Announcements - 9 Oct 2014

1. Prayer

2. **Exam 2 results**
   a. Median Score: As in the email yesterday, your score on Max includes the curve.
   b. Curve:
   c. Exams will be returned soon, our office assistant should put them in the boxes near N357 ESC sometime today.
   d. I'll post my solutions to the class website soon (probably this afternoon)

3. If you have questions about the exam that you'd like to discuss with me:
   a. Please look over your own exam & the posted solutions to figure things out as best you can, before coming to talk to me.
   b. Talking to me during my office hours is best
   c. Individual appointments can also be made

"Which of the problems from last night's HW assignment would you most like me to discuss in class today?"

Momentum Review

<u>Equations</u>

Definition: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Conservation Law (blueprint): $\sum \vec{\mathbf{p}}_{before} = \sum \vec{\mathbf{p}}_{after}$ (if no …. external forces)

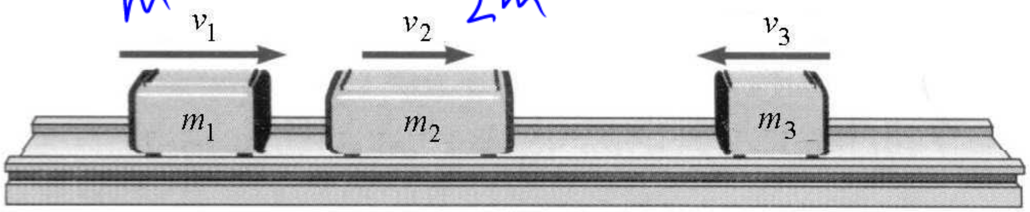
From warmup

To solve 2-dimensional collision problems, you should:

a. treat momentum as a vector
b. treat momentum as a scalar

Worked Problem

m 2m m



$v_f \rightarrow$

4m

Three carts collide on an air track. Carts m_1 and m_3 have the same mass, and m_2 has twice the mass. The initial velocities are 1.5 m/s, 1 m/s, and -1 m/s. The three stick together. What is the combined velocity after the collision? (magnitude and direction)

$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\frac{\cancel{m}(1.5\,m/s) + (2\cancel{m})(1\,m/s) - \cancel{m}(1\,m/s)}{4} = 4\cancel{m}(v_f)$$

$$v_f = \frac{1.5 + 2 - 1}{4}\ m/s$$

$$= \boxed{.625\ m/s}$$

Answer: 0.625 m/s, to the right

Limitation

Like conservation of energy, conservation of momentum is a "before" and "after" law which doesn't tell you about: time

If you want to know about time, you have to know

forces /

accelerations

kg · m/s

Impulse

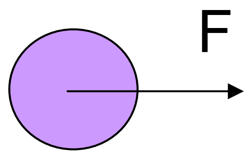
Concept: A force exerted for a brief time causes a change in momentum
The "impulse" is the force × time.

N · s

$$\boxed{\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}}$$

"Impulse equation" (focusing on one object)

Derivation:



$$\Sigma\vec{F} = m\vec{a}$$

$$F = m\frac{\Delta v}{\Delta t}$$

$$F\Delta t = m\Delta v$$

$$F\Delta t = \Delta p$$

When to use?

Worked Problem

Two steel balls (same mass, m = 0.050 kg) bounce off of each other. High speed photography reveals that the two balls are in contact for 0.015 s. Before the collision, the left ball is traveling to the right at 2 m/s and the right ball is traveling to the left at 2 m/s. After the collision, the left ball is traveling to the left, still at 2 m/s, and the right ball is traveling to the right at 2 m/s.

