together carts now have?



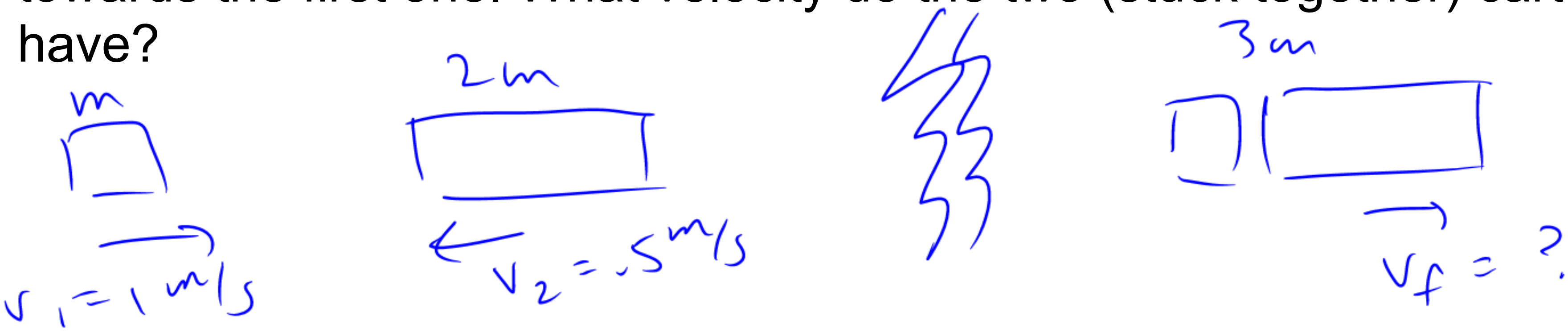
$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$m v_1 = (3m) v_f$$

$$v_f = \frac{1}{3} v_1$$

$\frac{1}{3}$ m/s

Demo Problem: A cart moving at 1 m/s runs into a second cart with *twice* the mass and sticks to it. The second cart is moving at 0.5 m/s towards the first one. What velocity do the two (stuck together) carts now have?



$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\frac{\cancel{m} v_1 - 2\cancel{m} v_2}{3} = \frac{(\cancel{3}\cancel{m}) v_f}{\cancel{3}}$$

$$v_f = \frac{1\text{ m/s} - 2(.5\text{ m/s})}{3} = \boxed{0\text{ m/s}}$$

Dr Colton's Guide:
How to solve Conservation of Momentum problems

1. Draw initial and final pictures

2. Draw *momentum* or *velocity* vectors (arrows) in each picture

3. Use $\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after}$ as "blueprint equation"

4. Divide into separate x- and y- equations if needed

5. Fill in both sides of blueprint equation(s) using initial and final pictures: one term in equation for each arrow in picture.

6. Reminder: be careful with signs! (Momentum is a **vector**)

The new blueprint

$$\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after} \quad \text{(if no external forces)}$$

Compare to previous two blueprint equations:

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$E_{before} = E_{after} \quad \text{(if no non-conservative forces)}$$

Similarities? Differences?

From warmup

Suppose Ralph is floating in outer space with no forces acting on him. He is at rest, so his momentum is zero. Now, he throws a ball. The ball goes one way, and he goes the other way. Before the collision, there was no momentum, and after the collision, there is plenty of momentum! Was momentum conserved?

“Think-pair-share”

- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

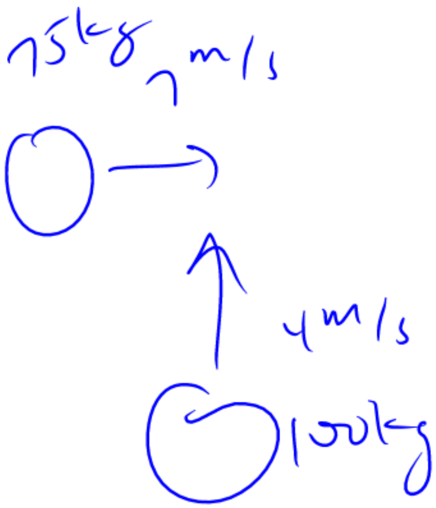
Clicker: I am now ready to share my answer if randomly selected.

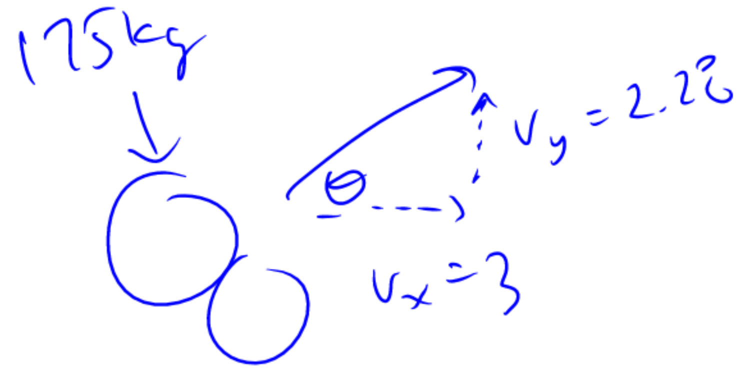
a. Yes

Note: you are allowed to "pass" if you would really not answer.

Worked Problem

In the new sport of “ice football”, a 100 kg defensive end running north at 4 m/s tackles a 75 kg quarterback running *east* at 7 m/s. There’s no friction. What is their combined velocity right after the tackle?





$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\sum p_{x\,bef} = \sum p_{x\,aft}$$

$$\sum p_{y\,bef} = \sum p_{y\,aft}$$

$$(75\text{ kg})(7\text{ m/s}) = (175\text{ kg})v_x$$

$$v_x = 3\text{ m/s}$$

$$(100\text{ kg})(4\text{ m/s}) = (175\text{ kg})v_y$$

$$v_y = 2.28\text{ m/s}$$

$$v_f = \sqrt{3^2 + 2.28^2} = \boxed{3.77\text{ m/s}}$$

$$\tan\theta = 2.28/3 \rightarrow \boxed{\theta = 37.3^\circ}$$

Answers: v_x = 3 m/s; v_y = 2.28 m/s; v = 3.77 at 37.3° north of east

Worked Problem

An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

Answers: v_x = 250 m/s; v_y = -75 m/s; v = 261 m/s at 16.7° south of east

Limitation

Like conservation of energy, conservation of momentum is a “before” and “after” law which doesn’t tell you about: ___________________________

If you want to know about ________, you have to know _______

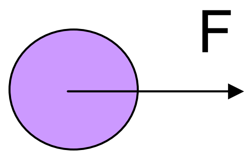
Impulse

Concept: A force exerted for a brief time causes a change in momentum

$$\boxed{\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}}$$

"Impulse equation" (focusing on one object)

Derivation:



$$\Sigma\vec{F} = m\vec{a}$$

$$F = m\frac{\Delta v}{\Delta t}$$

$$F\Delta t = m\Delta v$$

$$F\Delta t = \Delta p$$

When to use?

Worked Problem

Two steel balls (same mass, m = 0.050 kg) bounce off of each other. High speed photography reveals that the two balls are in contact for 0.015 s. Before the collision, the left ball is traveling to the right at 2 m/s and the right ball is traveling to the left at 2 m/s. After the collision, the left ball is traveling to the left, still at 2 m/s, and the right ball is traveling to the right at 2 m/s.

What was the contact force during the collision?

Answer: 13.33 N

From warmup, do as clicker quiz

A ping-pong ball moving forward with a momentum *p* strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

a. greater than *p*
b. less than *p*
c. equal to *p*

What about if ping-pong ball “thuds” and falls flat?

Demo: Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?