

Announcements – 11 Nov 2014

1. Prayer
2. Exam 3 starts two weeks from today!
 - a. Covers Ch 9-12, HW 18-24
 - b. Starts Tues before Thanksgiving
 - c. Late fee on Wed after Thanksgiving, 3 pm
 - d. Closes on Thursday after Thanksgiving, 3 pm
3. Photo contest submissions due on Dec 5

“Which of the problems from last night's HW assignment would you most like me to discuss in class today?”

Worked Problem

cold $\left| \begin{array}{c} \xrightarrow{q} \\ \xleftarrow{q} \end{array} \right|$ hot

You foolishly decide to build the walls of your new house out of solid aluminum, 5 cm thick. As a result, in the wintertime heat leaks out like a sieve. How much money will this cost you each day? The inside temp is 70° F (21.1° C), the average outside temperature is 25° F (-3.9° C). The surface area is 280 m². The gas company charges you \$0.89 per “therm” (1.055×10^8 J). Only count heat loss through conduction.

$$\frac{Q}{t} = k \frac{A \Delta T}{l}$$

$$Q = t \cdot k \frac{A \Delta T}{l}$$

$$Q = \frac{(24 \cdot 3600 \text{ s})(238 \frac{\text{W}}{\text{cm} \cdot \text{K}})(280 \text{ m}^2)(21.1^\circ \text{C} - -3.9^\circ \text{C})}{0.05 \text{ m}} \times \frac{\$0.89}{1.055 \cdot 10^8 \text{ J}}$$

