

Announcements – Thurs, 23 Oct 2014

1. **Exam 2** starts Oct 30, a week from today.
 - a. Late fee on Monday Nov 3, after 2 pm
 - b. Closes on Tuesday Nov 4, 2 pm
 - c. Jerika exam reviews, both in room C295 ESC:
 - i. Wed Oct 29 7 - 8:30 pm
 - ii. Thurs Oct 30 5:30 - 7 pm
 - d. Exam covers through today's lecture
 - i. Ch. 5, 6, 7.1-7.3, 8
 - ii. HW 10-17

2. Equations from last time:

Energy: $KE_{trans} = \frac{1}{2}mv^2$

Force: $\sum \vec{F} = m\vec{a}$

$KE_{rot} = \frac{1}{2}I\omega^2$

$\sum \tau = I\alpha$

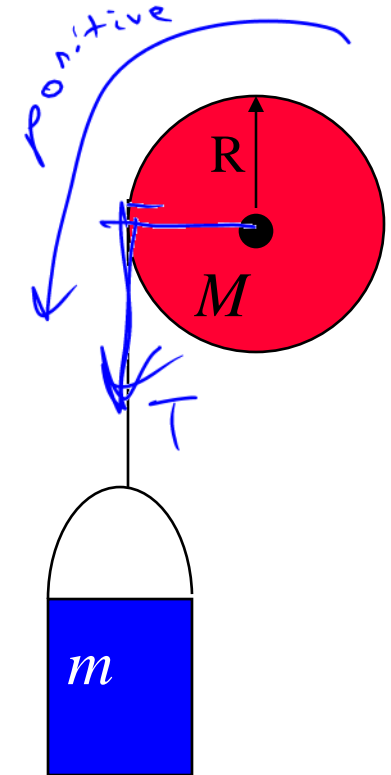
3. Equation from today:

Momentum: $p = mv$

ang. momentum = ??
 $I \cdot \omega$?

“Which of the problems from last night's HW assignment would you most like me to discuss in class today?”

Worked problem from last time: A falling mass starts a cylinder rotating (not a “massless pulley”). What is the acceleration of m ?



Draw FBDs

Write equations, plug in $\alpha = a/r...$

Cylinder

$$\alpha = \frac{a}{R}$$

Pail

$$\Sigma \tau_p = I \alpha$$

$$\Sigma F = ma$$

$$TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$$mg - T = ma$$

Solve simultaneous equations for a (and T , if desired)

$$T = \frac{1}{2}Ma$$

$$mg - \frac{1}{2}Ma = ma$$

$$mg = ma + \frac{1}{2}Ma$$

$$mg = a \left(m + \frac{1}{2}M\right)$$

Answer: $a = \frac{m}{m + M/2}g$

$$a = \frac{mg}{m + \frac{M}{2}}$$

$\theta = \omega r$ $\Sigma = \omega r$ $v = \omega r$ $a = \alpha r$

What if you just want to know v_f (given a distance d)?

$E_{\text{before}} + \cancel{W} = E_{\text{after}}$

$PE_g = KE_{\text{bucket}} + KE_{\text{cylinder}}$

$mgd = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
 $\frac{1}{2}(\frac{1}{2}Mr^2)(\frac{v^2}{r^2})$

