Announcements – 14 Oct 2014

1. Prayer

"Which of the problems from last night's HW assignment would you most like me to discuss in class today?"

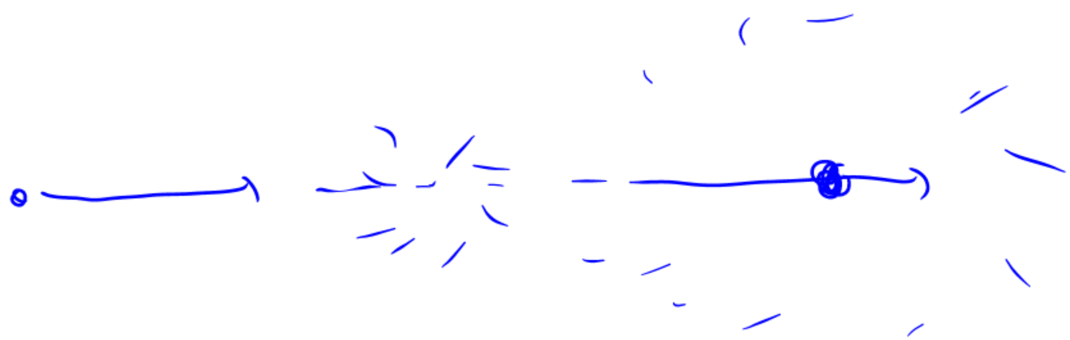
Center of Mass

What is the center of mass?

How does the center of mass move?

