Physics 123 section 2
Winter 2010
Instructor: Dallin S. Durfee
Exam #2, March 4-6

CID _____

*Allowed: One 8.5x11" sheet of hand written notes (no photocopies) and an un-programmed calculator. No time limit.*

 To receive full credit, show *all* work clearly and write neatly. If you wish to get partial credit on problems with incorrect answers, be sure to solve all questions algebraically first, then plug in numbers (with units) to get the final answer.
 Unless otherwise instructed, give all numerical answers in SI units. Give all numerical answers to *3 significant digits*. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final 3 significant digits may be off - be especially careful when subtracting two similar numbers!
 You may use the front and back of the test pages to do your work, but do not do work for one problem in space allotted for another problem. Be sure to work all of the problems. The point breakdown for all of the problems is listed in the column to the right.
 To remind you, standard units in the SI (mks) system include the meter, the kilogram, the second, the Newton, and the Pascal.
 HINTS: Work carefully and don't make mistakes! Try to *understand* each problem before you begin to *solve* the problem, and remember that drawing pictures and diagrams can be a big help. If you don't know how to work a problem, skip it, and then come back to it later in a different frame of mind. Remember to work all of your problems algebraically first, then put in numbers when necessary. Check your units when you are done, and make sure that your answers make sense. It also helps if you let some quantity in your problem go to infinity or zero to check that the equations you solve have the appropriate behavior in those limits.

| Problems | |
|---|---|
| 1. | _____ / 11 |
| 2. | _____ / 8 |
| 3. | _____ / 10 |
| 4. | _____ / 13 |
| 5. | _____ / 8 |
| Ex. | _____ / 3 |
| Total: | _____ / 50 |

*Fourier Transforms in Time:*

$$a_0 = \text{anything since } \sin(0) = 0$$
$$a_{m>0} = \frac{2}{T_0} \int_0^{T_0} f(t) \sin(m\omega_0 t) dt$$
$$b_0 = \frac{1}{T_0} \int_0^{T_0} f(t) dt$$
$$b_{m>0} = \frac{2}{T_0} \int_0^{T_0} f(t) \cos(m\omega_0 t) dt$$

*Euler's Relation:*

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

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1. *Guitars Rule:* The high B string on a guitar is tuned such that its fundamental frequency is 246.9 Hz. In a set of one particular brand of strings (D'Addario ProArté normal tension classical guitar strings), this string has a diameter of 0.818 mm. This string is strung onto a classical guitar with a scale length (the distance between the bridge and the nut, the two points where the string is fixed) of 25.5 inches (0.6477 m). According to the manufacturer, when I install this string on this guitar I will have to put the string under a tension of 51.55 N to get it to play in tune on this guitar.

   (a) What is the wavelength $\lambda$(in meters), the wavenumber $k$ (in radians per meter), and the angular frequency $\omega$ (in radians per second) of the fundamental mode of the string (also known as the first harmonic)? Assume that the string is correctly tuned to play a B at a frequency of 246.9 Hz.

   (b) What is the speed at which waves travel on the string?

   (c) What is the linear mass density of the string $\mu$?

   (d) If I install the string on a child's guitar with a scale length of only 18 inches, what tension will I have to put on the string in order to get it to play in tune?

Problem 1 continued...

2. *Patriotic Trains:* You are standing next to the railroad tracks when a train speeds by, tooting its horn. You notice that as the engine passes, the pitch that you hear goes down, making the first two notes of the national anthem. You remember that your band teacher in high school taught you to use different songs to remember what different intervals sound like, and you recall that the national anthem starts off with two notes differing by a minor third — which is just three half steps.

   **(a)** If the tone after the train has passed sounds three half steps lower than the tone when it is approaching, what is the ratio of the two frequencies that you hear? (In other words, if you hear a frequency $f_a$ as the train approaches and $f_p$ after the train has passed, what is $f_a/f_p$?)

   **(b)** How fast is the train traveling?

   Assume that the speed of sound is 343 m/s, and use equal temperament, and remember that there are 12 half steps in an octave.

Problem 2 continued...

**3.** *Before You Lost Your Hearing:* You arrive early to get a good seat at a concert. Just for fun you brought along a sound-level meter. While you are waiting, the sound engineer is testing the speakers by playing a 400 Hz sine wave through them. Assume that the speakers are emitting spherical waves.

   **(a)** She first turns off all of the speakers except one which is 3.91 meters in front of you. You note that your sound-level meter reads 83.2 dB. What is the intensity of the sound waves (in $W/m^2$) at the location of your meter? Note that the threshold of hearing is $I_0 = 10^{-12}$ $W/m^2$.