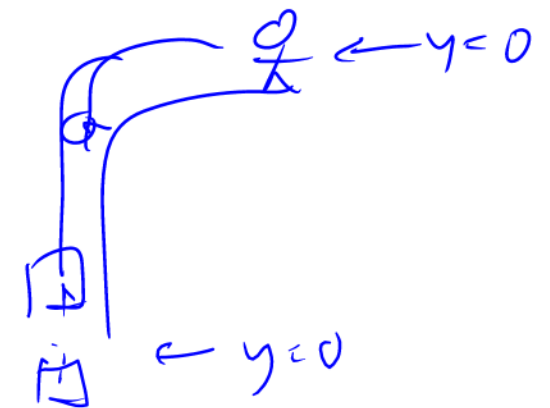
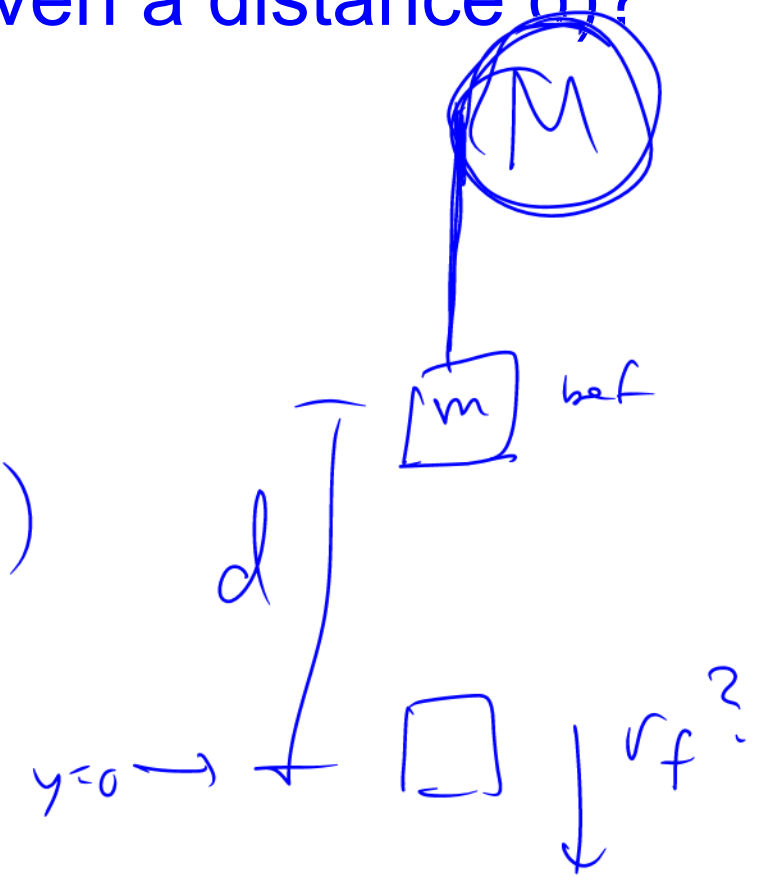
$mgd = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$

$mgd = v^2(\frac{1}{2}m + \frac{1}{4}M)$

$\frac{1}{2}m + \frac{1}{4}M$

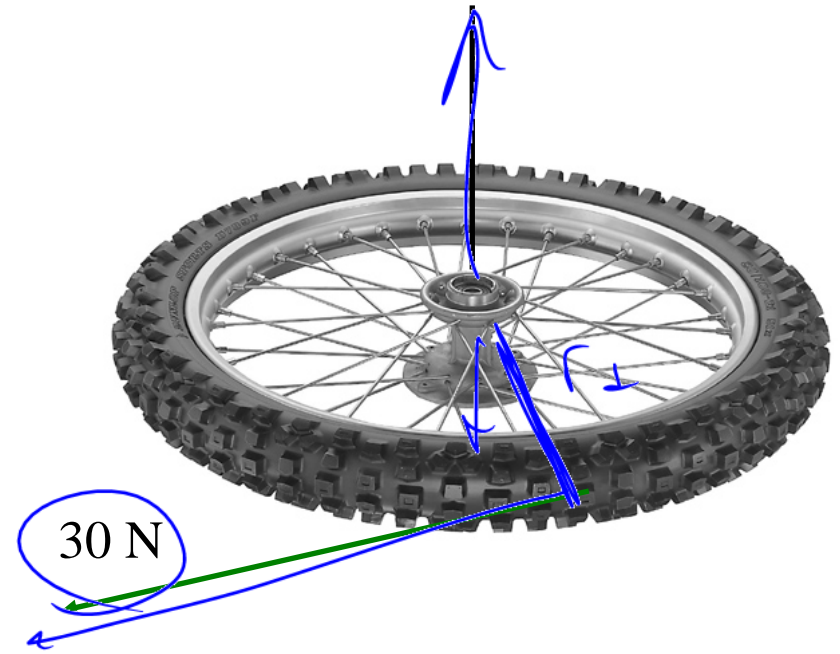
$v = \sqrt{\frac{mgd}{\frac{1}{2}m + \frac{1}{4}M}}$

