Announcements – Oct 21, 2014

1. **Exam ~~3~~ 2** starts Oct 30, a week from Thursday.
   a. Late fee on Monday Nov 3, after 2 pm
   b. Closes on Tuesday Nov 4, 2 pm
   c. Jerika exam reviews, both in room C295 ESC:
      i. Wed Oct 29 7 - 8:30 pm
      ii. Thurs Oct 30 5:30 - 7 pm
   d. Exam covers through lecture 16 (Thursday)
      i. Ch. 5, 6, 7.1-7.3, 8
      ii. HW 10-17

2. **Remaining exam material:** complete the angular quantities
   a. Distance $x \rightarrow \theta$
   b. Velocity $v \rightarrow \omega$
   c. Acceleration $a \rightarrow \alpha$
   d. Force $F \rightarrow \tau$
   e. Mass $m \rightarrow$?? (today)
   f. KE $\frac{1}{2} mv^2 \rightarrow$?? (today)
   g. Momentum $mv \rightarrow$?? (next time)

“Which of the problems from last night's HW assignment would you most like me to discuss in class today?”

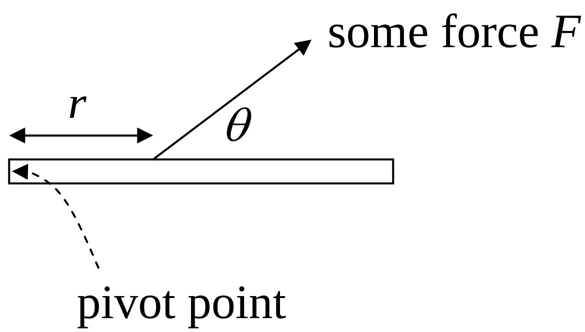
Review of Torques

lever arm

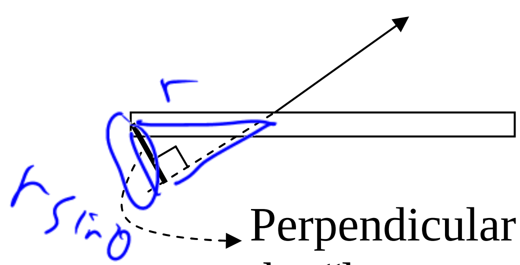
Definition of torque (about a pivot point):

$$\boxed{\tau_p = r_\perp F = rF_\perp = rF\sin\theta}$$

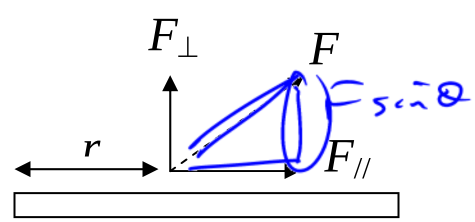
(careful with angles, make sure you get perpendicular component)



Positive/negative:
- Produces a **clockwise** rotation = **negative**
- Produces a **counter-clockwise** rotation = **positive**



Perpendicular distance $r_\perp$,
the "lever arm" of the force:
$r_\perp = r\sin\theta$