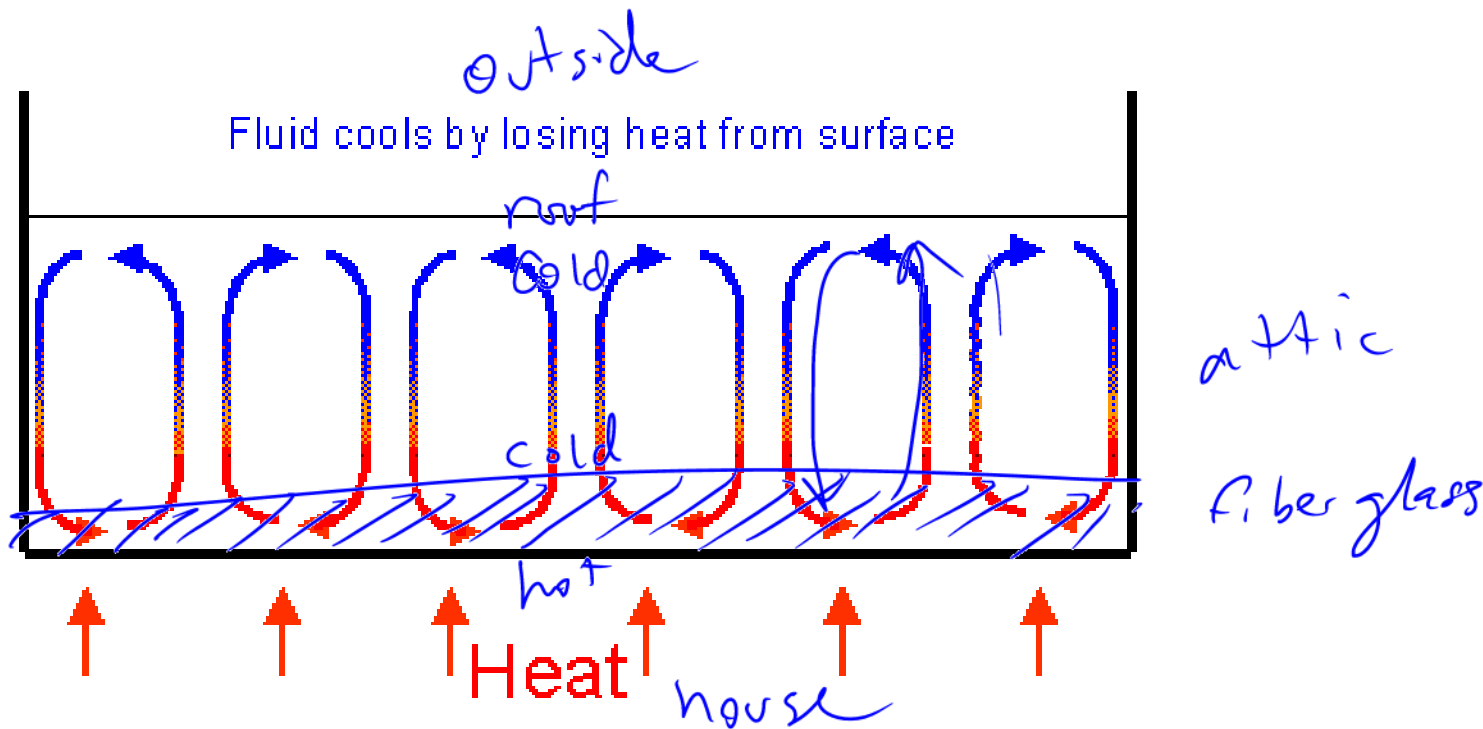
$$= \$24,286$$

Answer: \$24,286. Yikes!

Thermal convection

If air is a good thermal insulator why use fiberglass in attics?

Material	k (J/s·m·°C)
Copper	397
Aluminum	238 ←
Iron	79.5
Glass	0.84
Wood	0.10
Air	0.0234



Convection cell

Warm, low density fluid rises
Cool, high density fluid sinks

σ Σ

Blackbody Radiation

Hot objects glow!

“Glow” carries away energy

$P_{lost} = e\sigma A(T_{object})^4$

Surface area (pointing to A)

e (circled in blue)

Power: watts = heat/time

$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$
 (“Stefan-Boltzmann constant”)

e : “emissivity” between 0 and 1

Aluminum, highly polished: $e \approx 0.05$ ←

Aluminum, anodized (black): $e \approx 0.8$ ←

→ Depends on material, surface, shape, temperature, etc.



From warmup

If the temperature of a "black body" doubles, how much does its rate of energy emission change?

a. $\times 2$

b. $\times 4$

c. $\times 8$

d. more

$\times 16$

2^4

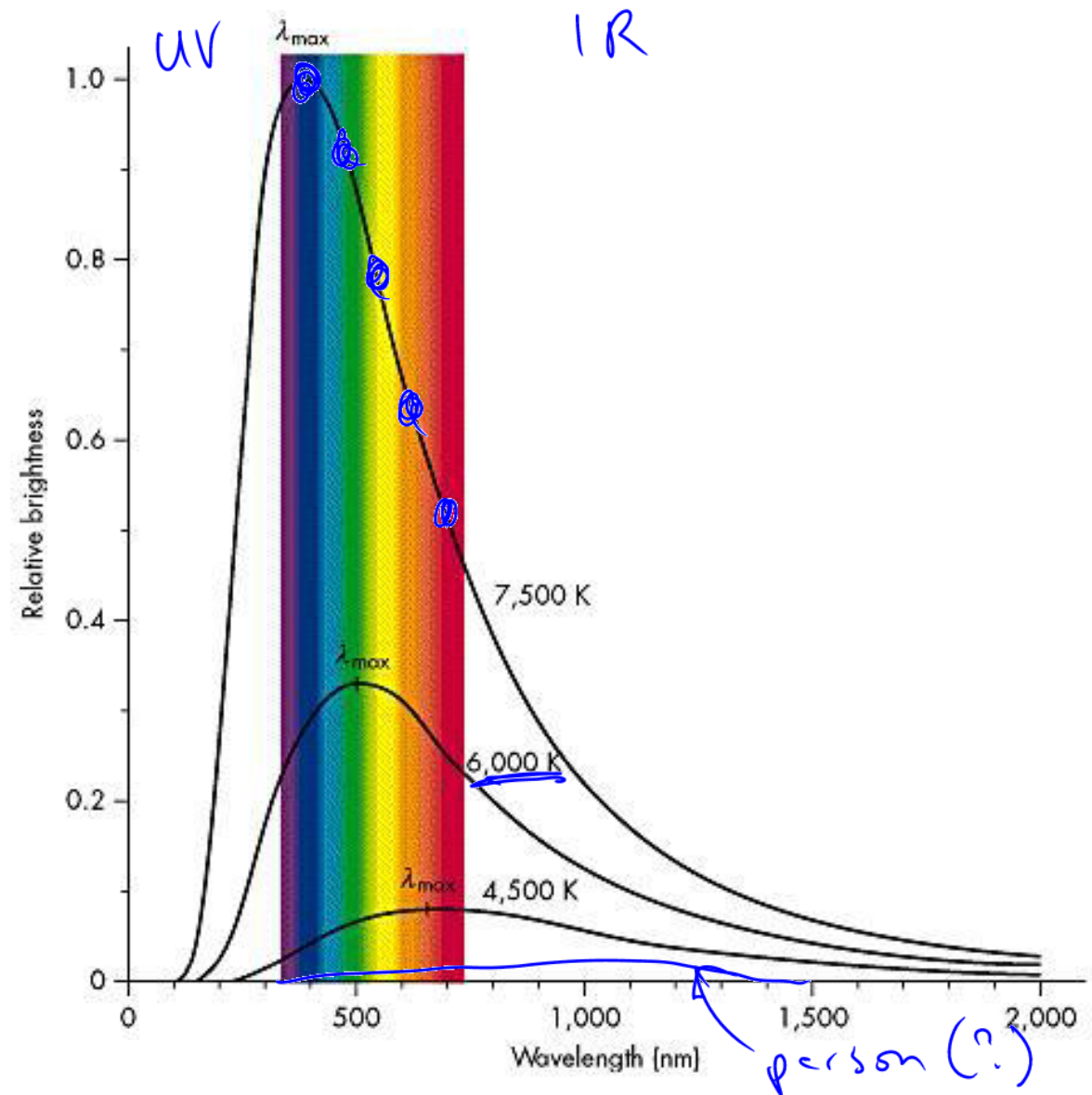
But wait! Surroundings are also glowing!