Answer: $v_f = \sqrt{\frac{mgd}{\frac{1}{2}m + \frac{1}{4}M}}$



Worked Problem

A bicycle tire ($r = 0.4 \text{ m}$, $I = 0.8 \text{ kg}\cdot\text{m}^2$) is hanging from a string from the ceiling, not moving. You push tangentially on the edge with a 30 N force for 0.3 seconds. What is ω_f ? (*Hint: because time is given, might be simplest to do it with N2, not energy.*)



$$F = ma$$

$$a = \frac{30 \text{ N}}{m}$$

$$v_f = \cancel{v_0} + at$$

Answer: 4.5 rad/s

$$\tau = I \alpha$$

$$(30 \text{ N})(0.4 \text{ m}) = (0.8 \text{ kg}\cdot\text{m}^2) \alpha$$

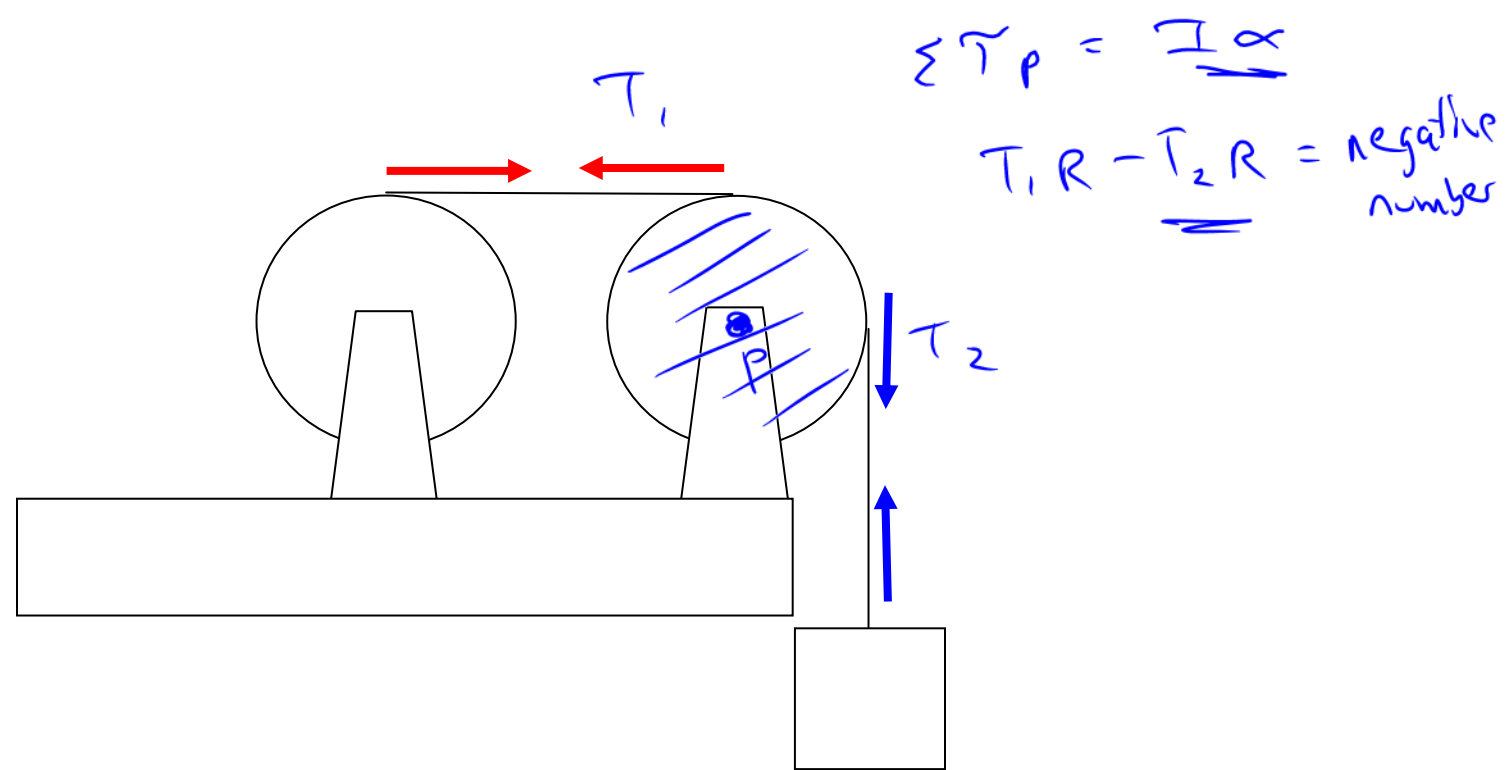
$$\alpha = \underline{\underline{15 \text{ rad/s}^2}}$$