Equilibrium

$$\sum F = 0$$

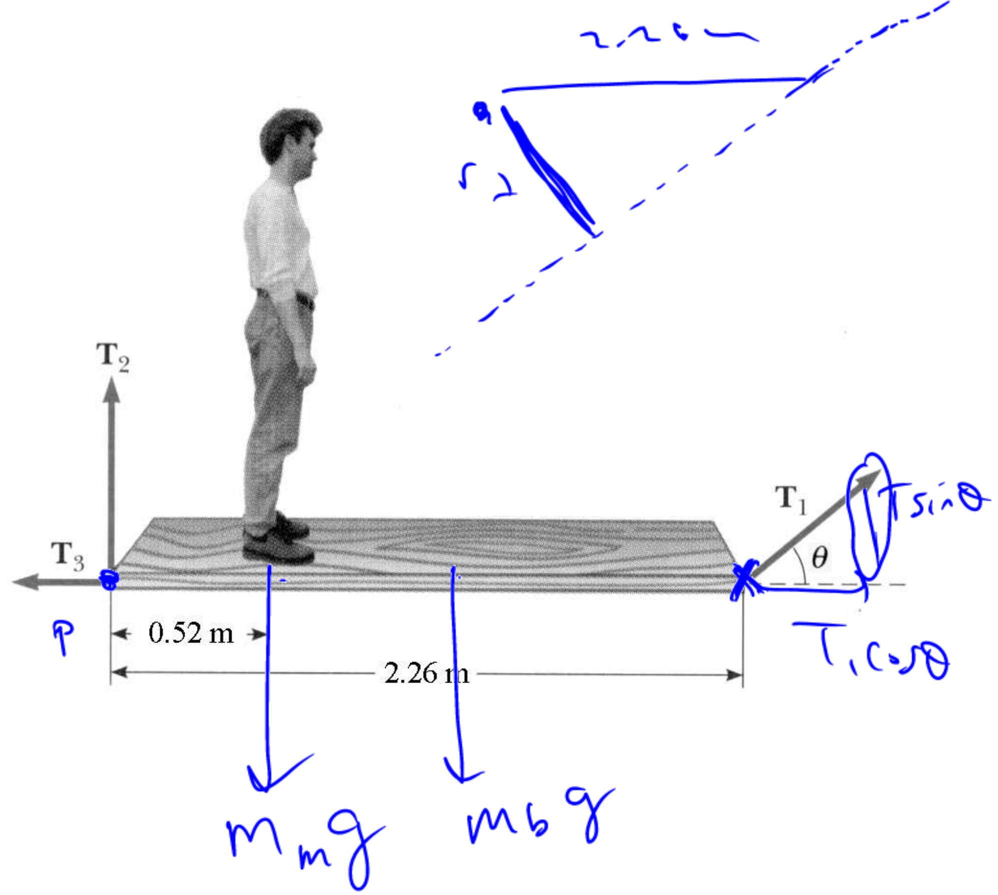
$$\sum \tau_p = 0$$

Translation:

- if an object is not speeding up or slowing down, there is no net force on it
- if an object is not speeding up or slowing down its *rotation*, there is no net *torque* on it.

One more equilibrium problem:

A uniform plank of length 2.26 m and mass 10 kg is balanced by three ropes as indicated in the figure, with $\theta = 35°$. A 75 kg person is standing 0.52 m from the left end. Find the tensions in all three ropes.



$$\Sigma F_x = 0$$

$$-T_3 + T_1 \cos\theta = 0$$

$$T_3 = T_1 \cos\theta$$
$$= (380.3\,N)\cos 35°$$
$$= \boxed{614.9\,N}$$

$$\Sigma F_y = 0$$

$$T_2 + T_1 \sin\theta - m_m g - m_b g = 0$$

$$T_2 = m_m g + m_b g - 380.3 \sin 35°$$
$$T_2 = \boxed{311.5\,N}$$

$$\Sigma \tau_p = 0$$

$$-(m_m g)(.52\,m) - (m_b g)(1.13\,m) + (T_1 \sin\theta)(2.26\,m) = 0$$

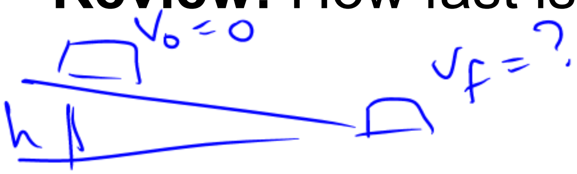
$$T_1 = \frac{m_m g \cdot .52 + m_b g \cdot 1.13}{\sin\theta \cdot 2.26}$$
$$= \boxed{380.3\,N}$$

Answers: 380.3 N, 311.5 N, 614.9 N

Rotational kinetic energy

Demo… a cart races a ball (video from warmup). Who wins? Why?

Review: How fast is **cart** going at bottom? (Energy)

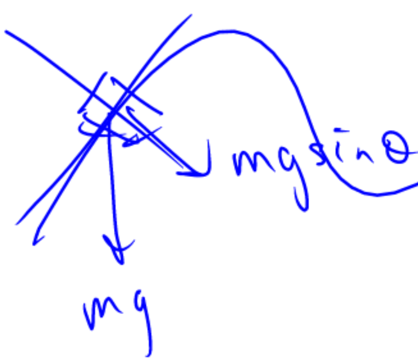


$$E_{bef} + \cancel{W} = E_{aft}$$

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v_f^2$$

$$v_f = \sqrt{2gh}$$

How long did it take to get there? (Kinematics)



