Announcements – Oct 16, 2014

1. Prayer
2. While waiting, see how many of these blanks you can fill out:

Centripetal Accel.:

Causes change in direction

but not speed

It points inward

Magnitude: a_c = v^2/r

How to use with N2: Always include on the right hand side

Tangential Accel.: Direction: tangent

Causes speed to change

Causes angular speed to change

Therefore, causes: angular acceleration

Definitions: θ = angle $\omega = \frac{\Delta\theta}{\Delta t}$ $\alpha = \frac{\Delta\omega}{\Delta t}$

Connecting eqns: arc length $s = \theta r$ $v_{tan} = \omega r$ $a_{tan} = \alpha r$

Angular Kinematic Equations: $x \rightarrow \theta$ $v \rightarrow \omega$ $a \rightarrow \alpha$

1. $\omega_f = \omega_0 + \alpha t$
2. $\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

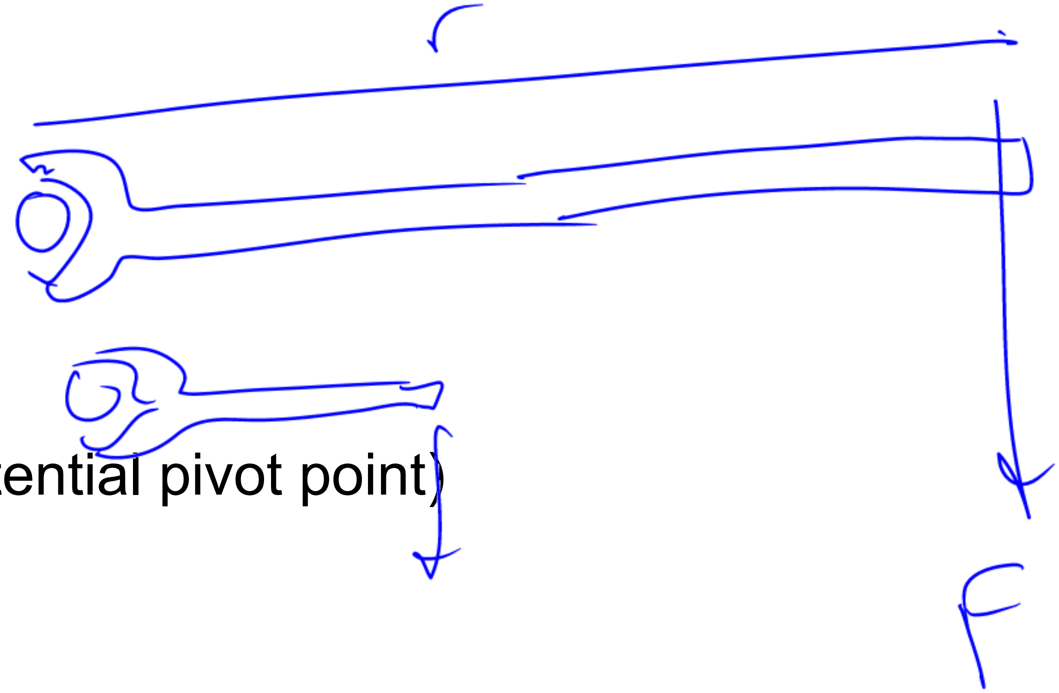
Angular acceleration is caused by torque

"Which of the problems from last night's HW assignment would you most like me to discuss in class today?"

Torque

Definition: $\tau = r_{\perp} F$

Measure *r* from a "pivot point" (or potential pivot point)



From warmup

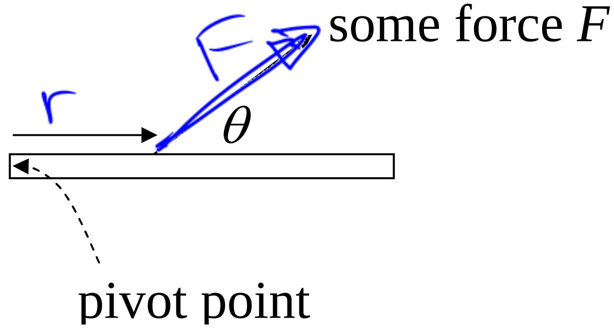
In order to apply the most torque to a bolt, you should:

a. use a wrench with a long handle
b. use a wrench with a short handle
c. there would be no difference