What was the contact force during the collision?

mass 1

$$\Delta p = p_f - p_i$$

$$F \cdot \Delta t = \Delta p$$

$$F \cdot (.015\,s) = -(.050\,kg)(2\,m/s) - (.05\,kg)(2\,m/s)$$

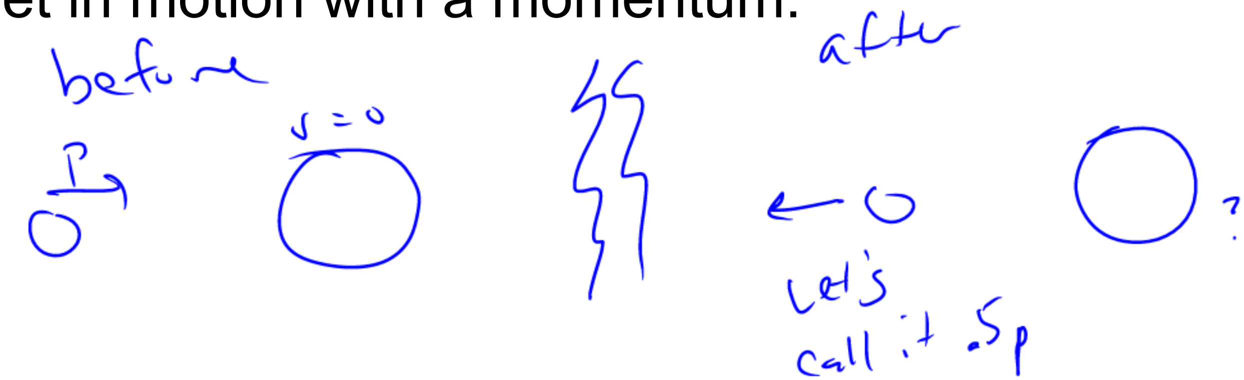
$$F = \frac{-.05 \cdot 2 - .05 \cdot 2}{.015}\,N = \boxed{-13.3\,N}$$

Answer: 13.33 N

From last lecture's warmup, do as clicker quiz

A ping-pong ball moving forward with a momentum *p* strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

a. greater than *p*
b. less than *p*
c. equal to *p*



$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$p + 0 = -.5p + p_{tennis}$$

$$p_{tennis} = 1.5p$$

What about if ping-pong ball "thuds" and falls flat?

$$p_{tennis} = p$$

Demo: Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?

Question

Is energy conserved in collisions? All? Some? None?

Special Case: “Elastic” Collisions

In some special collisions, energy is also conserved!

Elastic collisions: no lost kinetic energy

→ they are “bouncy”

(but not all bouncy-looking collisions are elastic… don’t assume)

Inelastic collisions: energy not conserved
kinetic
→ some energy is lost

Perfectly inelastic collisions: max. amount of KE is lost

→ stick together

From warmup

In addition to momentum, kinetic energy is also typically conserved in collisions.

a. true
b. false

Elastic Collisions

hard

$\Sigma KE_{before} = \Sigma KE_{after}$

$\rightarrow$ This is in addition to $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

The two equations can be put together to give:

$$\boxed{\left(\vec{v}_1 - \vec{v}_2\right)_{before} = \left(\vec{v}_2 - \vec{v}_1\right) after}$$ **"velocity reversal equation"**

easier

Derivation of velocity reversal equation

Cons. mom

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2f} - v_{2i}\right)$$

Cons. energy

$$\tfrac{1}{2} m_1 v_{1i}^{\;2} + \tfrac{1}{2} m_2 v_{2i}^{\;2} = \tfrac{1}{2} m_1 v_{1f}^{\;2} + \tfrac{1}{2} m_2 v_{2f}^{\;2}$$

$$m_1\left(v_{1i}^{\;2} - v_{1f}^{\;2}\right) = m_2\left(v_{2f}^{\;2} - v_{2i}^{\;2}\right)$$

$$m_1\left(v_{1i} + v_{1f}\right)\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2f} + v_{2i}\right)\left(v_{2f} - v_{2i}\right)$$

$2 = 2$ $\qquad$ $3 = 3$

$$\frac{2}{3} = \frac{2}{3}$$

Divide the two equations.

$$\frac{\cancel{m_1}\left(v_{1i} + v_{1f}\right)\cancel{\left(v_{1i} - v_{1f}\right)}}{\cancel{m_1}\cancel{\left(v_{1i} - v_{1f}\right)}} = \frac{\cancel{m_2}\left(v_{2f} + v_{2i}\right)\cancel{\left(v_{2f} - v_{2i}\right)}}{\cancel{m_2}\cancel{\left(v_{2f} - v_{2i}\right)}}$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

Dr. Colton's guide, cont.