$$\omega_f = \cancel{v_0} + \alpha t$$

$$\omega_f = (15 \frac{\text{rad}}{\text{s}^2})(0.3 \text{ s})$$

$$= \boxed{4.5 \frac{\text{rad}}{\text{s}}}$$

Clicker quiz



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is no slipping of the rope on either wheel; both wheels accelerate at the same rate. The tension in the rope is

- Largest between the disks (red arrows)
- Largest above the mass (blue arrows)
- The same in both places.

(What's the difference with our old "massless pulleys"?)

Angular momentum

Imagine a mass m on a thin rod moving in a circle, with constant speed v . It has linear momentum

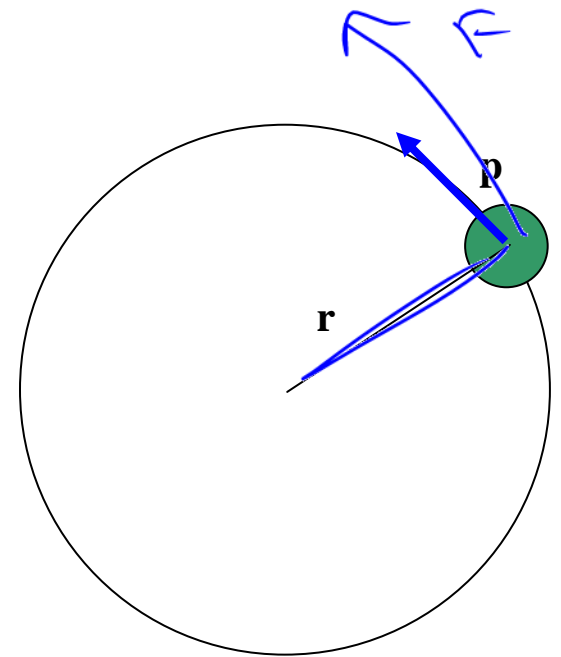
$$\vec{p} = \underline{mv}$$

Is \vec{p} constant? No

Is magnitude of \vec{p} constant? Yes

What do we need in order to affect magnitude of \vec{p} ?

Tangential Force \rightarrow torque



Derivation of Angular Momentum

Force-momentum relationship

Start with Newton 2:

$$\sum \vec{F} = m\vec{a}$$

$$= m \frac{\Delta v}{\Delta t}$$

$$\sum F = \frac{\Delta(mv)}{\Delta t}$$

If not F_{net}

then no $\Delta(mv)$

$\rightarrow p$ is cons.

**Momentum is conserved
if no net external force**

Torque-ang. mom. relationship

$$\sum \tau = I\alpha$$

$$= I \frac{\Delta \omega}{\Delta t}$$

$$\sum \tau = \frac{\Delta(I\omega)}{\Delta t}$$

if no τ_{net}

then no $\Delta(I\omega)$

$\rightarrow I\omega$ is cons

Define

$$L = I\omega$$

units of L ?

$$\text{kg} \frac{\text{m}^2}{\text{s}}$$

**Angular momentum is conserved
if no net external torque**

Conservation of Angular momentum blueprint

$$\boxed{\Sigma L_{bef} = \Sigma L_{aft}} \rightarrow \text{if and only if } \textit{no net external torque}$$

