

Allowed: One 8.5x11" sheet of hand written notes (no photocopies) and an un-programmed calculator. No time limit.

To receive full credit, please show *all* work clearly and write neatly. If you wish to get partial credit on problems with incorrect answers, be sure to solve all questions algebraically first, then plug in numbers (with units) to get the final answer. Unless otherwise instructed, give all numerical answers in SI units. Give all numerical answers to 3 significant digits.

For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final 3 significant digits may be off - be especially careful when subtracting two similar numbers!

You may use the front and back of the test pages to do your work, but do not do work for one problem in space allotted for another problem. Be sure to work all of the problems. The point breakdown for all of the problems is listed in the column to the right.

To remind you, standard units in the SI (mks) system include the meter, the kilogram, the second, the Newton, and the Pascal.

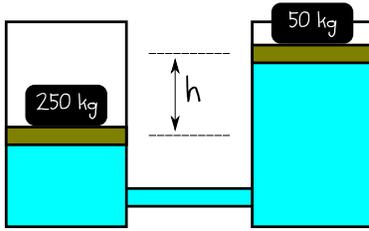
HINTS: Work carefully and don't make mistakes! Try to *understand* each problem before you begin to *solve* the problem, and remember that drawing pictures and diagrams can be a big help. Remember to work all of your problems algebraically first, then put in numbers when necessary. Check your units when you are done, and make sure that your answers make sense. It also helps if you let some quantity in your problem go to infinity or zero to check that the equations you solve have the appropriate behavior in those limits.

Problems	
1.	_____ / 5
2.	_____ / 8
3.	_____ / 11
4.	_____ / 10
5.	_____ / 8
6.	_____ / 8
Ex.	_____ / 3
Total:	_____ / 50

Constants, equations, and material properties you may find useful:

Universal Gas Constant	$R = 8.31451 \frac{\text{J}}{\text{K mol}}$
Boltzmann's Constant	$k_B = 1.38065 \times 10^{-23} \frac{\text{J}}{\text{K}}$
Avogadro's Number	$N_A = 6.02214 \times 10^{23}$
Stephan-Boltzman Constant for Blackbody Radiation	$\sigma = 5.6696 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$
Acceleration of Gravity Near the Earth	$g = 9.80 \frac{\text{m}}{\text{s}^2}$
Absolute Zero	$T_0 = -273.15 \text{ }^\circ\text{C} = 0 \text{ K}$
Density of Water	$\rho_{water} = 1000 \frac{\text{kg}}{\text{m}^3}$
Density of Type 316 Stainless Steel	$\rho_{316} = 0.29 \frac{\text{lb}}{\text{inches}^3}$
Density of Aluminum	$\rho_{Al} = 2.70 \times 10^3 \frac{\text{kg}}{\text{m}^3}$
Absolute Zero	$T_0 = -273.15 \text{ }^\circ\text{C} = 0 \text{ K}$
Specific Heat of Water	$c_{water} = 4186 \frac{\text{J}}{\text{kg} \cdot \text{ }^\circ\text{C}}$
Specific Heat of Ice	$c_{ice} = 2090 \frac{\text{J}}{\text{kg} \cdot \text{ }^\circ\text{C}}$
Specific Heat of Glass	$c_{glass} = 837 \frac{\text{J}}{\text{kg} \cdot \text{ }^\circ\text{C}}$
Specific Heat of Aluminum	$c_{Al} = 900 \frac{\text{J}}{\text{kg} \cdot \text{ }^\circ\text{C}}$
Thermal Linear Expansion Coefficient of Aluminum	$\alpha_{Al} = 24 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
Thermal Linear Expansion Coefficient of Copper	$\alpha_{Cu} = 17 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
Thermal Linear Expansion Coefficient of Iron	$\alpha_{Fe} = 12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$
Latent Heat of Fusion for Water	$L_{water} = 3.33 \times 10^5 \frac{\text{J}}{\text{kg}}$
Latent Heat of Vaporization for Water	$L_{water} = 2.26 \times 10^6 \frac{\text{J}}{\text{kg}}$
Latent Heat of Vaporization for Liquid Nitrogen	$L_{LN_2} = 1.98 \times 10^5 \frac{\text{J}}{\text{kg}}$
Atmospheric Pressure	$P_0 = 1.013 \times 10^5 \text{ Pa}$
Volume of a Sphere	$V_{sphere} = \frac{4}{3} \pi r^3$
Surface Area of a Sphere	$A_{sphere} = 4 \pi r^2$

1. A *Water Balance*: Two identical pistons are filled with water, as shown below. The surface area of each piston is 0.25 m^2 . They are connected together with a pipe, so that water can flow freely between them. A 50 kg weight is placed on one of the pistons, and a 250 kg weight is placed on the other. Find the difference in the height of the two pistons when the system is in equilibrium (labeled h in the figure).