$$P_{\text{gained}} = e\sigma A (T_{\text{surroundings}})^4 \quad \text{absorbed by the object}$$

Net power radiated away = $P_{\text{lost}} - P_{\text{gained}}$

$$\begin{aligned} &= e\sigma A T_{\text{object}}^4 - e\sigma A T_{\text{surrounding}}^4 \\ &= e\sigma A (T_{\text{obj}}^4 - T_{\text{surr.}}^4) \end{aligned}$$

“Color” of emission, IR thermometers



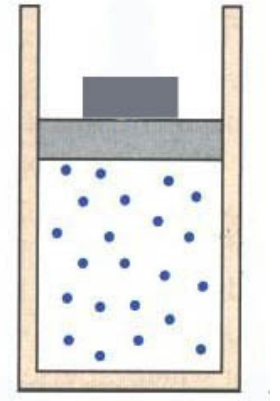
Gases

Boyle's Law: Hold T constant, increase P...

Volume decrease

Charles' Law: Hold P constant, increase T...

Volume increase



Émile Clapeyron, 1834: Combined the experimental results into...

$$PV = Nk_B T$$

Ideal gas law! “Physics version”

$$k_B = 1.381 \times 10^{-23} \text{ J/K}$$

Boltzmann's constant

Important:

P in pascal

V in m^3

T in Kelvin

N is number of molecules

Back to microscopic view of heat...

Ideal gases

1. Molecules bounce off each other like superballs (elastic)
2. They do not stick (no attractive forces)
3. Never condense into liquids or solids
4. Are like “frictionless surfaces”, “massless pulleys”, “perfect fluids”, etc.

A good model for real gases as long as the gas is far from condensing

From warmup

Suppose we have two jars of gas: one of helium and one of neon. If both jars have the same volume, and the two gases are at the same pressure and temperature, which jar contains the greatest number of gas molecules? (Both gases obey the ideal gas law. The mass of a neon molecule is greater than the mass of a helium molecule.)

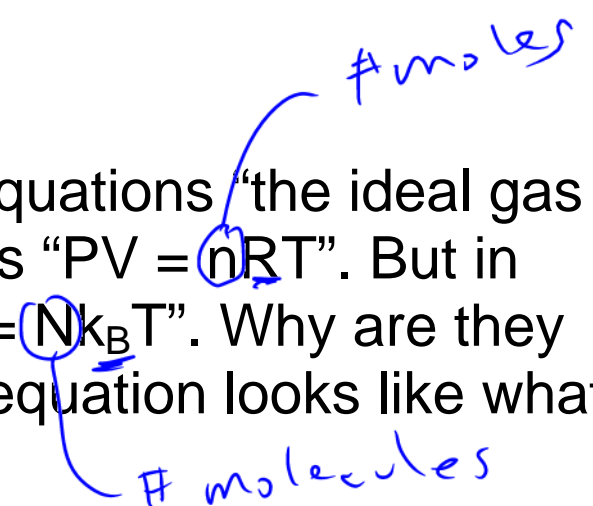
- a. jar of helium
- b. jar of neon
- ☒ c. same number

$$pV = Nk_B T$$

↑
same

From warmup

Ralph is confused...the book calls two different equations “the ideal gas law”. In equation 10.8 (8th edition), the equation is “ $PV = nRT$ ”. But in equation 10.11 (8th edition), the equation is “ $PV = Nk_B T$ ”. Why are they both called the ideal gas law, when only the first equation looks like what he learned in chemistry?