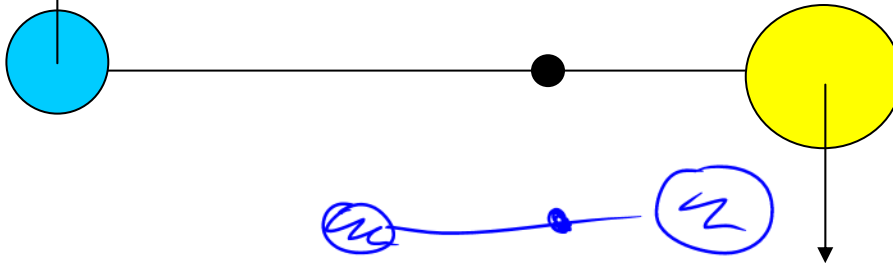
compare

$$\Sigma \vec{p}_{bef} = \Sigma \vec{p}_{aft} \quad \text{if no external force}$$

Problem

$$I = I_1 + I_2$$
$$= m_1 r_1^2 + m_2 r_2^2$$

$$L_{\text{before}} = L_{\text{after}}$$
$$I_{\text{before}} \omega_{\text{before}} = I_{\text{after}} \omega_{\text{after}}$$



Two space stations are connected by a cable. They are rotating about their center of mass. Someone in the blue station pulls the cable in so they are each closer to the center of rotation. What happens?

$$I \rightarrow$$

therefore $\omega \uparrow$

Demo: Hoberman sphere

Clicker quiz

Is rotational kinetic energy conserved in the Hoberman sphere? The final KE is _____ as the initial KE:

- a. more
- b. less
- c. the same

Hint: is there any non-conservative work done?

From warmup

Rotating stool, student with weights. What happens to her moment of inertia as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same



What happens to her rotational speed as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same

What happens to her rotational kinetic energy as she pulls in the weights?

- a. increases
- b. decreases
- c. remains the same

Demo: Spinning chair

Worked Problem



A skater has an initial ω of 2 rad/s and $I = 30 \text{ kg}\cdot\text{m}^2$. When she brings in her arms, $I = 10 \text{ kg}\cdot\text{m}^2$. What is her final ω ?

$$L_{\text{bef}} = L_{\text{aft}}$$
$$I_0 \omega_0 = I_f \omega_f$$
$$(30 \text{ kg}\cdot\text{m}^2)(2 \text{ rad/s}) = (10 \text{ kg}\cdot\text{m}^2) \omega_f$$

$$\omega_f = 6 \text{ rad/s}$$

How much work did it take to do this?

$$E_{\text{bef}} + W = E_{\text{aft}}$$
$$W = E_{\text{aft}} - E_{\text{bef}}$$
$$= \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_0 \omega_0^2$$
$$= \frac{1}{2} (10)(6^2) - \frac{1}{2} (30)(2^2) \quad \text{J}$$
$$= \underline{100 \text{ J}} - \underline{60 \text{ J}} = \boxed{120 \text{ J}}$$

Answers: 6 rad/s, 120 J

Videos

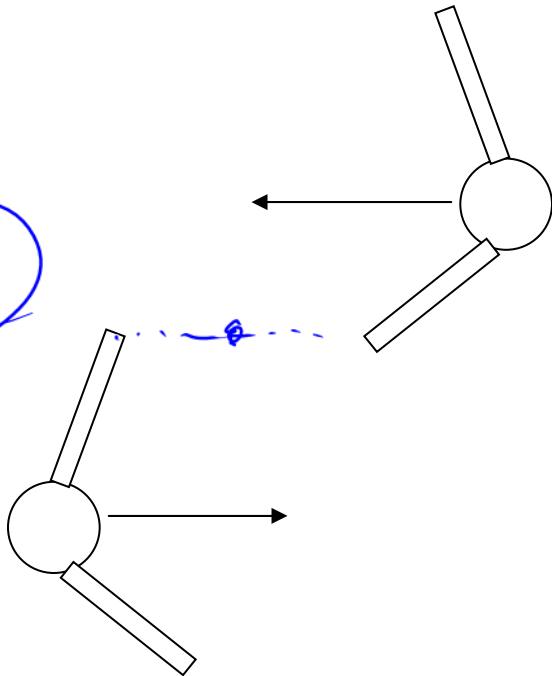
- marbles and funnel
- train on circular track
- pocket watch

