Announcements – 13 Nov 2014

A Cappella Jam
Tues 7⁰⁰ pm
Varsity Theater

1. Prayer

2. Exam 3 starts on ~~Tues Nov 25~~ *Mon Nov 24*
    a. Covers Ch 9-12, HW 18-24
    b. Late fee on Wed after Thanksgiving, 3 pm
    c. Closes on Thursday after Thanksgiving, 3 pm
    d. Jerika review sessions, both in C295 ESC
        i. Sat Nov 22, 10 - 11:30 am (before Thanksgiving)
   ii. Mon Dec 1, 5:30 - 7 pm   (after Thanksgiving)

3. Thanksgiving week:
    a. Homework is due on Monday, as usual
    b. Tuesday is a virtual Friday → we don't have class
    c. No classes on Wednesday
    d. Testing Center not open on Wed, Thurs, Fri, or Sat

4. Final Exam info - see email

"Which of the problems from last night's HW assignment would you most like me to discuss in class today?"

Molecular View of Pressure (cont. from last time)

Baseball problem: You throw baseballs (mass 145 g) at a wall (area 9 m^2), at a speed of 85 mph (38 m/s). The collisions are elastic, and last for 0.05 seconds. (This is the time the ball is in contact with the wall.) A baseball hits the wall every 0.5 seconds.
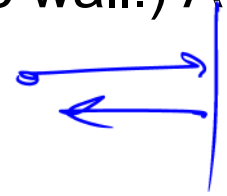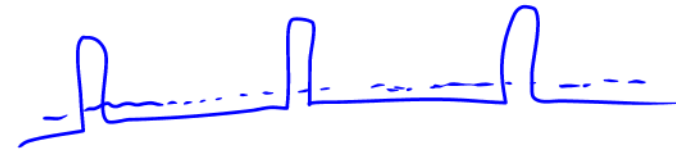
(a) How much force is generated by each hit? (Use impulse)

$$F\Delta t = \Delta p \quad 2mv$$

$$F = \frac{2(0.145\,\text{kg})(38\,\text{m/s})}{0.05\,\text{s}} = 220.4\,\text{N}$$

$t_{collision}$

(b) How much force is there, on average?

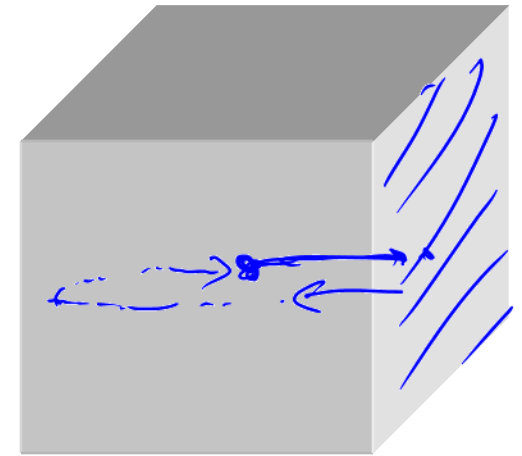$$F_{ave} = F_{peak}\frac{0.05\,\text{s}}{0.5\,\text{s}} = 22.04\,\text{N}$$

(c) How much overall pressure is generated by the balls?

$$P = \frac{F_{ave}}{A} = \frac{22.04\,\text{N}}{9\,\text{m}^2} = 2.449\,\text{Pa}$$

The actual problem

A cube filled with gas (focus on x-direction for now)

Molecules (mass m) hit the right wall, at a speed of v_x. Elastic collisions. How much pressure is generated by the molecules?