- 2. *Banging On Stuff:*** The handle of your favorite hammer has come loose. Being a physicist, you decide to fix it for good. The head of the hammer is made of Iron, and you make a new handle of aluminum which is just a tiny bit too big to fit into the hole in the head. Then you cool the aluminum handle in liquid nitrogen until it just fits. When everything warms up, the handle is stuck tightly in the head.
- (a) The hole in the head is circular with a diameter of 2 cm and the handle is cylindrical with a diameter of $2\text{ cm} + \delta$ at 25°C . What should δ be so that the handle just fits when it is cooled down to -196°C (the boiling point of liquid nitrogen).
 - (b) If the handle is 15 cm long, how many Joules of heat must be removed from the handle to reduce its the temperature one degree Celsius?
 - (c) How much liquid nitrogen (in kg) will boil off as the handle is cooled one degree Celsius?

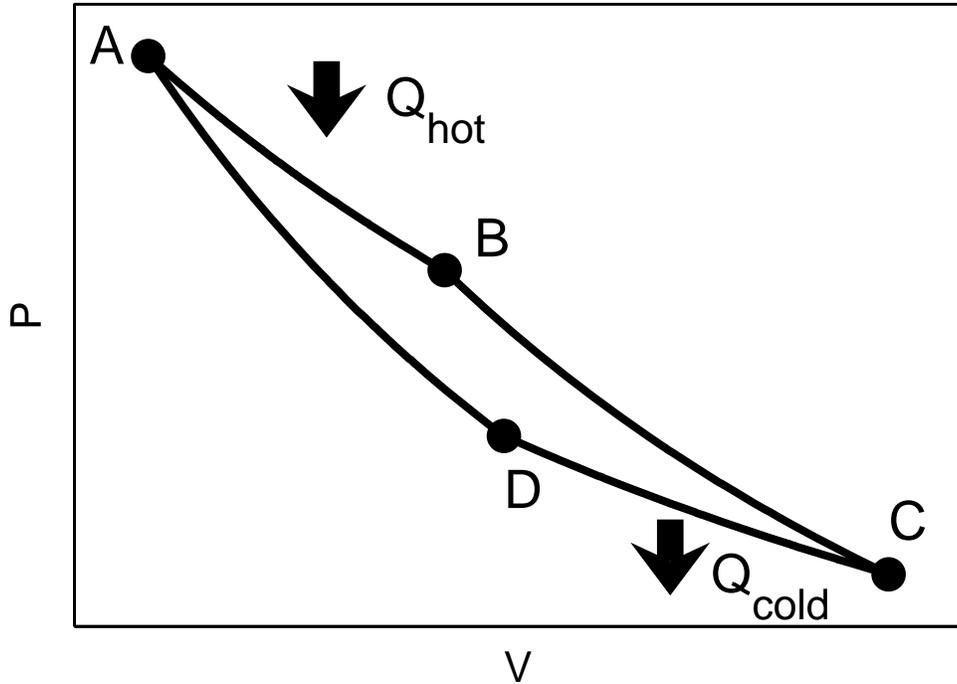
- 3. *Tiny Bubbles:*** A diver at a depth of 30 meters releases a spherical air bubble which is 1.00 cm in diameter. As the bubble rises to the surface, it expands. Just before it reaches the surface of the water, the air bubble has a diameter of 1.59 cm. The water temperature at the surface is 300 Kelvin. *Assuming that the bubble stays intact, remains in thermal equilibrium with the water around it, and maintains its spherical shape...*
- (a) What is the temperature of the water at a depth of 30 meters?
 - (b) What is the buoyant force on the bubble right after it is released?
 - (c) How much will the internal energy of the air in the bubble increase as it rises to the surface?
 - (d) How much will the entropy of the air in the bubble change as it rises to the surface?

Assume that the air molecules in the bubble have 5 degrees of freedom.

Problem 3 continued

4. *A Boring Heat Engine Problem Without a Good Story to Go With It:* Consider an ideal gas of 2×10^{22} molecules with a γ of $5/3$ used to make an ideal Carnot engine running between two thermal reservoirs at temperatures of 27°C and 770°C . The smallest that the volume of the gas gets during the cycle is $0.14 \times 10^{-3} \text{ m}^3$.

(a) On the P-V diagram of the cycle below (not shown to scale) label each step of the process as isobaric, isothermal, isovolumetric, or adiabatic.



- (b) What is the pressure of the gas when it is at its minimum volume?
- (c) What is the volume of the gas at point D?
- (d) What is the efficiency of this engine?
- (e) What is the total change in entropy of the ideal gas in the engine as the engine goes through one complete cycle?