Food for thought: two skaters joining hands

Before

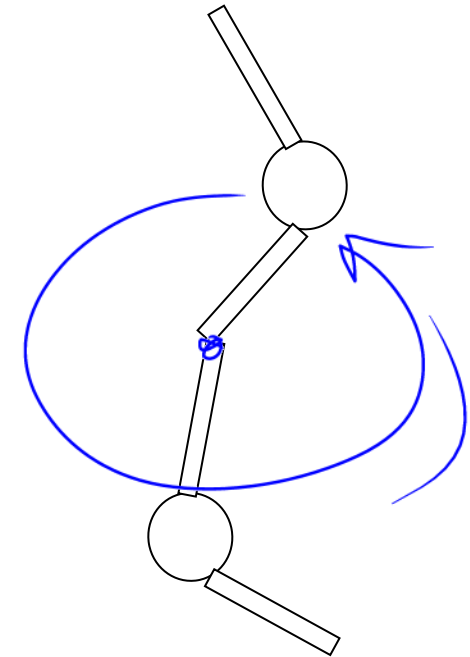
Any L?

yes



After

Any L?



Angular momentum conserved \Leftrightarrow No external torque
(system=both skaters)

Clicker quiz: Is there an external torque here? I.e. was angular momentum conserved?

- a. Yes external torque/ang. mom. not conserved
- b. No external torque/ang. mom. is conserved

"Hidden" angular momentum

$$\boxed{L = r_{\perp} p} \quad (= r p_{\perp} = r p \sin\theta)$$

Derivation:

$$\tau = r_{\perp} F$$

$$\tau = r_{\perp} \left(\frac{\Delta p}{\Delta t} \right)$$

$$\tau = \frac{\Delta(r_{\perp} p)}{\Delta t}$$

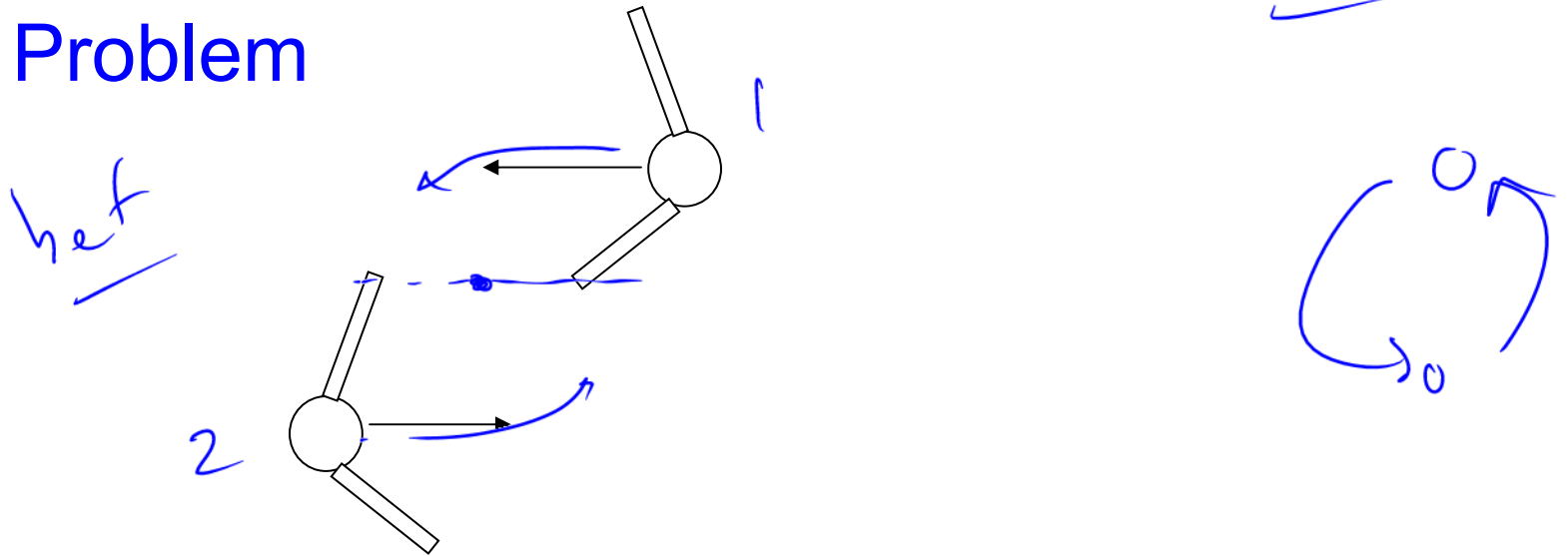
$$F = \frac{\Delta p}{\Delta t}$$
$$\Delta p = F \Delta t$$

$$\tau = \frac{\Delta L}{\Delta t}$$

But remember τ also $= \Delta L / \Delta t$!