L

(a) How much force is generated by each hit?

$$F_{peak} = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{t_{collision}}$$

$$t_{between} = \frac{distance}{v} = \frac{2L}{v_x}$$

(b) How much force is there from one molecule, on average?

$$F_{ave} = F_{peak} \cdot \frac{t_{collision}}{t_{between}}$$

$$\simeq \left(\frac{2mv_x}{t_{colli}} \right) \left(\frac{t_{coll}}{2L/v_x} \right) = \left(\frac{mv_x^2}{L} \right)$$

Answers: $\dfrac{2mv_x}{time\ of\ collision}$; $\dfrac{mv_x^2}{L}$

Colton - Lecture 22 - pg 4

(c) How much pressure is generated by the molecules?

$$P = \frac{F_{ave}}{area} = \frac{\frac{mv_x^2}{L}}{L^2} = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V}$$

$$\frac{\frac{1}{2}}{4} = \frac{1}{8}$$

(d) Expand to N molecules, and 3 dimensions ($v_x = v_y = v_z$). $P = ?$

$$P_{tot} = N\left(\frac{mv_x^2}{V}\right)$$

$$\boxed{P = \frac{N \cdot m \frac{1}{3}v^2}{V}}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$= v_x^2 + v_x^2 + v_x^2$$

$$v^2 = 3v_x^2$$

$$v_x^2 = \boxed{\frac{1}{3}v^2}$$

Answers: $\dfrac{mv_x^2}{V}$; $\dfrac{Nm\left(\frac{1}{3}v^2\right)}{V}$

$$PV = Nm\left(\tfrac{1}{3}v^2\right)$$

→ What does this remind you of?

$$PV = N\boxed{k_B T}$$

$$\frac{3}{2} \cdot m\tfrac{1}{3}v^2 = k_B T \,\frac{3}{2}$$

$$\boxed{\tfrac{1}{2}mv_{ave}^2 = \tfrac{3}{2}k_B T}$$

$$k_B = 1.38 \cdot 10^{-23}\ \text{J}/k$$

$$R = N_A k_B$$

$$8.31\ \tfrac{J}{mol\,k}$$

"kinetic theory equation"

$$T = \frac{m}{k_B} \frac{1}{3} v^2$$

$$\rightarrow \quad \frac{1}{2} mv^2 = ? \quad \frac{3}{2} k_B T$$

$$v_{ave} = \sqrt{\frac{3 k_B T}{m}}$$

This is in my "list of important equations". Put it on your note card for the exam!

What is *m*?

mass of a molecule (individual) in kg

Not the molar mass

From warmup (last time)

An ideal gas has a mixture of heavy and light molecules at the same temperature. The molecules with the most [translational] KE are…

    a. heavy

    b. light

    c. same

$$\underbrace{\frac{1}{2}mv^2} = \frac{3}{2}k_B T$$

Worked Problem

$m = \dfrac{.032 \, kg}{6.022 \cdot 10^{23}}$

How fast are the oxygen molecules traveling in this room? (300 K)
molar mass = 32 g/mol, or m = 5.31×10^{-26} kg

$$\frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

$$v = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3 \cdot \left(1.38 \cdot 10^{-23} \frac{J}{K}\right)(300 k)}{5 \cdot 31 \cdot 10^{-26}}}$$

$$= \boxed{483.846 \; m/s}$$

Answer: 483.46 m/s (= 1081 mph!)

Demos

Kinetic theory machine

Molecular speed

Clicker quizzes (review)

1. Which molecules have the most kinetic energy?
   a. The heavy ones
   b. The light ones
   c. Same

2. Which molecules have the fastest speed?
   a. The heavy ones
   b. The light ones
   c. Same

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT$$

$$KE = \frac{3}{2}k_BT$$

Internal energy, *U*

$$k_B = \frac{R}{N_A}$$

U = KE$_{ave}$ per molecule × number of molecules

$$= \left(\frac{3}{2} k_B T \right) \times N$$

$$= \frac{3}{2} \cdot N k_B T = \frac{3}{2} \frac{N R}{N_A} T$$

(this is for monatomic gases only, I'll explain why in a minute)

$$U = \frac{3}{2} n R T$$

$$\uparrow$$

$$8.31 \, J/mol\,k$$

Translational kinetic energy

$$KE_{tot} = \boxed{\frac{1}{2}mv^2 = \frac{3}{2}k_B T}$$

$KE_x = \quad \dfrac{1}{2}k_B T$

$KE_y = \quad \dfrac{1}{2}k_B T$

$KE_z = \quad \dfrac{1}{2}k_B T$

Degrees of Freedom

Each "degree of freedom" has energy of $\dfrac{k_B T}{2}$

This is called the "Equipartition theorem". It's only briefly mentioned in your book, and not by name. See page 390, Section 12.2 in 8th edition:

> "The total kinetic energy of a system is shared equally among all of its <u>independent parts</u>, on the average, once the system has reached thermal equilibrium."

<u>Independent parts</u>: larger for molecules with more than one atom
- rotational KE
- ~~vibrational KE~~

→ such molecules have more internal energy

Internal energy of diatomic gases

Near room temperature, turns out vibrational modes aren't activated (quantum mechanics)

Rotational degrees of freedom: 2

$$KE_{trans} = \frac{3}{2} k_B T \qquad\qquad \frac{1}{2} m_v{}^2$$

$$KE_{rot} = \frac{2}{2} k_B T \qquad\qquad \frac{1}{2} I \omega^2$$

$$KE_{tot} = \frac{5}{2} k_B T$$

$$\boxed{U = \frac{5}{2} n R T} \quad \text{diatomic molecules} \quad \left(= \frac{5}{2} N k_B T \right)$$

$$\frac{3}{2} n R T \quad \text{monatomic}$$

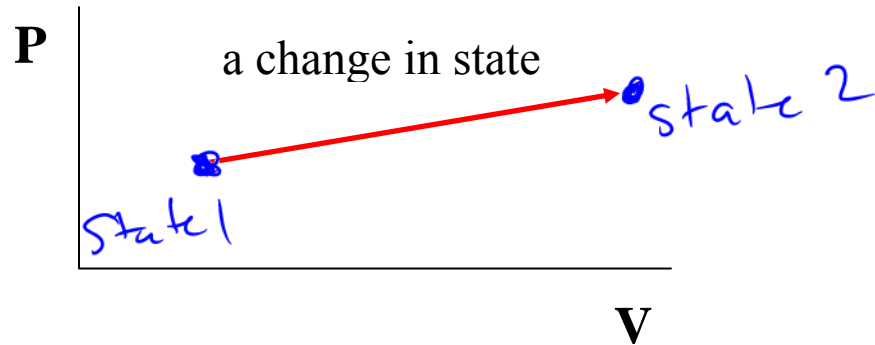# *Change* in internal energy

Monatomic:

$$U = \frac{3}{2} n R T$$

$$\Delta U = \frac{3}{2} n R \, \Delta T$$

Diatomic:

$$\Delta U = \frac{5}{2} n R \, \Delta T$$

P-V diagrams

$$PV = nRT$$

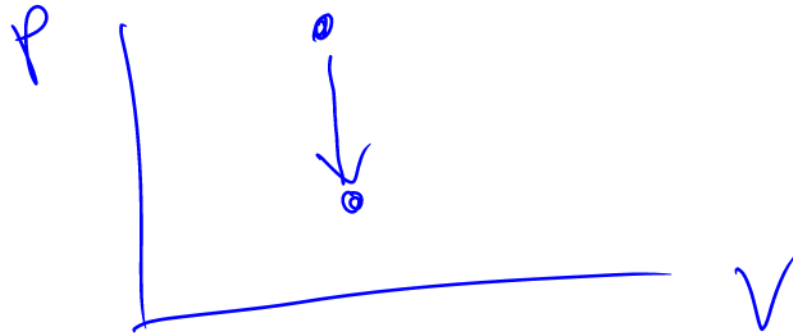**P**

a change in state

• state 2

state 1

V

State postulate: any
two independent
variables determine the
state: P, V, T, etc.