$$\Sigma F_x = ma_x$$

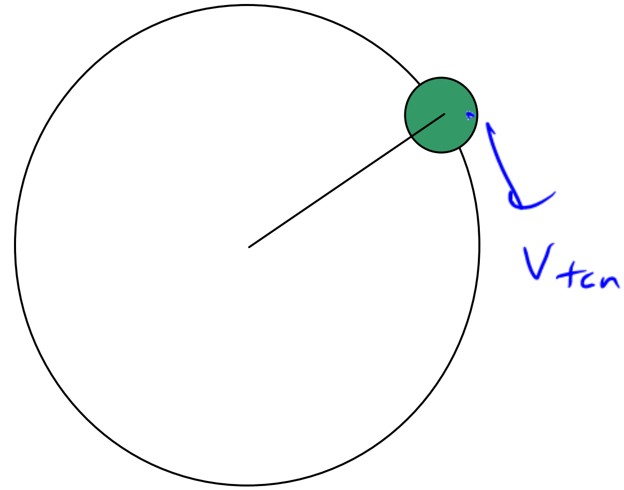
$$\cancel{m}g\underline{\sin\theta} = \cancel{m}a_x$$

$$v_f = \cancel{v_0} + at$$

$$t = \frac{v_f}{a}$$

→ **What's different about the ball?**

Kinetic energy of a **“point mass”** rotating in a circle:



$$KE = \frac{1}{2} m v_{tan}^2$$

$$\rightarrow v_{tan} = \omega r$$

Write in terms of ω:

$$= \frac{1}{2} m \omega^2 r^2$$

$$= \frac{1}{2} \left(m r^2 \right) \omega^2$$

$$I = m r^2$$

$$KE_{rot} = \frac{1}{2}\left(\text{something}\right)\omega^2$$

→ what’s the something?

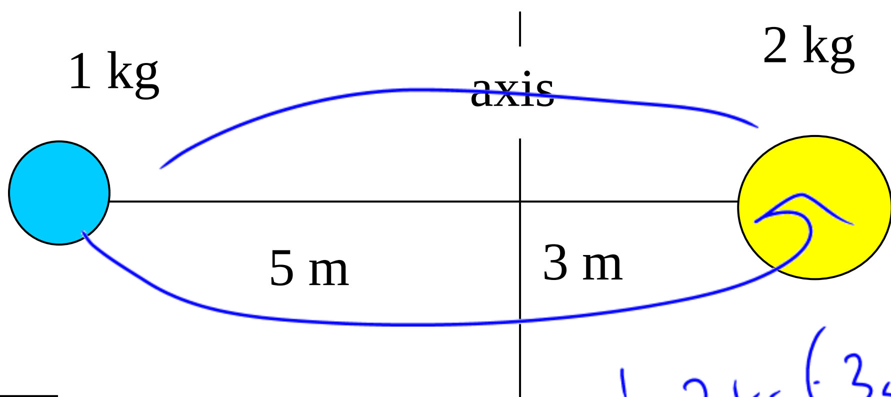
"Moment of inertia"

$\boxed{I_{pt\,mass} = mr^2}$ (rotating in a circle; r = radius of circle)

compare to $\frac{1}{2}mv^2$

Kinetic energy in terms of I and ω: $\boxed{KE_{rot} = \frac{1}{2} I \omega^2}$

Moment of inertia for two masses? (connected with a rod)