$$L = r_{\perp} p$$

Worked Problem



The skaters have 0.7 m arms and are each 62 kg. They come together at 3.5 m/s. How fast (rad/s) are they turning afterwards?

$$\sum L_{\text{bef}} = \sum L_{\text{aft}}$$

$$r_{\perp 1} p_1 + r_{\perp 2} p_2 = I_{\text{tot}} \omega_f$$

$$\frac{(.7\text{m})(62\text{kg} \cdot 3.5\text{m/s}) + (.7\text{m})(62\text{kg} \cdot 3.5\text{m/s})}{2 \cdot 62\text{kg} (.7\text{m})^2} = \frac{I_1 + I_2}{2mr^2} \omega_f$$

Answer: 5 rad/s

$$\omega_f = 5 \frac{\text{rad}}{\text{s}}$$

L is a vector

With **no external torques...**

...both magnitude and direction of L stay the same

Demo: gyroscope

With **external torques?**

From warmup: <http://science.howstuffworks.com/gyroscope1.htm>

Ralph watched the video with the bicycle wheel, but became very confused. He had learned that angular momentum is conserved, but in this case isn't the angular momentum of the wheel constantly changing in direction as the wheel spins around. What's up?



“Think-pair-share”

- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

Clicker: I am now ready to share my answer if randomly selected.

a. Yes

Note: you are allowed to "pass" if you would really not answer.

Demo

Bicycle wheel

Demo: Angular momentum with external torque

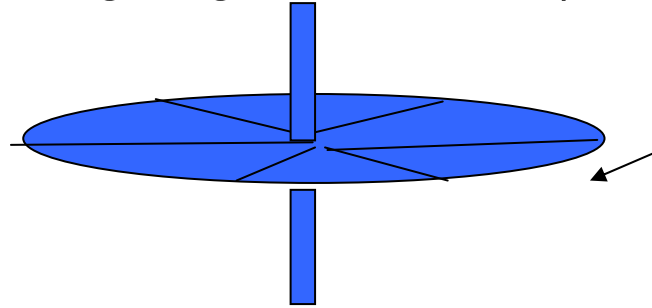
(wacky briefcase)

Demo: briefcase

To fully describe what happens to angular momentum with external torque takes more math than we have... just understand that strange things can happen. 😊

Clicker quiz

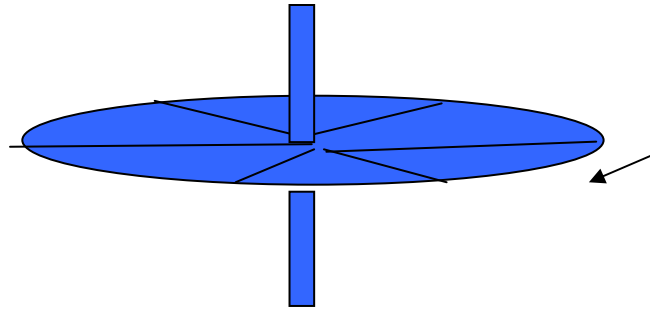
José sits still on frictionless ice, holding a bicycle wheel that's already spinning. Viewed from above it is going **clockwise** (CW).



If he grabs on to the wheel edge firmly and stops it from spinning he will:

- Start to turn CW (viewed from the top)
- Start to turn CCW
- Remain sitting without turning

Clicker quiz



José still on frictionless ice holding this spinning wheel. Viewed from above it is going **clockwise** (CW).