From warmup

A gas has its pressure reduced while its volume is kept constant. What does this look like on a PV diagram?

a. a horizontal line going to the right
b. a horizontal line going to the left
c. a vertical line going up
d. a vertical line going down

Clicker quiz

A gas in a piston expands from point A to point B on the P-V plot, via either path 1 or path 2. Path 2 is a "combo path," going down first and then over.

P   A T_A
2   1
    B T_B
V
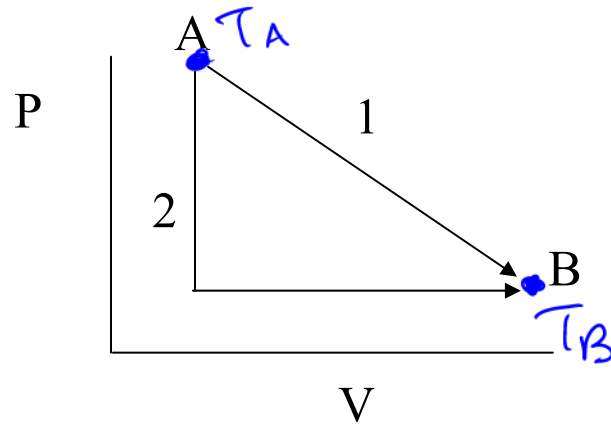
$$\Delta U = \frac{3}{2} n R \Delta T$$

$$\left(\frac{5}{2}\right)$$

$$= \frac{3}{2} n R \left(T_f - T_i\right)$$

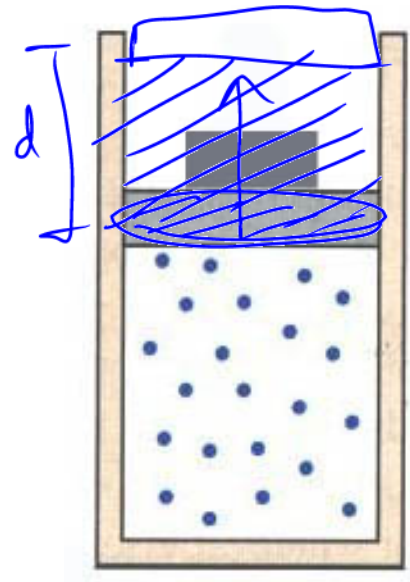The process in which ΔU from A to B is the greatest (magnitude) is:
 a. path 1
 b. path 2
 c. same

Describe both paths:

$$P = F/A \qquad F = P \cdot A$$

Work done by a gas, constant pressure

1 m^3 of an ideal gas at 300 K supports a weight in a piston such that the pressure in the gas is 200,000 Pa (about 2 atm). The gas is heated up. It expands to 3 m^3. How much work did the gas do as it expanded?

How do you know it did work? It exerted a force over a distance!