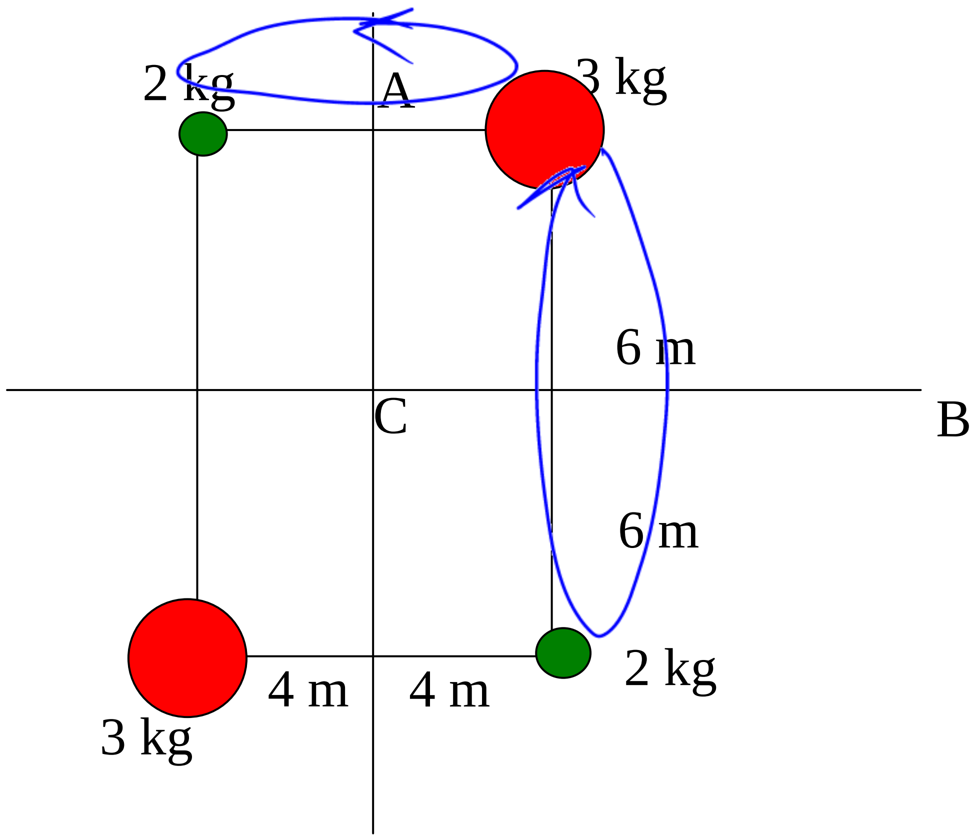
$\boxed{I = I_1 + I_2 + \ldots}$

$\frac{1}{2} \cdot 2\,kg\,(3\,m)^2 + \frac{1}{2}(1\,kg)(5\,m)^2$

$21.5\ kg \cdot m^2$

$2\cdot 4^2 + 3\cdot 4^2 + 3\cdot 4^2 + 2\cdot 4^2$

Clicker quiz



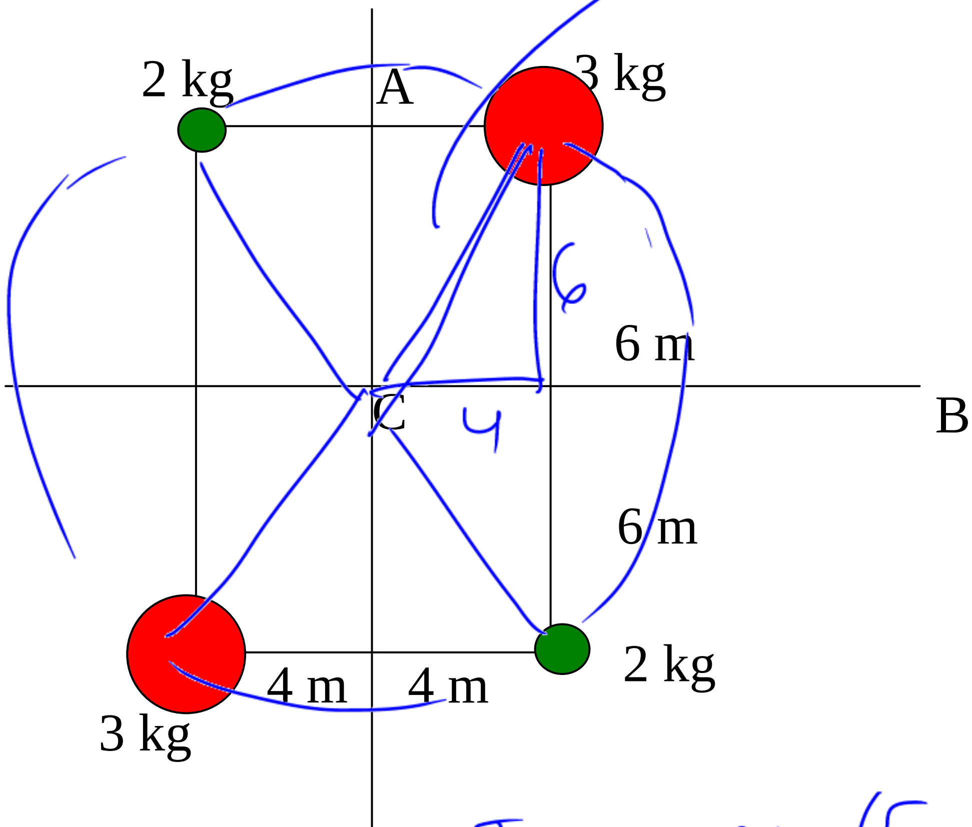
Tip: If size of object is much smaller than rotation radius, treat it as a "point mass"

Does *I* change when you rotate about axis A vs. axis B?

a. About axis A has larger *I*
b. About axis B has larger *I*
c. They have the same *I*