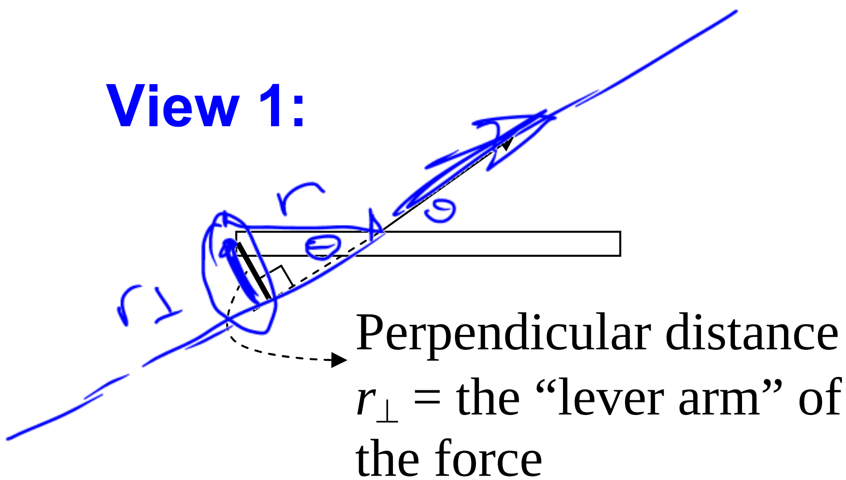
Demo: T-handle torque

Torque tug-of-war

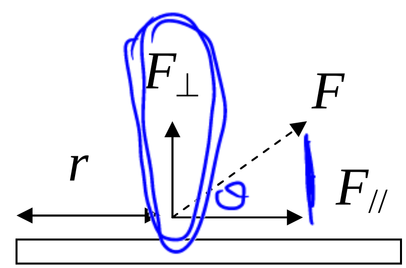
"Lever Arm"



View 1:



View 2:



$r \cdot F_\perp$

$F \sin\theta$

$r_\perp = r \sin\theta$

$$\tau = r_\perp F = rF_\perp = rF\sin\theta$$

$\tau = r_\perp F$

$= r \sin\theta F$

$\tau = r \cdot F \sin\theta$

$= r F_\perp$

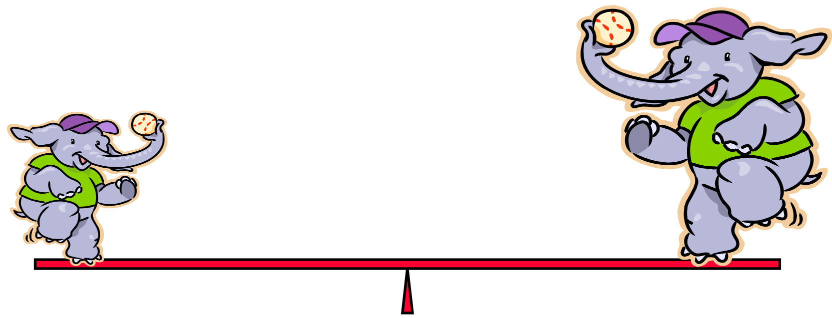
Torque Summary

$$\tau = r_{\perp}F = rF_{\perp} = rF\sin\theta$$

→ **but be careful about which angle you call θ!**

Note: If you are familiar with vector cross products, you can write it like this: $\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$