If, instead of stopping the wheel, he carefully turns it over so it is going CCW (viewed from the top), he will start to:

- Turn CW, but slower than in the previous problem
- Turn CCW, but slower than in the previous problem
- Turn CW, but faster than in the previous problem
- Turn CCW, but faster than in the previous problem
- Remain sitting without turning

Demos: rotating platform, bicycle wheel

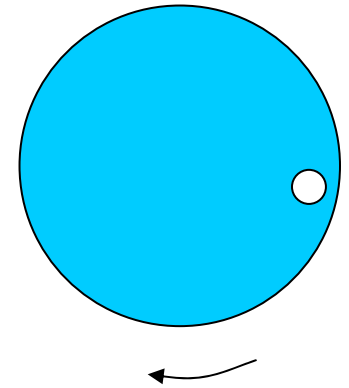
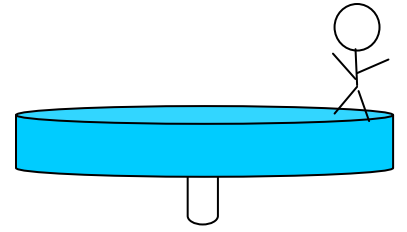
Demo: double bicycle wheels

Clicker quiz

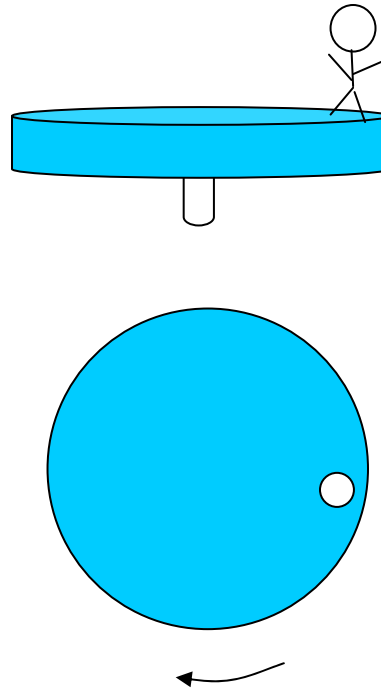
What will happen to the **rotational speed ω** of the merry-go-round if the girl...

...walks towards the center?

- a. it slows down
- b. it stays same speed
- c. it speeds up



Clicker quiz

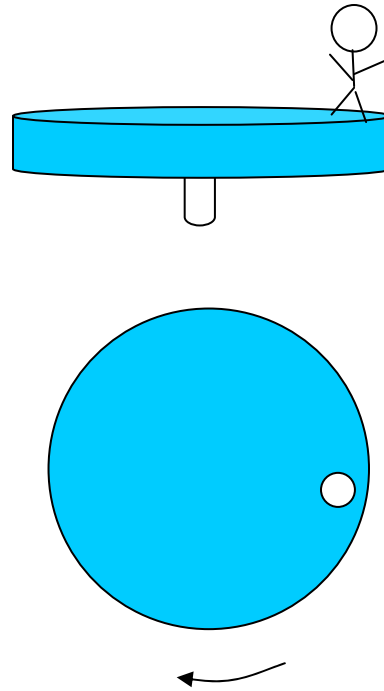


...starts running opposite to the spinning so she is at rest vs the ground?

- a. it slows down
- b. it stays same speed
- c. it speeds up

HINT: Sometimes it's easier to think of the **forces (torques)** she puts on the merry-go-round to change, rather than conservation of L .

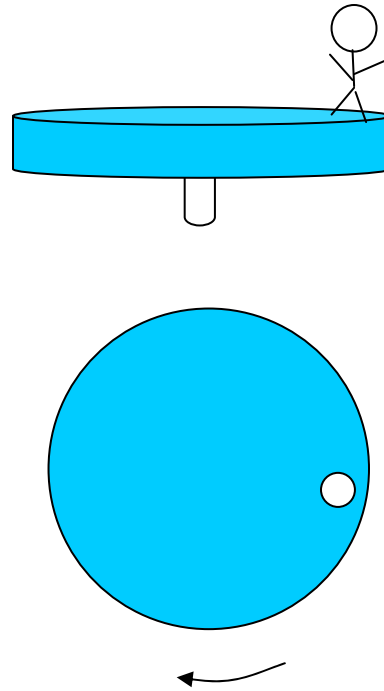
Clicker quiz



...slips off when she steps on a frictionless icy part?

- a. it slows down
- b. it stays same speed
- c. it speeds up

Clicker quiz



...throws her shoe off tangentially in the direction she's moving?

- a. it slows down
- b. it stays same speed
- c. it speeds up