$\sqrt{6^2+4^2} = \sqrt{52}$ m

Worked problem



What's the total moment of inertia about axis C? (C is into the page)

$$I_{tot} = 2\,kg\cdot(\sqrt{52}\,m)^2 \quad 3\,kg\cdot(\sqrt{52}\,m)^2 + \dots \quad \dots$$

$$= 52\,(2+3+3+2)$$

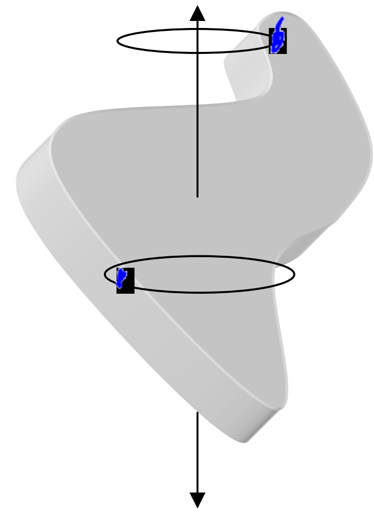
$$= \boxed{520\ kg\,m^2}$$

Answer: I_{tot} = 520 kg·m^2

Demo

Variable "I-rotator"

“Extended” objects



Must add up mr^2 *for each bit of mass* in the object

Which bits of mass contribute the most to *I*?

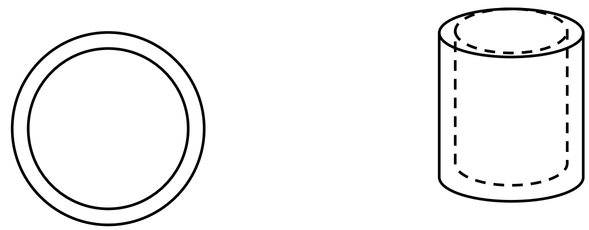
From warmup. Moment of inertia is biggest for:

a. compact objects
b. objects that are spread out
c. neither; doesn’t depend on shape

Demo: Long “I-bars”

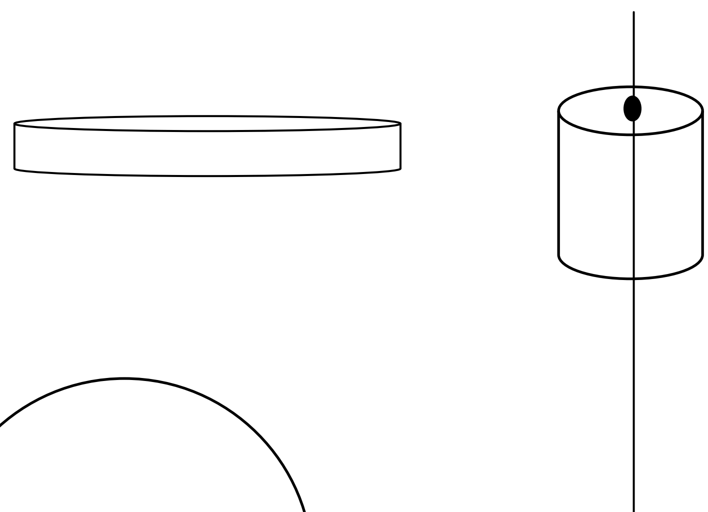
Which of these objects will have the largest I?