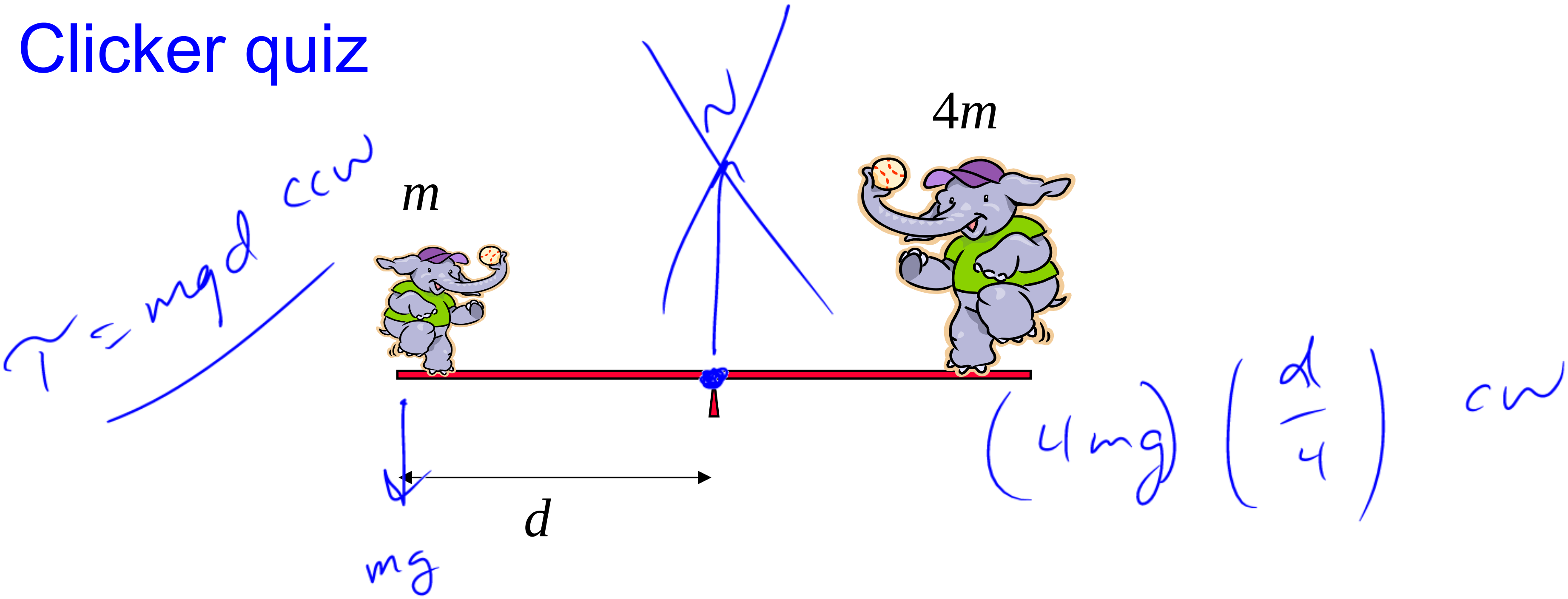
From warmup



Two people sit on a seesaw. They sit in positions such that the seesaw is balanced in a horizontal position. The two people must weigh the same amount.

a. true
b. false

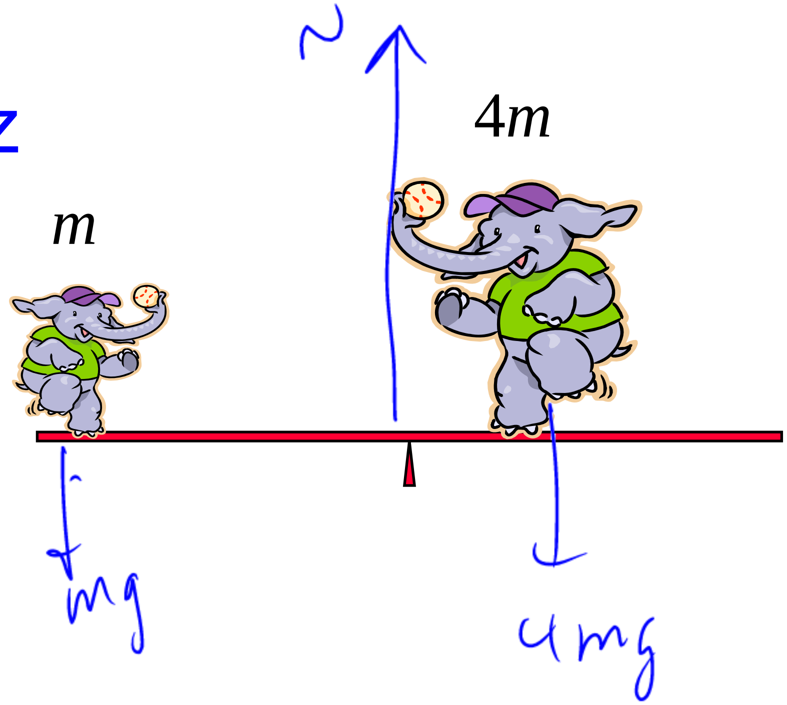
Clicker quiz



Where should the large elephant stand in order to balance the seesaw? (big elephant mass = 4× the little elephant mass)