like a single object

Demo: Foam object



Worked Problem

An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

$\tan\theta = \frac{75}{250}$

$\theta = 16.7^\circ$

$\Sigma P_{bef\,x} = \Sigma P_{aft\,x}$

$(20\,kg)(100\,m/s) = (8\,kg)\,v_x$

$\frac{(20\,kg)(100\,m/s)}{8\,kg}$

$v_x = 250\,m/s$

$\Sigma P_{bef\,y} = \Sigma P_{aft\,y}$

$0 = (12\,kg)(50\,m/s) - 8\,kg \cdot v_y$

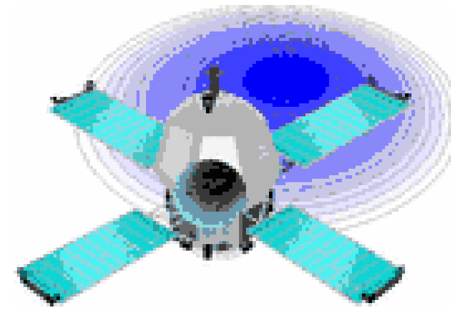
$v_y = \frac{12 \cdot 50\,m/s}{8}$

$v_y = 75\,m/s$

$v = \sqrt{250^2 + 75^2} = 261\,m/s$

Answers: v_x = 250 m/s; v_y = -75 m/s; v = 261 m/s at 16.7° south of east

Circular Motion



Demo: Bicycle wheel

Complicated motion of rotating body:
Different r, v, a's for different parts

But same ___ang velocity___

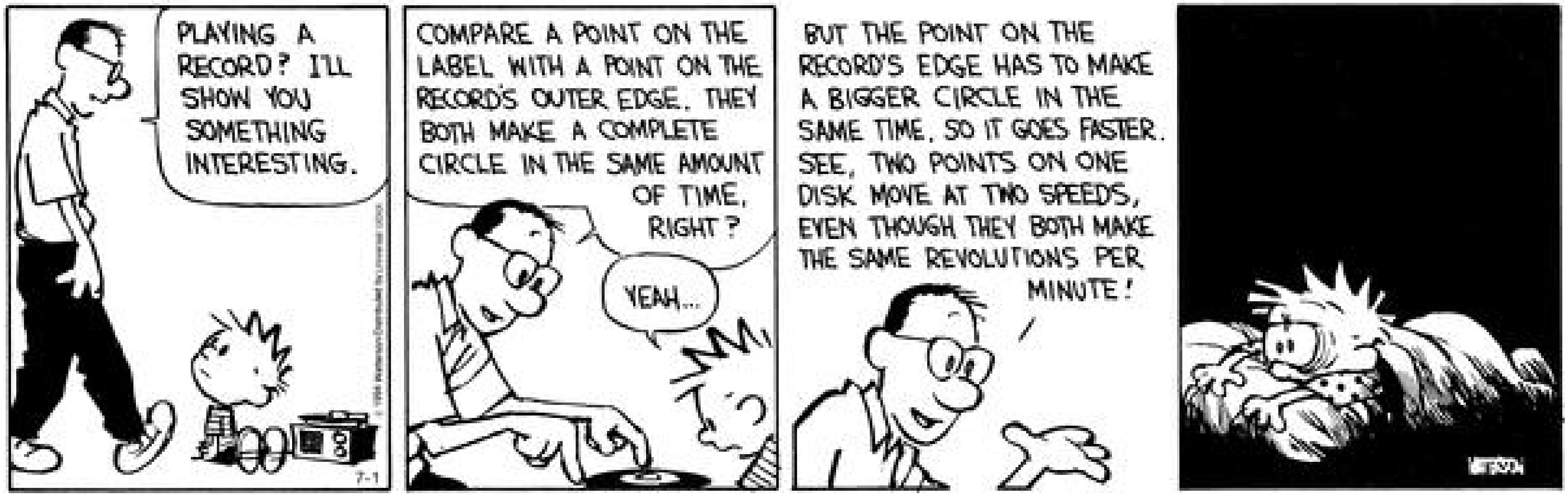
or revs/min

From warmup

Which has greater linear speed, a horse near the outside rail of a merry-go-round or a horse near the inside rail?

a. outside horse
b. inside horse
c. both the same

Calvin & Hobbes, Bill Watterson



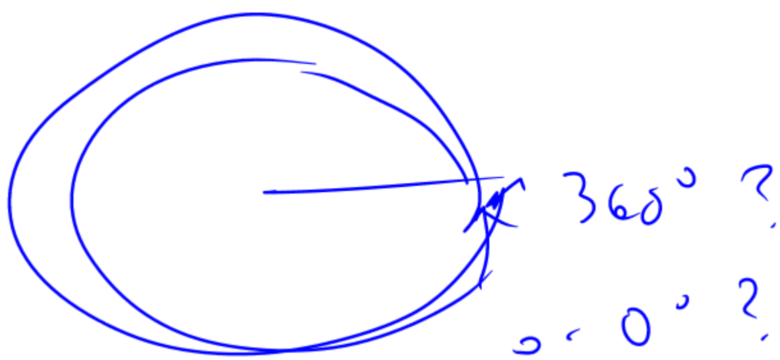
Do revolutions relate to angles?

Question: Which angle is greatest:

a. 30 revolutions
b. 30^o
c. 30 radians

1 rev = 360°

1 rad ≈ 60°



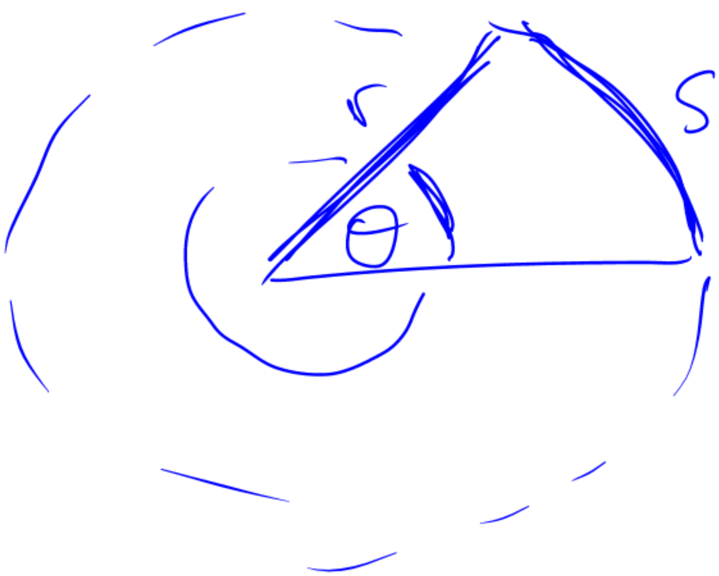
Definition of radian

How many radians in one circumference? 2π rad

How many radians in 360°? $360° = 2\pi$ rad

→ I will not give you these conversion factors on exam!