#7. If it's a 1D elastic collision then in addition to conservation of momentum, also use…

$$\boxed{\left(\vec{v}_1 - \vec{v}_2\right)_{before} = \left(\vec{v}_2 - \vec{v}_1\right) after}$$

"velocity reversal equation"

Careful with signs! "Right = positive, left = negative" applies

From warmup

Ralph is confused. He's looking at what I call the "velocity reversal equation" in the book. The textbook says "This equation, in combination with conservation of momentum, will be used to solve problems dealing with perfectly elastic head-on collisions." What confuses Ralph, is why won't we need to use the conservation of energy equation? Shouldn't energy conservation be important in elastic collisions? Help him understand why we don't use this: $\frac{1}{2}m_1 {v_{1i}}^2 + \frac{1}{2}m_2 {v_{2i}}^2 = \frac{1}{2}m_1 {v_{1f}}^2 + \frac{1}{2}m_2 {v_{2f}}^2$

"Think-pair-share"

- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

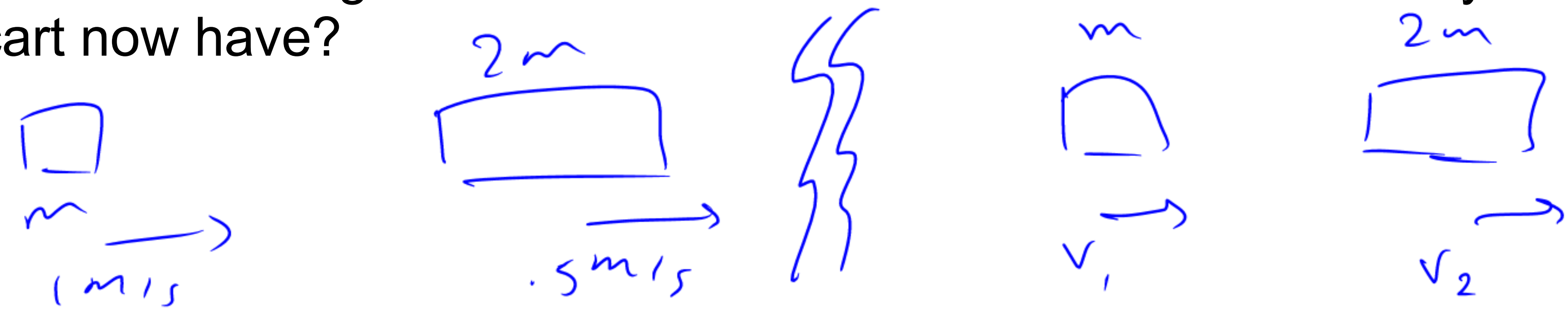
Clicker: I am now ready to share my answer if randomly selected.

a. Yes

Note: you are allowed to "pass" if you would really not answer.

Demo Problem

A cart moving at 1 m/s bounces elastically off of a second cart of twice the mass which is moving at 0.5 m/s in the same direction. What velocity does each cart now have?



$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\cancel{m}(1) + 2\cancel{m}(.5) = \cancel{m} v_1 + 2\cancel{m} v_2$$

$$2 = v_1 + 2v_2$$

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$1 - .5 = v_2 - v_1$$

$$.5 = v_2 - v_1$$

$$v_2 = v_1 + .5$$

$$2 = v_1 + 2(v_1 + .5)$$

$$2 = v_1 + 2v_1 + 1$$

$$1 = 3v_1$$

$$v_1 = .333 \text{ m/s}$$

$$v_2 = .333 + .5$$

$$v_2 = .833 \text{ m/s}$$

Answer: v_1 = 0.333 m/s; v_2 = 0.833 m/s

Demo Problem

A cart moving at 1 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have?