a. *d*
b. *d*/2
c. *d*/4
d. *d*/6
e. *d*/8

Clicker quiz



$\Sigma F_y = 0$

$N - mg - 4mg = 0$

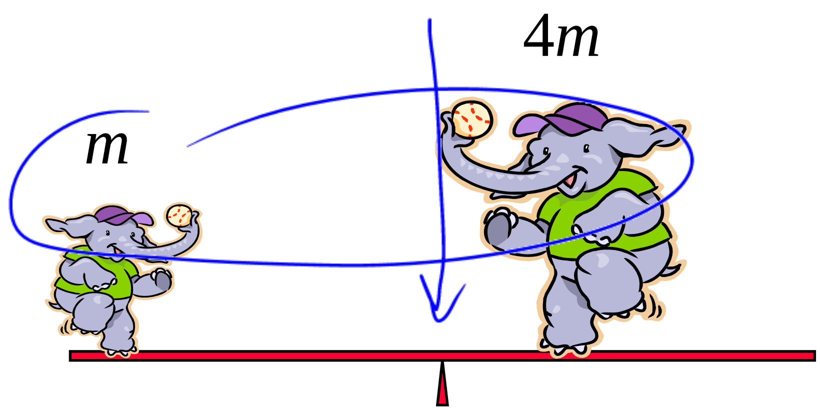
$N = 5mg$

When the see-saw is balanced, what is the upwards force from the pivot point? (Or, equivalently, the downward force *on* the pivot point.)

a. *mg*
b. 4*mg*
c. 5*mg*
d. 6*mg*
e. 8*mg*

Center of mass

Where is the center of mass of the elephants?



Demos: Center of mass (balanced objects)

Equilibrium

What concepts are involved?

1. If an object is not moving ("translational equilibrium"), then…

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

2. If an object is not ***rotating*** ("rotational equilibrium"), then…

$$\Sigma \tau = 0$$

A new blueprint equation!

From warmup

If an object is in equilibrium:

a. the net force on it must be zero
b. the net torque on it must be zero
c. both of the above
d. neither of the above

Blueprint advice

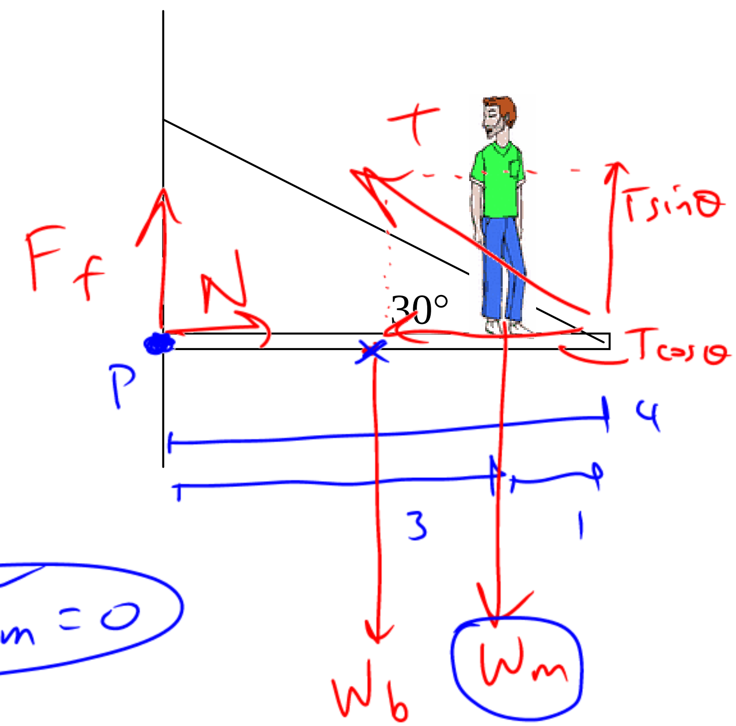
$$\boxed{\Sigma \tau = 0}$$ if it has no ang. accel.

Think carefully about the pivot point

and the sign of the torques

Worked problem

A 1500 N man is standing on a board supported by a wall and a rope. He is 1 meter from the right end. The board weighs 800 N and is 4 meters long. What is the tension in the rope?



1. Draw all of the forces present.
Note: gravity acts at the *center of mass*

1b. Divide forces into components

2. Use Σ**F** blueprint equation(s)

$\Sigma F_x = 0$

$N - T\cos\theta = 0$

$\Sigma F_y = 0$

$F_f + T\sin\theta - W_b - W_m = 0$

3. Use Στ blueprint equation

→ which point to use as the "pivot point"?