How many radians in an arc of length "s"?



θ in rad $= \frac{s}{r}$

i.e. $\frac{\theta}{2\pi} = \frac{s}{2\pi r}$

$s = \theta\,(\text{in rad}) \cdot r$

What is angular speed? (aka angular velocity)

ω

Clicker quiz: The symbol ω, used for angular velocity, is pronounced:

a. "al-pha"
b. ~~"double-you"~~
c. "gam-ma"
d. "om-e-ga"
e. "pi"

α A
γ Γ
ω Ω
π Π

From warmup

Which has greater *angular* speed, a horse near the outside rail of a merry-go-round or a horse near the inside rail?

a. outside horse
b. inside horse
c. both the same

Angular quantities

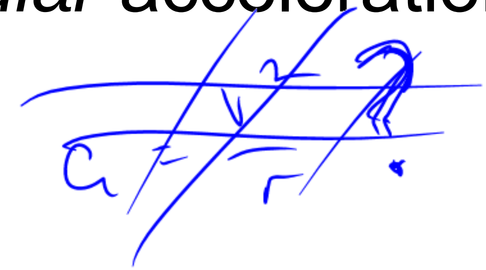
ang. displacement $\Delta\theta = \theta_f - \theta_i$ rad

ang. average velocity $\omega_{ave} = \dfrac{\Delta\theta}{\Delta t}$ $\dfrac{rad}{s}$ $V_{ave} = \dfrac{\Delta x}{\Delta t}$

ang. average acceleration $\alpha_{ave} = \dfrac{\Delta\omega}{\Delta t}$ $\dfrac{rad}{s^2}$

Units?

Kinematic equations (for constant *angular* acceleration)



Substitutions:

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow \alpha$

Regular kinematic	Angular kinematic
Definition: $v_{ave} = \dfrac{\Delta x}{\Delta t}$	$\omega_{ave} = \dfrac{\Delta \theta}{\Delta t}$
Definition: $a_{ave} = \dfrac{\Delta v}{\Delta t}$	$\alpha_{ave} = \dfrac{\Delta \omega}{\Delta t}$
For constant *a*:	For constant α
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0{}^2 + 2a(x - x_0)$	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

Some Guidance, a.k.a. What I Do

1. Pretend a problem involves regular distances & velocities, and figure out how you would solve it
2. Then use the corresponding angular equations

Worked Problem

Friction slows down a 10 cm radius spinning top with angular deceleration of 7 rad/s^2. It was initially spinning at 6 rad/s.

How many radians/degrees/revolutions will it turn before stopping?

"Translated problem": Friction slows down a block, a = -7 m/s^2. It was initially travelling at 6 m/s. How far will it go before stopping?

6 m/s

$\Delta x = ?$

$v_f^2 = v_0^2 + 2a\Delta x$

$\omega_f^2 = \omega_0^2 + 2\alpha\,\Delta\theta$

$0^2 = \left(6\,\frac{rad}{s}\right)^2 + 2\left(-7\,\frac{rad}{s^2}\right)\Delta\theta$

$\Delta\theta\left(14\,\frac{rad}{s^2}\right) = 36\,\frac{rad^2}{s^2}$

$\Delta\theta = \frac{36}{14}\,rad = 2.57\,rad$

$\times \frac{360°}{2\pi\,rad} = 147°$

$2.57\,rad \times \frac{1\,rev}{2\pi\,rad} = .411\,rev$

Answer: 2.57 rad, 147.3°, 0.409 rev

Worked Problem, cont.