Hoop/cylindrical shell



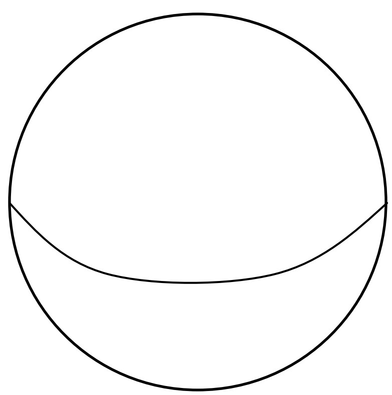
larger I

Solid disk/cylinder



smaller I

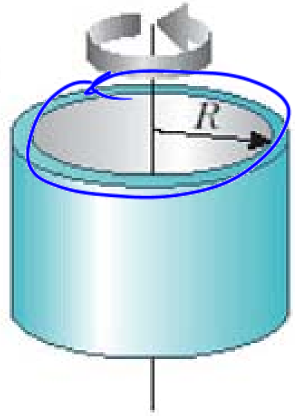
Solid sphere



smallest I

Hoop or thin cylindrical shell
$I = MR^2$

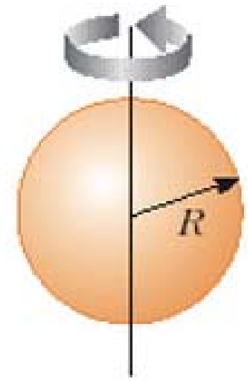
$1mr^2$



Solid sphere

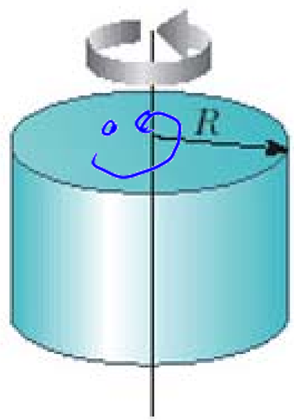
$I = \frac{2}{5} MR^2$

$.4mr^2$

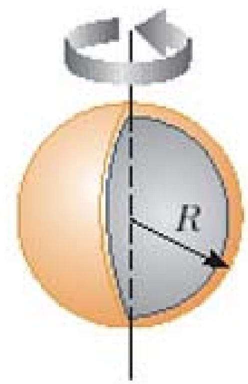


Solid cylinder or disk
$I = \frac{1}{2} MR^2$

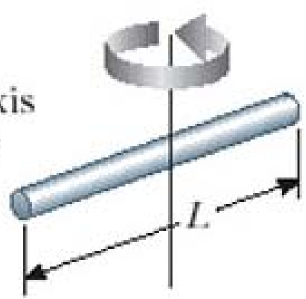
$.5mr^2$



Thin spherical shell
$I = \frac{2}{3} MR^2$

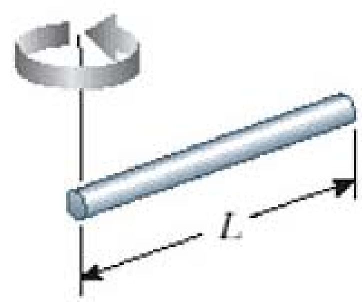


Long thin rod with rotation axis through center
$I = \frac{1}{12} ML^2$



Long thin rod with rotation axis through end

$I = \frac{1}{3} ML^2$



Clicker quiz

Which kind of rolling object will be moving the fastest at the bottom of an incline?

a. Hoop
b. Solid disk
c. Sphere
d. Hoop and disk will tie for fastest
e. They will all tie for fastest

Additional question: Which object will get to the bottom first?

Demo: Moment of inertia races

Hoop vs. sphere
Hoop vs. disk
Big disk vs. little disk
Big hoop vs. little hoop
Big sphere vs. little sphere

Clicker quiz

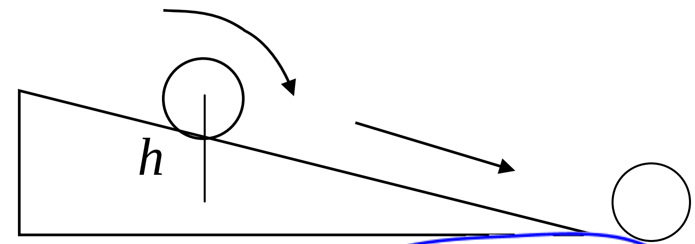
If they continued on, which would go the farthest up a hill on the other side?

a. Hoop
b. Solid disk
c. Sphere
d. All would end at the same height

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Worked Problem

An object with moment of inertia *I* rolls down a height *h* without slipping. Find the speed at bottom.



$$PE_i = KE_f + KE_{rot,f}$$

$$\omega_f = \frac{v_f}{r}$$

$$v_f = \omega_f \cdot r$$

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f^2}{r^2}\right)$$

$$2\cancel{m}gh = v_f^2\left(\cancel{\tfrac{1}{2}}^{1}\cancel{m} + \cancel{\tfrac{1}{2}}\frac{I}{mr^2}\right)$$

$$2gh = v_f^2\left(1 + \frac{I}{mr^2}\right)$$

$$v_f = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

Answer: $v = \sqrt{\frac{2gh}{1 + I/mR^2}}$

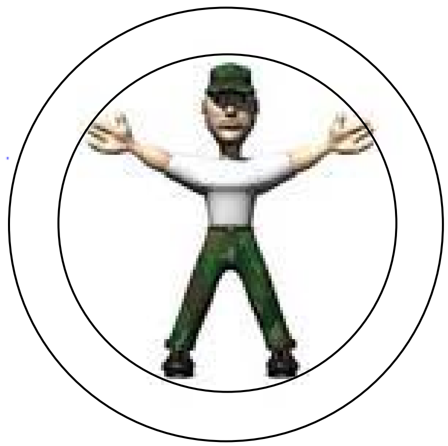
Clicker quiz

Mary and Fred are rolling a large tire down a hill. Mary says it will go faster if Fred gets inside the tire as shown and rolls down with it. Fred's not sure. What do you think?