“**Pair share**”—I am now ready to share my neighbor’s answer if called on.
a. Yes

Ideal gas law

in K

$$PV = nRT$$

“Chemistry version”
(useful for Physics, too...)

Pa m³ { $R = 8.314 \text{ J/mol}\cdot\text{K}$

Universal gas constant

→ don't use $R = 0.0821 \text{ liter}\cdot\text{atm/mol}\cdot\text{K}$

Important:

P in pascal

V in m³

T in Kelvin

n is number of *moles*

Connection: $R = N_A \times k_B$

↑ $(6.022 \cdot 10^{23}) (1.38 \cdot 10^{-23})$

Avogadro's Number

$$N_A = 1 \text{ mole} = 6.622 \cdot 10^{23} = \text{"Avagadro's number"}$$

$$\# \text{ moles} = \# \text{ molecules} / N_A$$
$$n = \underline{N / N_A}$$

ex. $MM_{Al} = 26.98 \frac{g}{mol}$

"molar mass": mass of one mole
(careful: commonly given in *grams*)

$$n = m_{tot} / MM$$

May need to
convert to kg!

$$n = \frac{1 \text{ kg}}{26.98 \frac{kg}{mol}} = \text{--- mol}$$

Clicker quiz

Which will shrink more when cooled to 77K? (I'll use liquid nitrogen)

a. helium balloon

b. air balloon

c. same

Demo: Liquid nitrogen and balloons

$$pV = nRT$$

from 300 K to 77 K

$$\sim \frac{1}{4}$$
$$V \sim \frac{1}{4}$$

Worked Problem

In an engine piston, with air at 1 atm, the volume is decreased from 200 cm^3 to 40 cm^3 , while the temperature increases from 300 K to 600 K. Find the final pressure.

$$pV = nRT$$

Method 1: Find N (or n)

$$n = \frac{pV}{RT} = \frac{(1.01 \cdot 10^5 \text{ Pa})(200 \cdot 10^{-6} \text{ m}^3)}{8.31 \frac{\text{J}}{\text{mol K}} \cdot 300 \text{ K}} = \underline{0.0081 \text{ moles}}$$

$$p = \frac{nRT}{V} = \frac{(0.0081 \text{ mol})(8.31 \frac{\text{J}}{\text{mol K}})(600 \text{ K})}{40 \cdot 10^{-6} \text{ m}^3} = \boxed{1.01 \cdot 10^6 \text{ Pa}}$$

Answer: $1.01 \times 10^6 \text{ Pa}$, 10 atm

Method 2: ratios

n is the same!

$$PV = nRT \rightarrow n = \frac{PV}{RT}$$

$$\frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right) \left(\frac{T_2}{T_1} \right)$$

$$= (1 \text{ atm}) \left(\frac{200 \cancel{\text{cm}^3}}{40 \cancel{\text{cm}^3}} \right) \left(\frac{600 \cancel{\text{K}}}{300 \cancel{\text{K}}} \right)$$

$$= \boxed{10 \text{ atm}}$$

Answer: $1.01 \times 10^6 \text{ Pa}$, 10 atm

Worked Problem

$$\underline{p} V = n R T$$

How much volume will 1 liter of liquid nitrogen fill when it becomes gas?

Density of LN = 0.807 g/cm³

Molar mass of N₂ = 28 g/mol

Temperature in this room = about 70° F (= 294.3 K)