$\Sigma \tau_p = 0$

$-W_b(2m) - W_m(3m) + T\sin\theta(4m) = 0$

4. Use the filled-in blueprints to solve for what you're looking for.

$$-w_b \cdot 2 - w_m \cdot 3 + T\sin\theta \cdot 4 = 0$$

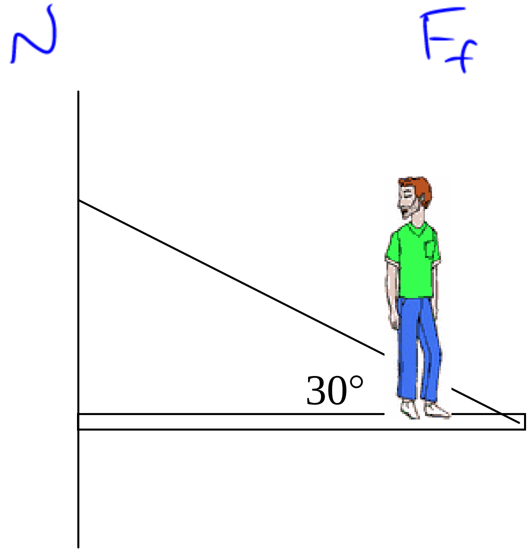
$$T\sin\theta \cdot 4 = \frac{2w_b + 3w_m}{4\sin\theta}$$

$$T = \frac{2(800\text{ N}) + 3(1500\text{ N})}{4\sin 30^\circ} = 3050\text{ N}$$

Answer: T = 3050 N

Additional question

What are the horizontal and vertical forces of the wall on the board?



$N - T\cos\theta = 0$

$N = T\cos\theta$

$= 3050\text{ N} \cdot \cos 30°$

$= \boxed{2641\text{ N}}$

$F_f + T\sin\theta - w_b - w_m = 0$

$F_f = w_b + w_m - T\sin\theta$

$= 800\text{ N} + 1500\text{ N} - 3050\text{ N}\cos 30°$

$= \boxed{775\text{ N}}$

Answers: F_x = 2641 N to right, F_y = 775 N up

From warmup

Ralph noticed that both torque and work are obtained by multiplying a force times a distance. He wants to know: how are they different? Do they have the same units? What can you tell Ralph to help him out?

$W = N \cdot m = J$ $\tau = N \cdot m$

"Think-pair-share"

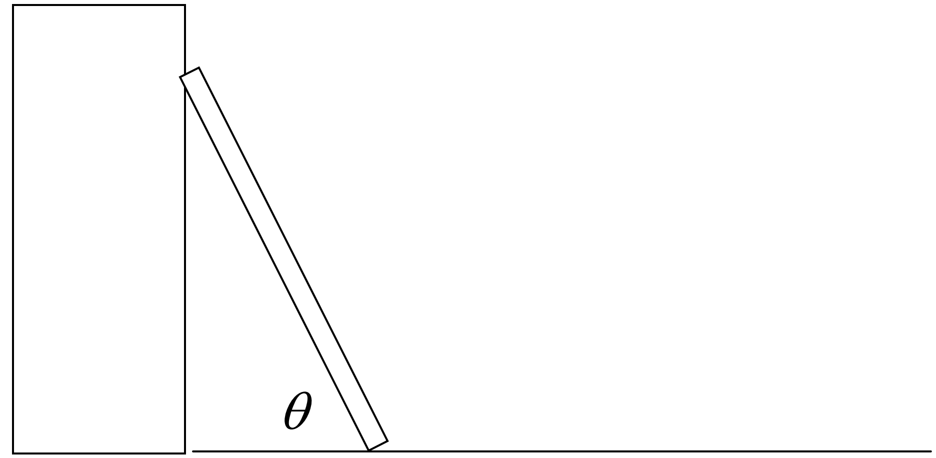
- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

Clicker: I am now ready to share my answer if randomly selected.

a. Yes

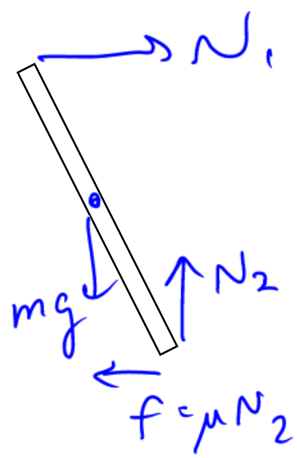
Note: you are allowed to "pass" if you would really not answer.

Problem:
(Like HW 15-2)



A ladder leans against a **frictionless** wall. The ground has static coefficient of friction μ. What's the smallest angle θ such that the ladder doesn't slip? Length of ladder is *d*, mass of ladder is *m*.