$$W_{by} = F \cdot d$$

$$= P \cdot A \cdot d$$

$$W_{by} = P \cdot \Delta V = (200000\,Pa)(3m^3 - 1m^3)$$

$$= \boxed{400000\ J}$$

$$P_{are} \cdot \Delta V$$

Result $\boxed{W_{by\ gas} = P\Delta V}$ (for constant P) 5th edition

$$W_{on\ gas} = -P\Delta V$$

Answer: 400,000 J

$W_{by\ gas} > 0$ when... gas expands

$$W_{on\ gas} < 0 \qquad "" \qquad ""$$

Work done <u>on</u> a gas

$$W_{\text{on gas}} = -P\Delta V$$

$6^{\text{th}} - 10^{\text{th}}$ editions

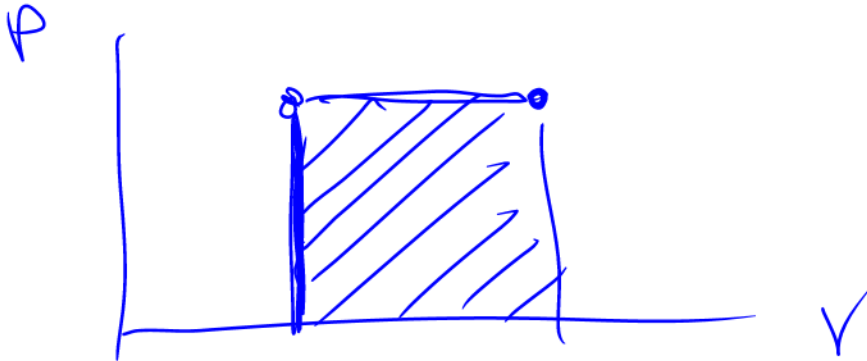(for constant P)

$W_{\text{on gas}} > 0$ when…   volume decreasing

$W_{\text{by gas}} < 0$ "   "
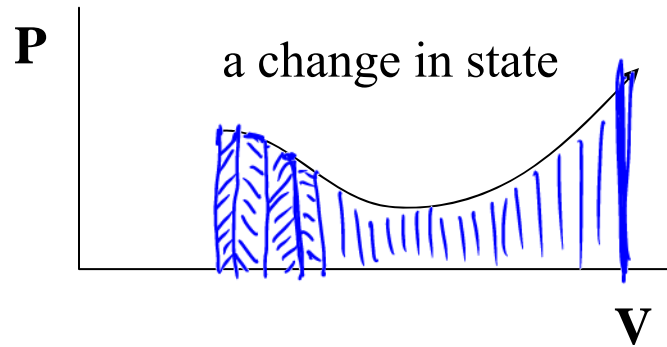
What is represented by $P\Delta V$ on a PV diagram?

Work done by a gas, changing pressure

What's the work if pressure is not constant?

P

a change in state

V

View #1:

$$W = P_{ave} \cdot \Delta V$$

P_{ave}

View #2: Area under the curve representing the state change

From warmup

Using a P-V diagram, the work done by a gas when expanding can be calculated via:

a. the area under the curve representing the process
b. the projection of the curve representing the process along the P-axis
c. the projection of the curve representing the process along the V-axis

Clicker Quiz

Same two paths as before.
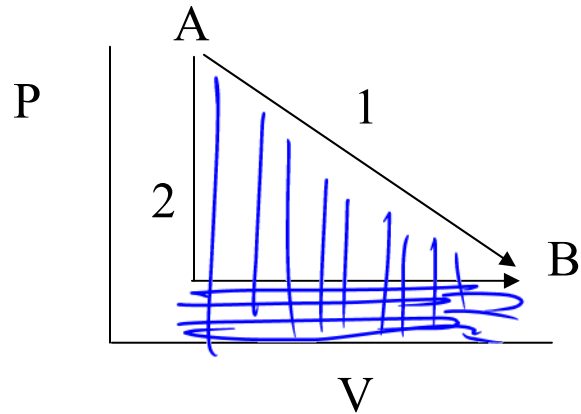
The gas does the most work in:
- a. path 1
- b. path 2
- c. same

1st Law of Thermodynamics

$$\Delta U = Q_{added} + W_{on\ system}$$

(note: 5th edition uses $-W_{by\ system}$)

System: the gas/object you are studying

What does it mean?? Use 5th edition version:

$\Delta U = Q_{added} - W_{by\ system} \rightarrow Q_{added} = \Delta U + W_{by\ system}$

Meaning of 1st Law:
Heat added can go either towards
* increasing internal energy (temperature), or
* doing work by the gas

From warmup

The first law of thermodynamics is a statement of:
a. conservation of energy
b. conservation of (regular) momentum
c. conservation of angular momentum
d. conservation of mass

From warmup

Ralph is confused because he knows that when you compress gases, they tend to heat up. Think of bicycle pumps, for example. So, how are isothermal processes possible? How can you compress a gas without its temperature increasing?