Problem 4 continued

5. *Superfluid Waterfalls*: It turns out that if you cool liquid helium to a low enough temperature, it loses all of its viscosity. Imagine that! It makes really weird things possible. Imagine that liquid helium is poured out of a cup and falls under gravity. Let's call the density of liquid helium ρ , and the acceleration due to gravity g .
- (a) At a particular point below the cup, the stream of liquid helium has a velocity v_0 . Find an equation for its velocity after falling a distance h further.
 - (b) Assume that the stream of liquid helium has a circular cross section. If the cross section of the stream at the same particular point below the cup has a diameter D_0 , what is the diameter of the cross section of the stream after it has fallen a distance h further? Assume that the liquid helium is incompressible.
 - (c) Now plug in $\rho = 1.45 \times 10^2 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$, $v_0 = 0.1 \text{ m/s}$, $D_0 = 1 \text{ cm}$ and $h = 1 \text{ m}$ and find the diameter of the stream after it has fallen one meter.

- 6.** *A Shiny New Thermos:* A thermos is basically a glass or plastic cup inside of another cup. The space in between them acts as insulation. In a really good thermos, the space in between is a vacuum, so that no heat can conduct or convect through it. But heat is still radiated by blackbody radiation. This is why the inner cup in most thermoses is made to be very shiny. To simplify matters, in this problem we'll assume an "ideal" thermos which is just a hollow glass sphere surrounded by vacuum. The glass sphere has an outer diameter of 8 cm and the hollow area inside the glass has a diameter of 7.5 cm. The density of the glass is $2.51 \times 10^3 \text{ kg/m}^3$ and the density of water is $1.00 \times 10^3 \text{ kg/m}^3$. The specific heat of water is $4186 \text{ J/kg} \cdot ^\circ \text{C}$, and the specific heat of the glass is $837 \text{ J/kg} \cdot ^\circ \text{C}$.
- (a) If the glass sphere is at a temperature of 25°C and we fill it with water at 95°C , what will the final temperature of the glass and the water? Assume that no heat is radiated away as it comes to equilibrium.
- (b) Let's see what would happen if the thermos wasn't shiny. If the glass sphere is painted black (such that the emissivity is 1) and held at a temperature of 90°C , what power will the sphere radiate into the vacuum?

Problem 6 continued...

Extra Credit Problem This is only worth 3 points, so make sure you've done the other problems as well as you can before you work this one!

- (a) (2 pts) A sphere of aluminum with radius r which is painted with perfectly black paint is placed in deep space. Find an equation for the temperature of the sphere at time t as a function of T_0 , its temperature at time $t = 0$. Assume that deep space has a temperature of 0 K (it actually has a temperature of around 3.14 K).
- (b) (1 pt) Consider the Carnot engine problem. If $Q_{hot} = 1000$ J, what is the volume at point B?

Before it is placed on the stove, the air, cooker, and water are at a temperature of 25 °C, and the air inside of the cooker is at atmospheric pressure.

(a) The cooker is placed on the stove and begins to absorb heat. How much heat does the water absorb as its temperature goes from 25°C to 100°C?

(a) The mass of the water is

$$m_{water} = \rho V = 10^3 \text{ kg/m}^3 \cdot 0.211 \times 10^{-3} \text{ m}^3 = 0.221 \text{ kg}.$$

So the heat it absorbs is

$$Q_{water} = m_{water} c_{water} \Delta T = 0.221 \text{ kg} \cdot 4186 \frac{\text{J}}{\text{kg} \cdot ^\circ \text{C}} (100^\circ \text{C} - 25^\circ \text{C}) = \boxed{6938.3 \text{ J}}.$$

(b) Calculate the change in the internal volume of the cooker when the system is heated from 25 to 100°C.

(b)

$$\Delta V_{cooker} = 3\alpha V_{cooker} \Delta T = 3 \cdot 24 \times 10^{-6} \text{ } ^\circ \text{C}^{-1} \cdot 0.450 \times 10^{-3} \text{ m}^3 \cdot 75^\circ \text{C} = \boxed{2.43 \times 10^{-6} \text{ m}^3}.$$

(c) What is the pressure of the air in the cooker when it reaches a temperature of 100°C? (Assume that thermal expansion does not have a significant effect on the volume of the air. Because of the weird properties of water and the fact that you are approaching a phase transition, the water's volume will change by about 4 percent. But for simplicity, let's assume that the volume occupied by the gas stays the same as we heat it up.)

(c)

$$P_0 V = nRT_0$$

and

$$P_f V = nRT_f,$$

so

$$\frac{P_f}{P_0} = \frac{T_f}{T_0} \Rightarrow P_f = P_0 \frac{T_f}{T_0} = 1.013 \times 10^5 \text{ Pa} \frac{100 + 273.15}{25 + 273.15} = \boxed{1.27 \times 10^5 \text{ Pa}}, \text{ or } 1.25 \text{ atm}.$$