   **(b)** If your sound-level meter was placed 5 meters from the speaker, what level would it read (in dB)?

   **(c)** Next the sound engineer turns on a speaker which is 4.71 meters in front of you, and you determine that this speaker generates a sinusoidal pressure oscillation at a frequency of 400 Hz with an amplitude of $1.24 \times 10^{-4}$ Pa at your location. Then she turns that speaker off and turns on one which is 10.4 meters in front of you, and you determine that this speaker generates a wave with an amplitude of $3.11 \times 10^{-4}$ Pa at your location. If both speakers are turned on at the same time, what would the amplitude of the resulting wave be? (Assume that the speakers are driven with the same electronic signal, such that they are both moving in the same direction at any given time. Also assume that the speed of sound is 343 m/s).

**4.** *A Triangle Wave:* Imagine that I have a generator that makes a wave like the image below. The wave is a triangle wave which oscillates between zero volts and $V_0$ with a period $T$. From $t = -T/2$ to $t = 0$ the wave is described by the equation $V = -2V_0 t/T$, and from $t = 0$ to $t = T/2$ the wave is described by the equation $V = +2V_0 t/T$.
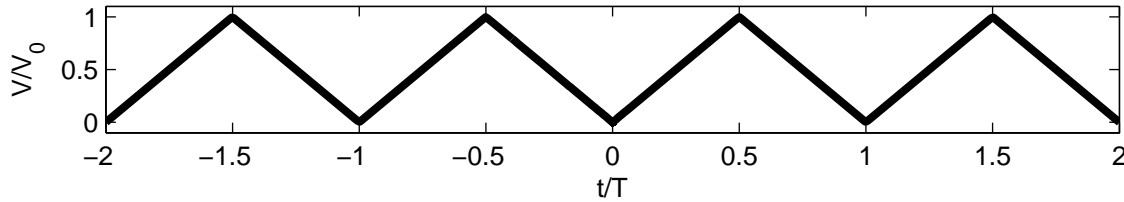


Note that

$$\int_0^a u\cos(u)du = -\int_{-a}^0 u\cos(u)du = \cos(a) + a\sin(a) - 1.$$

**(a)** Imagine that I filter the wave to get a DC signal (i.e., I only keep the $b_0$ term), what DC voltage will I have generated? Give your answer in terms of the given quantities ($V_0$ and $T$) and fundamental constants (like $\pi$, etc.).

Even a really good filter will still let some of the non-zero frequency components through, causing a small amount of ripple in our DC voltage. If we know the properties of our filter, to predict what the ripple will be we just need to know the amplitudes of the different sines and cosines that make up the triangle wave.

**(b)** Find $a_n$ for an arbitrary $n$. Give your answer in terms of the given quantities ($V_0$ and $T$) , the integer $n$, and fundamental constants (like $\pi$, etc.).

**(c)** Find $b_n$ for an arbitrary $n$. Give your answer in terms of the given quantities ($V_0$ and $T$) , the integer $n$, and fundamental constants (like $\pi$, etc.).

**(d)** What is the angular frequency $\omega_n$ of the cosine wave that has the amplitude $b_n$? Give your answer in terms of the given quantities ($V_0$ and $T$ ), the integer $n$, and fundamental constants (like $\pi$, etc.).

Problem 4 continued...

**5.** *Back To The Guitar Theme:* Joe Satriani is warming up for a concert. He is holding his guitar, which is plugged into two identical speakers. One speaker is placed 10 meters to his left, and the other is placed 10 meters to his right. To make this problem simpler, we will pretend that the guitar generates perfect sine waves[1].

   **(a)** Joe Satriani plays a note at a frequency of 831 Hz. The pressure waves of the two sounds from the two speakers add constructively precisely where he is standing. He then starts to walk to the left, and he hears the sound get soft and then starts to get louder again. He stops when he reaches the next maximum in sound intensity. How far did he walk? Assume that the speed of sound is 343 m/s and that both speakers are emitting waves of the same amplitude.

   **(b)** Joe Satriani can play really really fast music - for a human. If he were superhuman, he might be able to play notes so fast that the bandwidth of the speakers would be the ultimate limit as to how fast he could make different notes. If his speakers can generate waves with frequencies from 0 to 20 kHz, what is the shortest note they can make? Give your answer in seconds.

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[1]

To do this problem correctly, you could take the Fourier transform of the actual sound waves and consider what happens to each sine wave independently.

**Extra Credit** *Physics Rocks:* This is only worth 3 points, so make sure you've done the other problems as well as you can before you work this one!

(3 pts) A string of length $L$ is fixed at both ends. The tension in the string is $T$, but the linear mass density $\mu$ changes along the length of the string according to the equation $\mu(x) = \mu_0(1 + x/L)^2$ where $x$ is the distance from one end of the string. What is the frequency of the lowest resonance of the string?