Atmospheric pressure in Provo = 0.85 atm

$$1000 \text{ cm}^3 \times \frac{0.807 \text{ g}}{1 \text{ cm}^3} \times \frac{1 \text{ mol}}{28 \text{ g}} = \frac{28.8}{\text{mol}}$$

$$V = \frac{nRT}{p} = \frac{(28.8 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol K}} \right) (294.3 \text{ K})}{0.85 \cdot 1.01 \cdot 10^5 \text{ Pa}}$$

$$= 0.821 \text{ m}^3$$

$$= \boxed{821 \text{ L}}$$

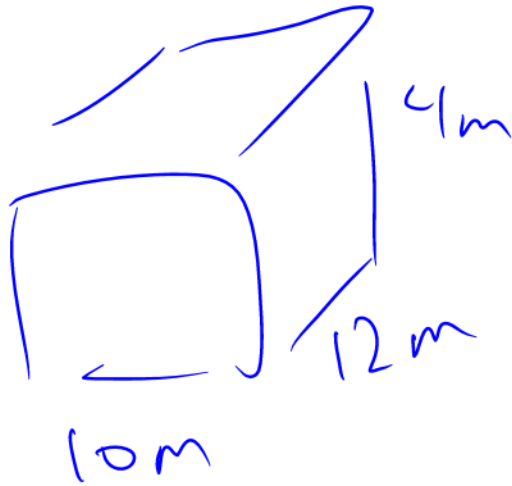
Demo: Liquid nitrogen tower

Answer: 821 L

$$pV = nRT$$

Worked Problem

What is the mass of all the air in this room? The average molar mass of the molecules in air (mainly nitrogen and oxygen) is 29.0 g/mol.



$$n = \frac{pV}{RT}$$

$$= \frac{(.85 \cdot 1.01 \cdot 10^5 \text{ Pa}) (10 \text{ m} \times 4 \text{ m} \times 12 \text{ m})}{(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) (300 \text{ K})}$$

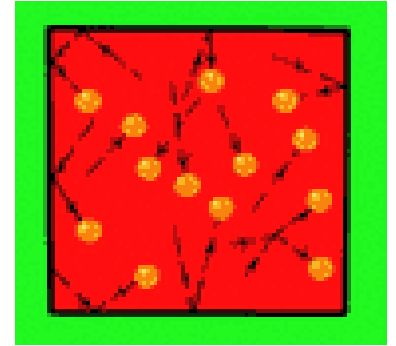
$$= 16500 \text{ mol} \times \frac{.029 \text{ kg}}{1 \text{ mol}}$$

$$= \boxed{4791 \text{ kg}}$$

Answer: more than you'd expect!

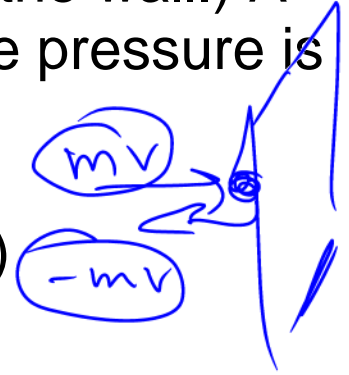
Molecular View of Pressure

Pressure: Comes from collision forces of molecules hitting wall



Related problem: You throw baseballs (mass 145 g) at a wall (area 9 m^2), at a speed of 85 mph (38 m/s). The collisions are elastic, and last for 0.05 seconds. (This is the time the ball is in contact with the wall.) A baseball hits the wall every 0.5 seconds. How much average pressure is generated by the baseballs? (We'll do this in three steps)

(a) How much force is generated by each hit? (Use impulse)