"Think-pair-share"
- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

Clicker: I am now ready to share my answer if randomly selected.
   a. Yes

Note: you are allowed to "pass" if you would really not answer.

Warning

Be careful with all the signs!!!

ΔU is positive if:

Q_{added} is positive if:

$W_{on\ system}$ is positive if:

Review

Internal energy

monatomic: $U = \frac{3}{2}nRT$, $\Delta U = \frac{3}{2}nR\Delta T$

diatomic, around 300K: $U = \frac{5}{2}nRT$, $\Delta U = \frac{5}{2}nR\Delta T$

Work

constant P: $W_{by} = P\Delta V$, $W_{on} = -W_{by}$

changing P: $W_{by} = P_{ave}\Delta V$, $W_{on} = -W_{by}$

in general: $W = \text{area under curve on PV diagram}$

1st Law of Thermodynamics

$$\Delta U = Q_{added} + W_{on\ system}$$

Isothermal "Contours"

Conceptual Exam Questions: Does temperature increase/decrease/ stay the same for some change in state? Is ΔU pos or neg?

PV = nRT
$\rightarrow$ How can you tell if two points are at the same temperature?
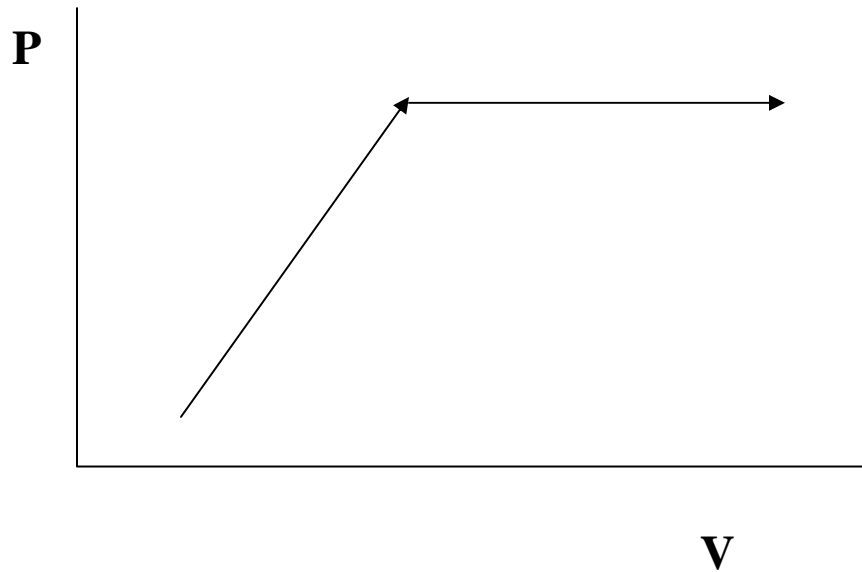$\rightarrow$ If temperature is constant, this gives a curve like xy = 3
                … or xy = 10 (for a higher temperature)

Contours of constant T: "isotherms"

Clicker Quizzes

Some random process

Clicker #1: Is ΔU (a) positive, (b) negative, or (c) zero?

Clicker #2: Is $W_{on\ gas}$ (a) positive, (b) negative, or (c) zero?

Clicker #3: Is Q_{added} (a) positive, (b) negative, or (c) zero? or (d) can't tell?

Worked Problem

A piston designed to keep the pressure constant ("isobaric") at 2 atm contains one mole of a monatomic ideal gas. The initial temperature is 300K and the initial volume is 0.0123 m^3. Heat is added, causing the gas to increase in temperature and also causing the piston to expand to 0.02 m^3. How much heat was that?

What if diatomic gas?

Answer: 3889 J

Isothermal Processes

P

$$W_{by\,gas} = nRT \ln\left(\frac{V_2}{V_1}\right) \quad \text{(isothermal)}$$

(From calculus)

area

V

ΔU $W_{on\,gas}$ Q