Friction slows down a 10 cm radius spinning top with angular deceleration of 7 rad/s^2.   It was initially spinning at 6 rad/s.

How long will it take to stop? 6 m/s $a = -7\ m/s^2$ $v_f = 0$

$t = ?$

$v_f = v_0 + at$

"Translated problem":

$$\omega_f = \omega_0 + \alpha t$$

$$0 = 6\,\frac{\text{rad}}{\text{s}} - 7\,\frac{\text{rad}}{\text{s}^2}\,t$$

$$7\,\frac{\text{rad}}{\text{s}^2}\,t = 6\,\frac{\text{rad}}{\text{s}}$$

$$t = \frac{6}{7}\ \text{s} = .86\ \text{s}$$

Answer: 0.86 s

From warmup

If a woman walks 1 meter around the circumference of a 1 meter radius circle, what is the angular measure of her travel?

a. 1/2 rad
b. 1 rad
c. 2 rad
d. $\pi/2$ rad
e. π rad
f. 2π rad

$\theta(\text{in rad}) = \frac{s}{r}$

Angular motion of the whole object vs. motion of a spinning **point**

$s = r\theta$ → $\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$

Angular displacement $\Delta\theta$ vs "distance around circumference", *s*

Angular velocity ω vs tangential speed *v*

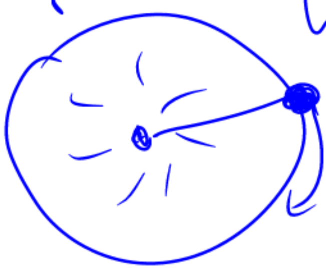
$v_{tan} = r\omega$

Angular acceleration α vs tangential acceleration *a*

$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$

$a_{tan} = r\alpha$

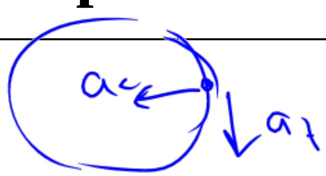
$r = .3\,m$



$\omega = 10\,\frac{rad}{s}$

$v_{tan} = (.3\,m)(10\,\frac{rad}{s}) = 3\,m/s$

Important: You must use radians if you want to use these equations



$a_c = \frac{v_{tan}^2}{r}$ $a_t = \frac{\Delta v}{\Delta t}$

Worked Problem, same situation as before

Friction slows down a 10 cm radius spinning top with angular deceleration of 7 rad/s^2. It was initially spinning at 6 rad/s.

Consider a point on the rim.

a. What is its initial velocity (m/s)? (magnitude, direction)

$v = r\omega = (.1\,m)(6\,rad/s)$ = $.6\,m/s$

b. What is its initial acceleration (m/s^2)? (magnitude, direction)

$a_{tan} = r\alpha = (.1\,m)(7\,rad/s^2) = -.7\,m/s^2$

$a_c = \frac{v^2}{r} = \frac{(.6\,m/s)^2}{.1} = 3.6\,m/s^2$

$a_{tot} = \sqrt{.7^2 + 3.6^2} = 3.667\,m/s^2$

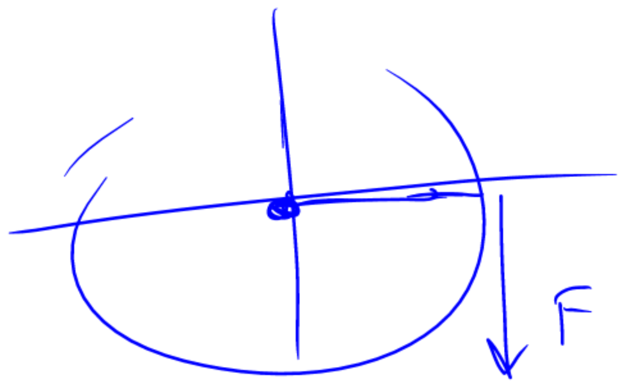
$\tan\theta = .7/3.6$ $\theta = 11°$

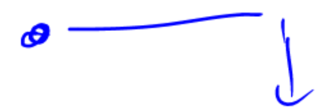
Answers: v_{tan} = 0.6 m/s; a_{tan} = -0.7 m/s^2; a_c = 3.6 m/s^2; a_{tot} = 3.667 m/s^2; dir = 11.0° away from inward