$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\cancel{m}(1) = \cancel{m}v_1 + \cancel{m}v_2$$

$$\boxed{1 = v_1 + v_2}$$

$$1 = v_1 + (1 + v_1)$$

$$1 = 1 + 2v_1$$

$$0 = 2v_1$$

$$\boxed{v_1 = 0}$$

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$1 - 0 = v_2 - v_1$$

$$\boxed{1 = v_2 - v_1}$$

$$v_2 = 1 + v_1$$

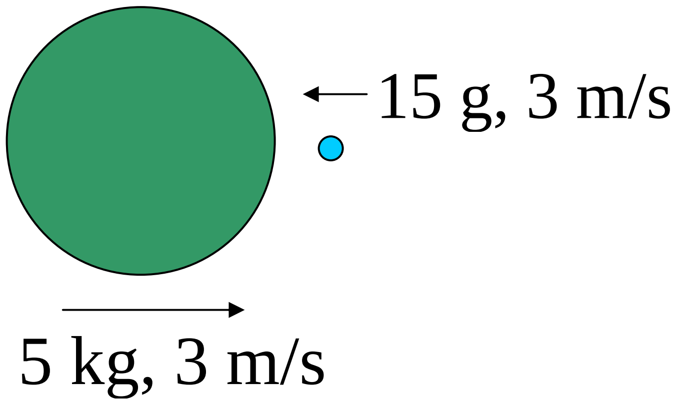
$$v_2 = 1 + 0$$

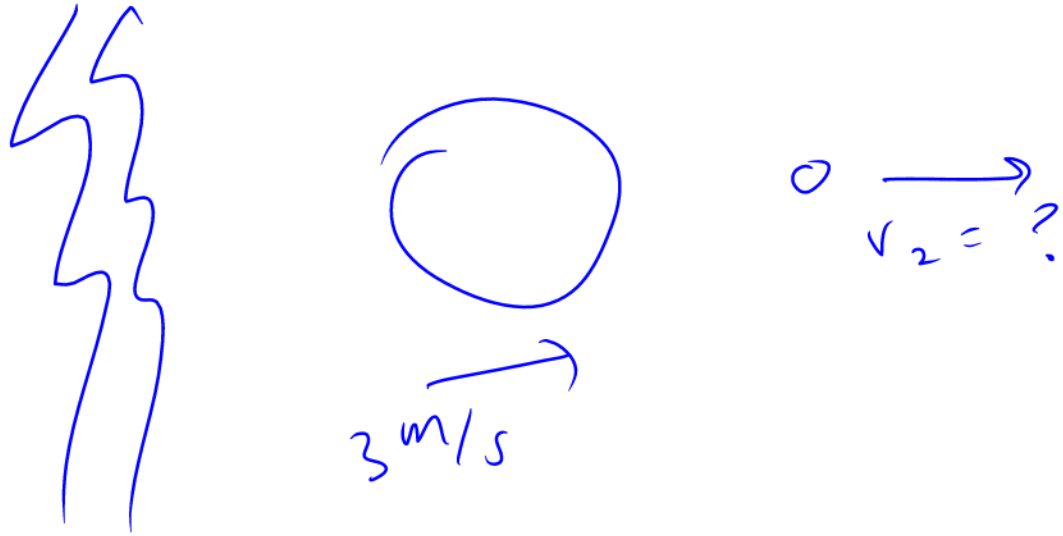
$$\boxed{v_2 = 1}$$

Demo: Newton's cradle

Worked problem

Elastic collision between big M and small m, bowling ball vs. marble





What are the final speeds?

Hint: $v_{\text{bowling ball final}} \approx v_{\text{bowling ball initial}}$. Use only velocity reversal eqn.

$$(v_1 - v_2)_{bef} = (v_2 - v_1)_{aft}$$

$$3 - (-3) = v_2 - 3$$

$$6 = v_2 - 3$$

$$v_2 = 9 \text{ m/s}$$

Answer: v_{marble} = 9 m/s

Demo

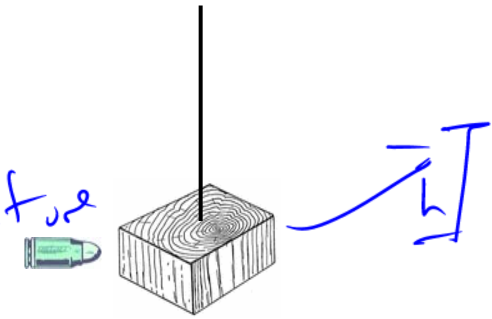
Velocity amplifier

Multi-step problems

→ Collision followed by something else

"Ballistic pendulum". A bullet of mass *m* and speed *v* embeds in a block of wood of mass *M* hanging from a string. How high do they rise?