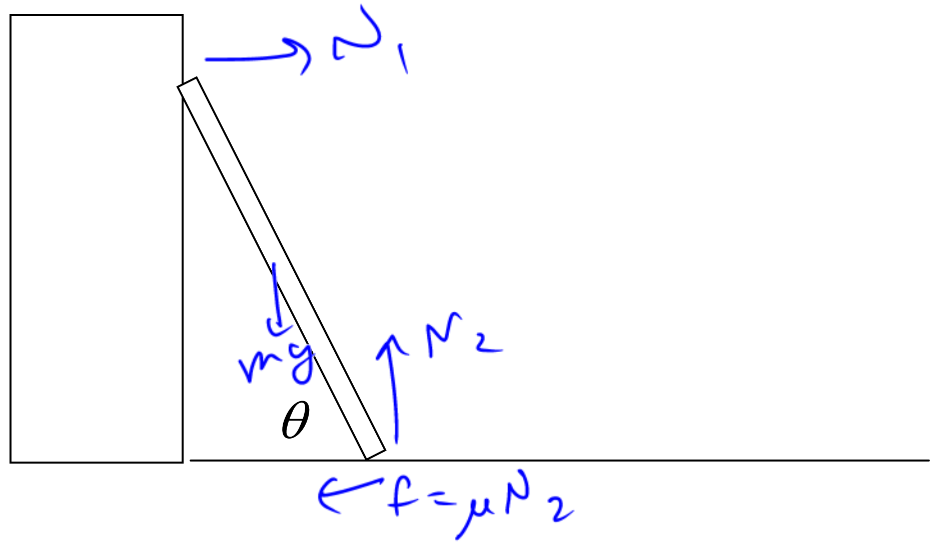
Draw a FBD of ladder:



Clicker quiz: I have done so

a. yes

Clicker quiz



The ground's frictional *force* is ________ compared to the wall's normal force.

a. more than
b. less than
c. the same
d. can't tell

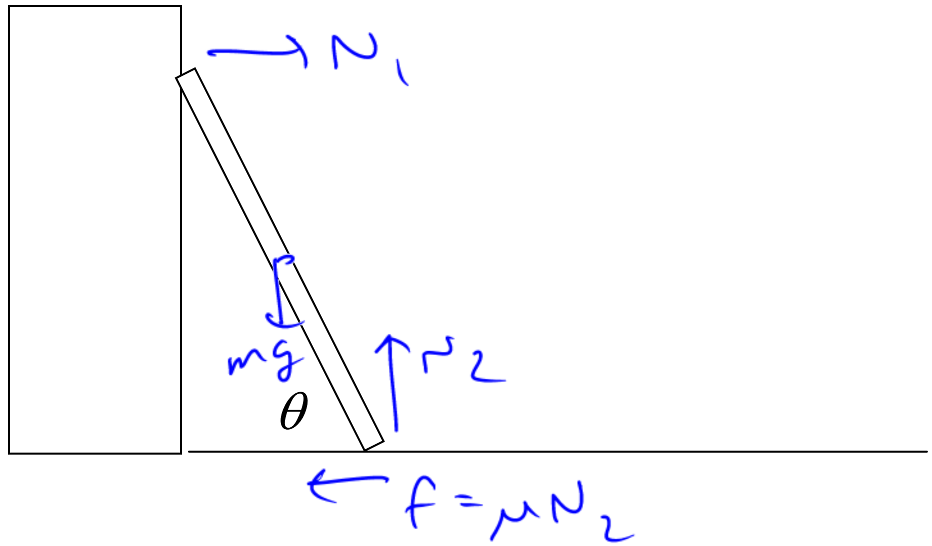
$\Sigma F_x = 0$

$N_1 - \mu N_2 = 0$ (f)

$N_1 = f$

$N_1 = \mu N_2$

Clicker quiz



The ground's normal *force* pushing upward is _________ compared to the weight.

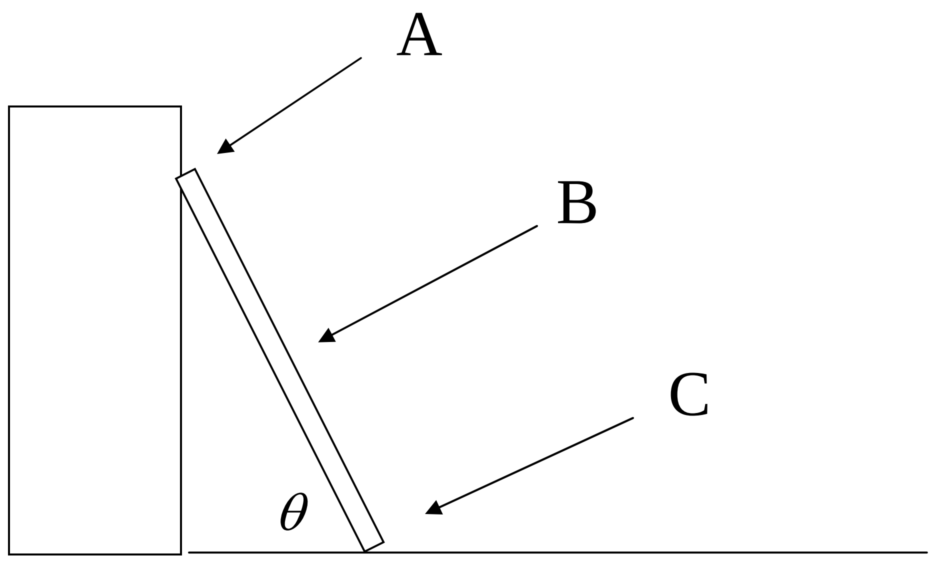
a. more than
b. less than
c. the same
d. can't tell