Intro to Torque

A force supplies a ***torque*** on an object when it is applied in such a way that could cause the object to ___spead up or slow down its rotation___

Definition: $\tau = r_{\perp} F$





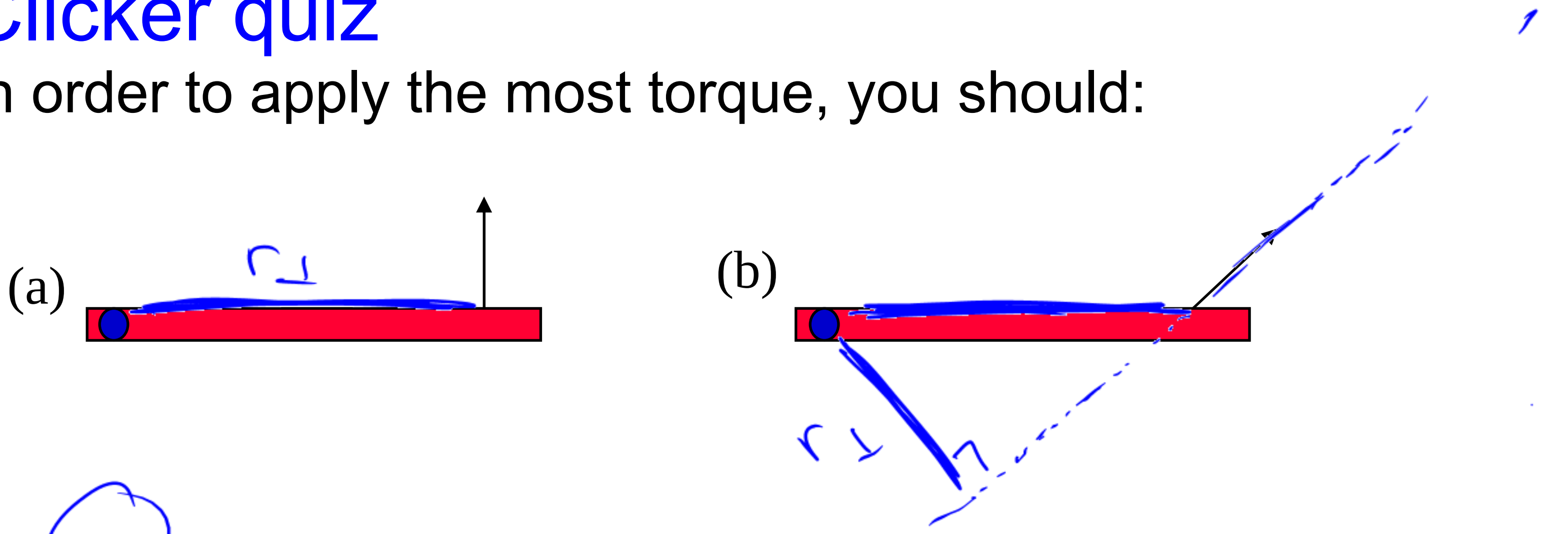
Note: where do you measure the distance *r* from?

If the object is rotating: from axis of rotation
If the object is standing still: you can choose

Above all, be ___consistent___

Clicker quiz

In order to apply the most torque, you should:



a. apply the force perpendicular to ***r***
b. apply the force at a 45° angle from ***r***
c. no difference

$\tau = r_{\perp} F$

Positive vs. negative torques:

Is torque a vector? Yes!