a. It will go faster
b. It will go slower
c. It will take the same time

initially

mr^2



now closer to

$.5mr^2$

Newton's second law for torques

$$\boxed{\sum \tau_p = I\alpha}$$

Compare $\Sigma F = ma$

From warmup. Angular acceleration will definitely increase if:

a. torque is decreased and moment of inertia is decreased
b. torque is decreased and moment of inertia is increased
c. torque is increased and moment of inertia is decreased
d. torque is increased and moment of inertia is increased

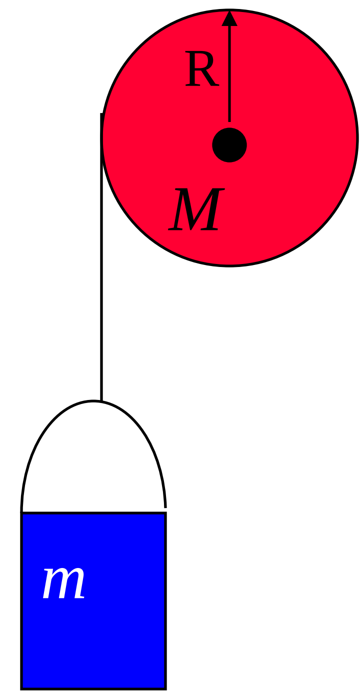
From warmup

Ralph heard his instructor say **"Moment of inertia plays the same role in rotational motion that mass does in linear motion."** This confuses him. What does it mean?

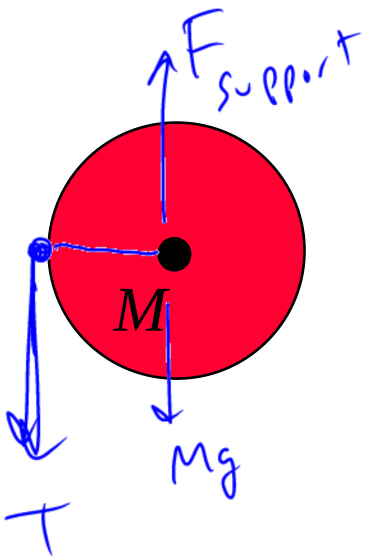
"Pair share"–I am now ready to share my neighbor's answer if called on.

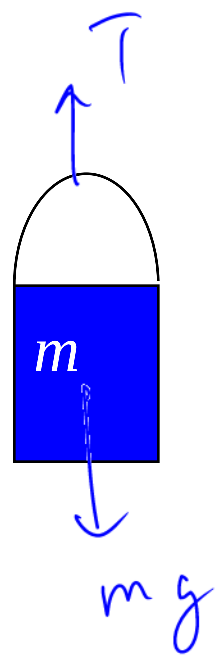
a. Yes

Worked problem: A falling mass starts a cylinder rotating (<u>not</u> a “massless pulley”). What is the acceleration of *m*?



Start with FBDs:





Write equations…

Cylinder

$\Sigma\tau_p = I\alpha$

$T \cdot r = \left(\tfrac{1}{2} M r^2\right) \alpha$

$\alpha \rightarrow a/r$

Pail

$\Sigma F = ma$

$mg - T = ma$

Make a connection between α and *a*:

$T r = \left(\tfrac{1}{2} M r^2\right)\left(\frac{a}{r}\right)$

$mg - T = ma$

Solve!