$\Sigma F_y = 0$

$N_2 - mg = 0$

$N_2 = mg$

Clicker quiz



To solve the problem, we need to use $\Sigma\tau = 0$… but about which point should we compute the torques?

a. A
b. B
c. C

Solved problem

$$\Sigma \tau_p = 0$$

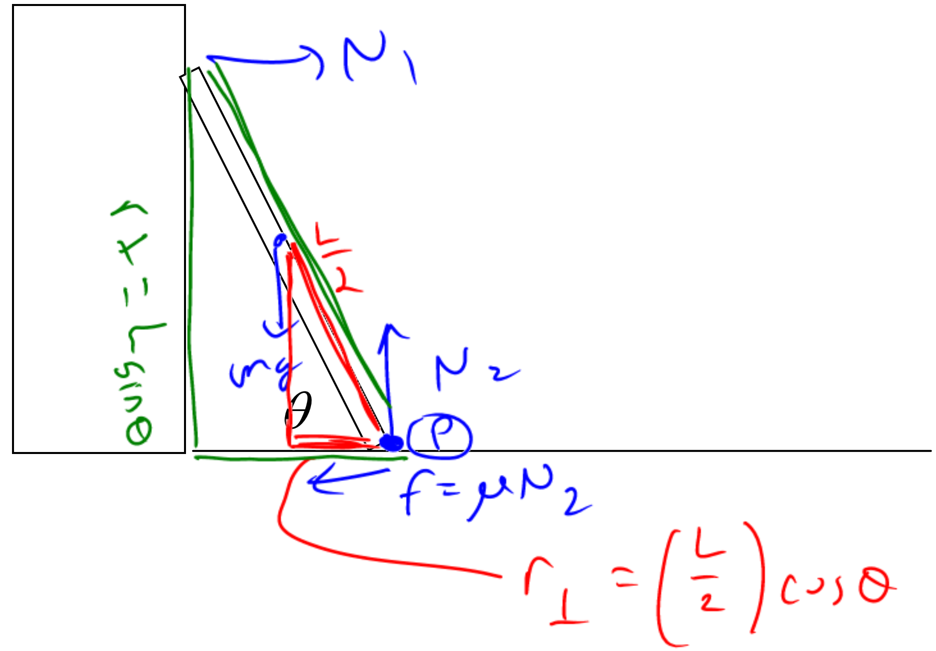
$$mg\left(\frac{L}{2}\cos\theta\right) - N_1 (L\sin\theta) = 0$$

$$\frac{1}{N_1}\frac{mg}{2}\cos\theta = N_1\frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{mg}{2\cdot N_1}$$

$$\tan\theta = \frac{mg}{2\cdot(\mu mg)} = \frac{1}{2\mu}$$

$$\theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$$



$$N_1 = \mu N_2 = \mu mg$$

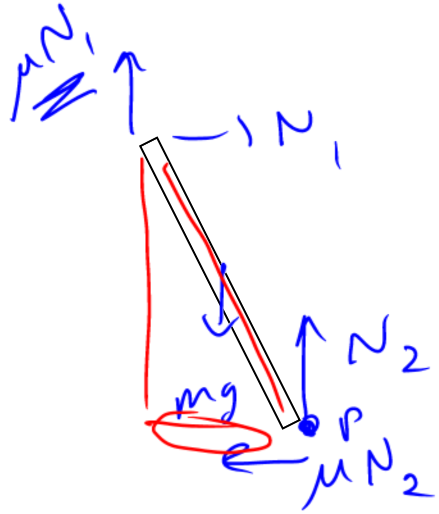
Numerical answers: if $\mu = 0.5 \rightarrow \theta = 45°$; $\mu = 0.7 \rightarrow \theta = 35.5°$; $\mu = 0.9 \rightarrow \theta = 29.1°$

Answer: $\theta = \tan^{-1}\left(1/(2\mu)\right)$

Modification

Suppose the wall ***also*** has friction, μ. What's the angle θ now? (Think: bigger or smaller?)