before



How not to do: $\cancel{\frac{1}{2}mv^2 = (m+M)gh}$

How to do:

1. **Collision part: *p* is conserved** (but KE is not!)
   i. This gets you the velocity right after the collision
2. **Motion part: Energy is conserved** (but *p* is not!)
   i. This gets you the height based on velocity

$\frac{1}{2}\cancel{M_{tot}}v^2 = \cancel{M_{tot}}gh$

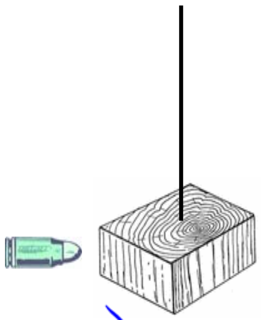
bef.

aft

p conserved

E conserved

Before | After/Before | After



$\vec{v}$

$$mv = (m+M)v_{int}$$

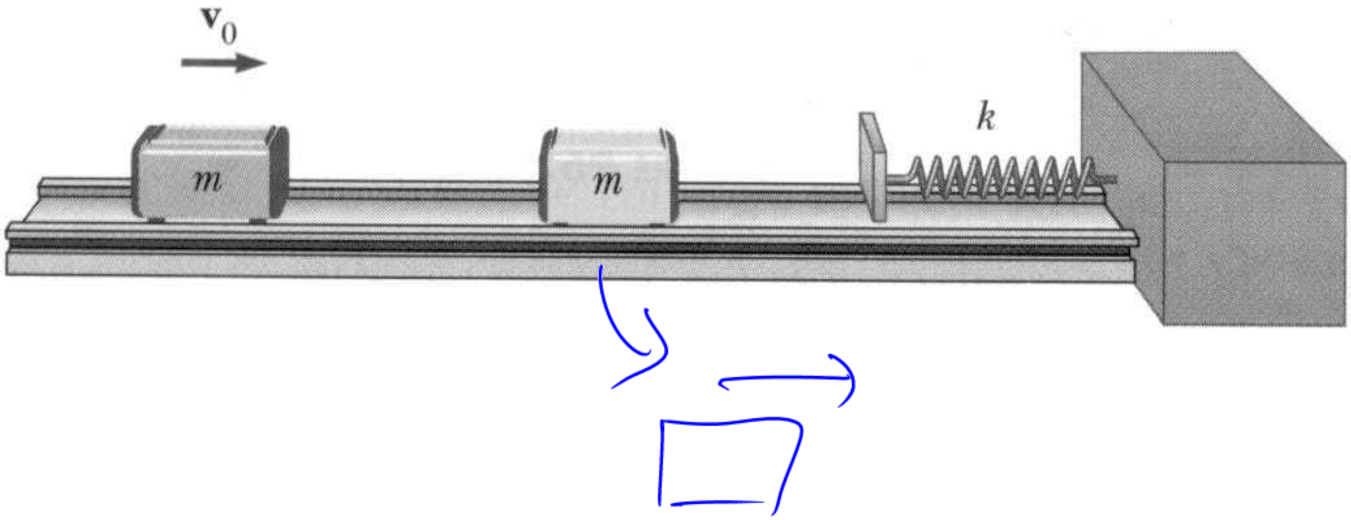
$$v_{int} = \frac{m}{m+M} \cdot v$$

$$h = \frac{1}{2}\frac{v^2}{g}$$

$$h = \frac{\frac{1}{2}\left(\frac{m}{m+M} \cdot v\right)^2}{g}$$

Answer: $h = \left(\frac{m}{m+M}\right)^2 \frac{v^2}{2g}$

HW 13-1: Very similar!



Explosions

Is momentum conserved?

yes!

What is the center of mass?

How does the center of mass move?

Center of Mass

Demo: Foam object

Demo: Spring apart gliders

Worked Problem

An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

Answers: v_x = 250 m/s; v_y = -75 m/s; v = 261 m/s at 16.7° south of east