$$a = \frac{m}{m + \frac{M}{2}}\, g$$

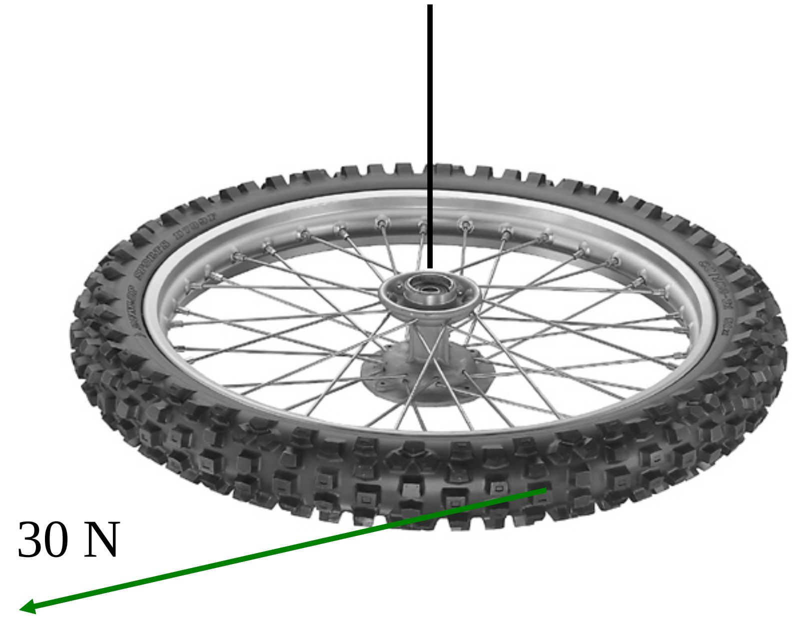
Answer: $a = \dfrac{m}{m + M/2} g$

What if you just want to know v_f (given a distance d)?

Answer: $v_f = \sqrt{\dfrac{mgd}{\frac{1}{2}m + \frac{1}{4}M}}$

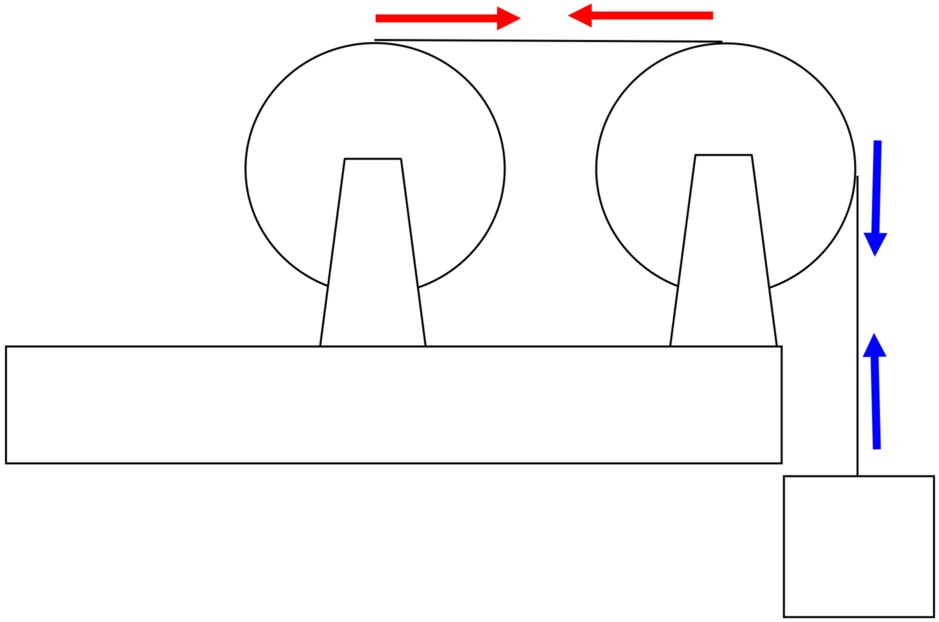
Worked Problem

A bicycle tire (r = 0.4 m, I = 0.8 kg·m^2) is hanging from a string from the ceiling, not moving. You push tangentially on the edge with a 30 N force for 0.3 seconds. What is ω_f? (*Hint*: because time is given, might be simplest to do it with N2, not energy.)



Answer: 4.5 rad/s

Clicker quiz



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is no slipping of the rope on either wheel; both wheels accelerate at the same rate. The tension in the rope is

a. Largest between the disks (red arrows)
b. Largest above the mass (blue arrows)
c. The same in both places.

(What's the difference with our old "massless pulleys"?)

Angular Correspondences Review

Kinematics	Distance: x	Angle: θ
	Velocity: v	Angular velocity: ω
	Acceleration: a	Angular acceleration: α
	$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
	$v^2 = v_0{}^2 + 2a(x - x_0)$	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$
Mass	Mass: m	Moment of inertia: I
Force/Newton's 2nd Law		
	Force: F	Torque: τ
	$\sum \vec{F} = m\vec{a}$	$\sum \tau = I\alpha$
Energy	$KE_{trans} = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$
Momentum…	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	Angular momentum??