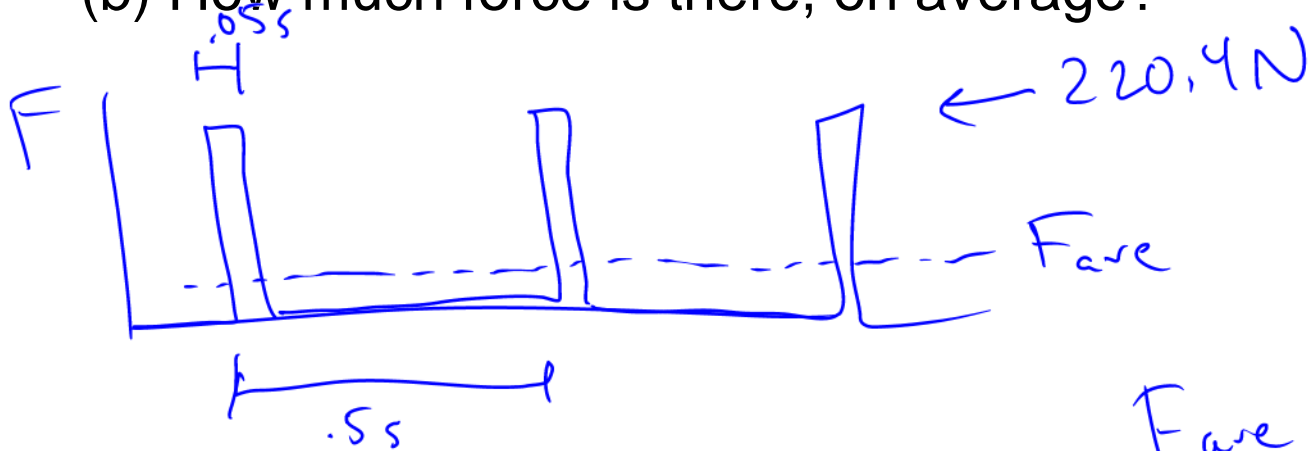
$$F \Delta t = \Delta p$$

$$F_{\text{collision}} = \frac{\Delta p}{\Delta t} = \frac{2mv}{t_{\text{collision}}} = \frac{2 \cdot (.145 \text{ kg})(38 \text{ m/s})}{.05 \text{ sec}}$$

$$= \boxed{220.4 \text{ N}}$$

Answer: 220.4 N

(b) How much force is there, on average?



$$F_{ave} = \left(\frac{.055}{.55} \right) (220.4 \text{ N}) = \boxed{22.04 \text{ N}}$$

(c) How much overall pressure is generated by the balls?

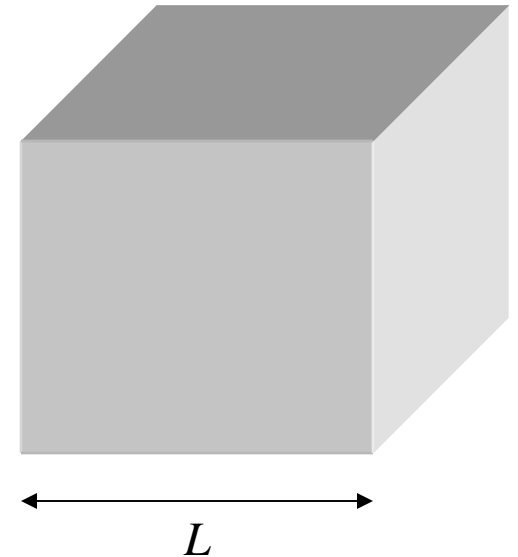
$$p = \frac{F_{ave}}{\text{area}} = \frac{22.04 \text{ N}}{9 \text{ m}^2} = \boxed{2.449 \text{ Pa}}$$

Answers: 22.04 N; 2.449 Pa

The actual problem

A cube filled with gas (focus on x-direction for now)

Molecules (mass m) hit the right wall, at a speed of v_x . Elastic collisions. How much pressure is generated by the molecules?



(a) How much force is generated by each hit?

(b) How much force is there from one molecule, on average?

Answers: $\frac{2mv_x}{\text{time of collision}}$; $\frac{mv_x^2}{L}$

(c) How much pressure is generated by the molecules?

(d) Expand to N molecules, and 3 dimensions ($v_x = v_y = v_z$). $P = ?$

Answers: $\frac{mv_x^2}{V}$; $\frac{Nm\left(\frac{1}{3}v^2\right)}{V}$

$$PV = Nm \left(\frac{1}{3} v^2 \right)$$

→ Does this remind you of anything?

$$T = \frac{m}{k_B} \frac{1}{3} v^2$$

$$\rightarrow \frac{1}{2} m v^2 = ?$$

This is in my “list of important equations”. Put it on your note card for the exam!

From warmup

An ideal gas has a mixture of heavy and light molecules at the same temperature. The molecules with the most [translational] KE are...

- a. heavy
- b. light
- c. same

Worked Problem

How fast are the oxygen molecules traveling in this room? (300 K)

molar mass = 32 g/mol, or $m = 5.31 \times 10^{-26}$ kg

Answer: 483.46 m/s (= 1081 mph!)

Kinetic Energy

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

$$KE_{\text{tot}} = \underline{\hspace{2cm}}$$

$$KE_x = \underline{\hspace{2cm}}$$

$$KE_y = \underline{\hspace{2cm}}$$

$$KE_z = \underline{\hspace{2cm}}$$

Degrees of Freedom

Each “degree of freedom” has energy of $\frac{k_B T}{2}$

This is called the “Equipartition theorem”. It’s only briefly mentioned in your book! And not by name!

See page 390, Section 12.2 in 8th edition:

“The total kinetic energy of a system is shared equally among all of its independent parts, on the average, once the system has reached thermal equilibrium.”