New FBD:



Equations:

$$\sum F_x = 0$$

$$N_1 = \mu N_2$$

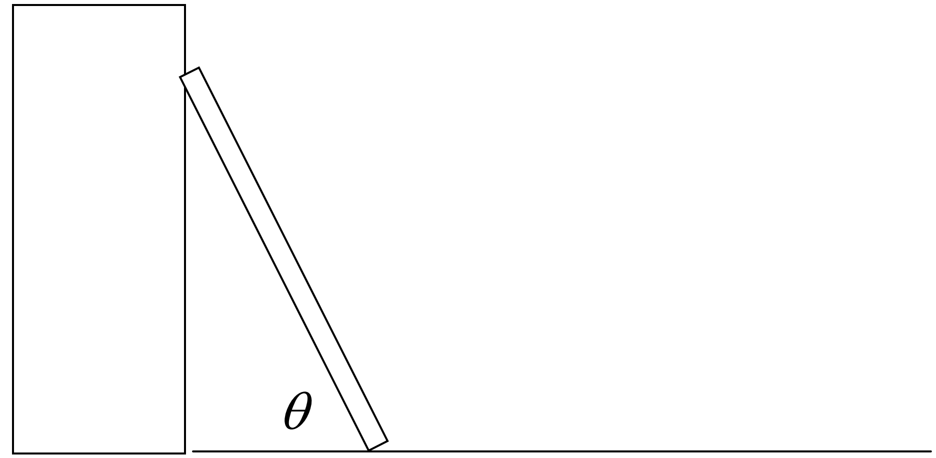
$$\sum F_y = 0$$

$$\mu N_1 + N_2 = mg$$

$$\sum \tau_p = 0$$

$$mg\frac{L}{2}\cos\theta - N_1 L\sin\theta - \mu N_1 L\cos\theta = 0$$

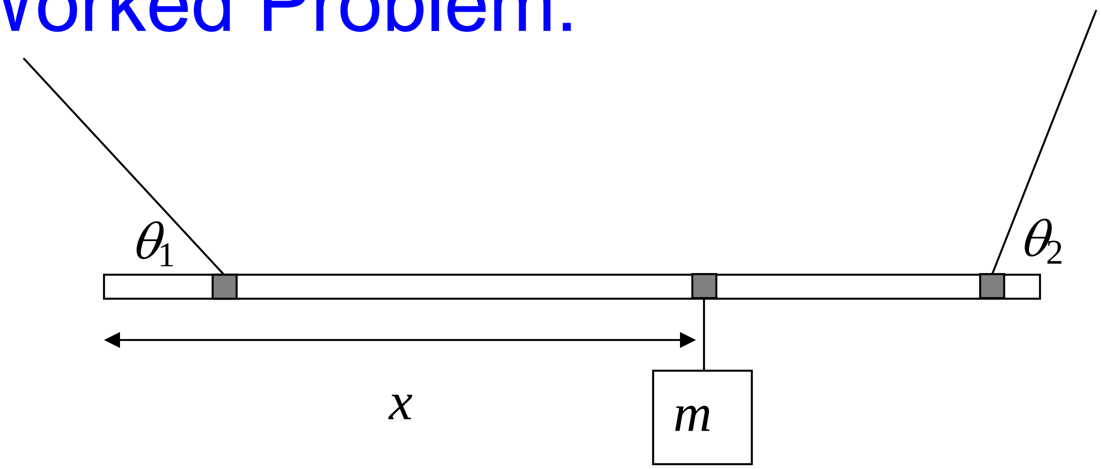
Solved problem



If $\mu = 0.5 \rightarrow \theta = 36.9°$; $\mu = 0.7 \rightarrow \theta = 20.0°$; $\mu = 0.9 \rightarrow \theta = 6.0°$

Answer: $\theta = \tan^{-1}\left(1/(2\mu) - \mu/2\right)$

Worked Problem:



A 0.4 kg meterstick is suspended from pulleys and support pillars (not shown) via two strings at θ_1 =41.4° and θ_2 = 60°, with tensions of 2 N and 3 N, respectively. The strings are attached at 10 cm and 5 cm from the two ends of the meterstick. The stick is *not* in equilibrium until an additional mass is hung from a point in the middle. Find the unknown x and m.

Answers: 0.171 kg, 38.2 cm