

*Allowed: One 8.5x11" sheet of hand written notes (no photocopies) and an un-programmed calculator. No time limit.*

To receive full credit, please show *all* work clearly and write neatly. If you wish to get partial credit on problems with incorrect answers, be sure to solve all questions algebraically first, then plug in numbers (with units) to get the final answer. Unless otherwise instructed, give all numerical answers in SI units. Give all numerical answers to *3 significant digits*.

For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final 3 significant digits may be off - be especially careful when subtracting two similar numbers!

You may use the front and back of the test pages to do your work, but do not do work for one problem in space allotted for another problem. Be sure to work all of the problems. The point breakdown for all of the problems is listed in the column to the right.

To remind you, standard units in the SI (mks) system include the meter, the kilogram, the second, the Newton, and the Pascal.

HINTS: Work carefully and don't make mistakes! Try to *understand* each problem before you begin to *solve* the problem, and remember that drawing pictures and diagrams can be a big help. Remember to work all of your problems algebraically first, then put in numbers when necessary. Check your units when you are done, and make sure that your answers make sense. It also helps if you let some quantity in your problem go to infinity or zero to check that the equations you solve have the appropriate behavior in those limits.

*Some Useful Equations:*

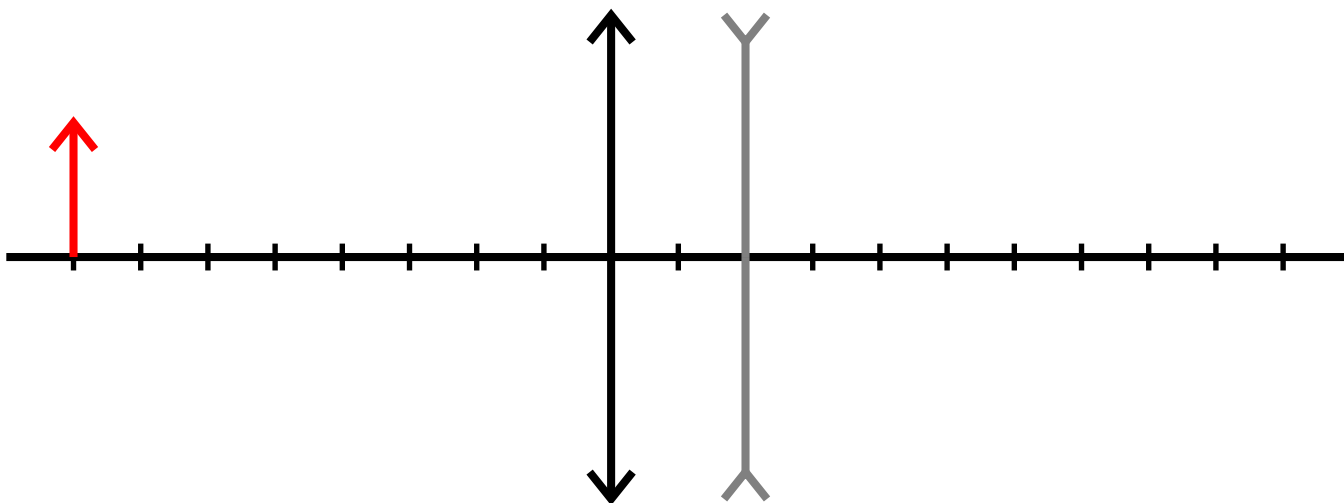
the volume of a sphere	$V = \frac{4}{3}\pi r^3$
the surface area of a sphere	$A = 4\pi r^2$
the speed of light in a vacuum	$c = 2.9979 \times 10^8 \text{ m/s}$
the index of refraction of water	$n_{\text{water}} = 1.333$

*Metric Prefixes:*

Larger	Smaller
k=kilo= $10^3$	c=centi= $10^{-2}$
M=Mega= $10^6$	m=milli= $10^{-3}$
G=Giga= $10^9$	$\mu$ =micro= $10^{-6}$
	n=nano= $10^{-9}$

Problems	
1.	_____ / 12
2.	_____ / 9
3.	_____ / 9
4.	_____ / 9
5.	_____ / 11
Ex.	_____ / 3
Total:	_____ / 50

1. You are using two lenses to make an image of your slinky. As shown in the figure below, you place your slinky 200 cm away from the first lens, which has a focal length of 75 cm. The second lens (shown in gray in the figure below) has a focal length of -125 cm and is placed 50 cm behind the first lens.



- (a) Draw rays on the diagram above to find the location and size of the final image of the slinky formed by the two lenses. You may remove the last page of this exam and fold it to make a straight edge to help you draw the rays.
- (b) Calculate the distance from the second lens to the final image.
- (c) If the slinky is 4.5 cm tall, how tall will its image be?
- (d) Is the image of the slinky real or virtual?
- (e) Is the image of the slinky upright or inverted?

Problem 1 continued...

2. Since CDs store data in fine reflective lines going around the disk, the disk acts like a reflective diffraction grating. One day I pointed a laser pointer with a wavelength of 650 nm at near normal incidence to a CD-ROM. I notice that the CD projected a series of dots on the wall behind me. The brightest dot was made by a beam of light coming almost directly back toward the laser pointer. The two dots closest to the bright central dot (the first-order diffraction peaks) were made by beams falling 20.2 degrees above and below the central beam. (Hint - for this problem you should use physics.)
- (a) What is the spacing between lines on the CD?
  - (b) At what angle from the central beam will the second-order diffraction peaks be (in degrees)?
  - (c) After calculating the spacing between the lines, imagine that I decide to use the first diffraction order from my CD to find the wavelength of a second laser pointer. I measure a wavelength of about 633 nm. If the beam has a diameter of 1 mm, what will the uncertainty in the measured wavelength be? (In other words, what will  $\Delta\lambda$  be? Think "Resolving Power.")

3. Some guy named Bill has perfect 20/20 vision. His sister Gzxeiye is nearsighted with a far point of 10 meters.
- (a) One sunny day, Bill picks up a newspaper as he leaves a 3D movie. He is still wearing red/blue 3D glasses. So with his left eye he can only see blue light with a wavelength of about 450 nm, and with his right eye he can only red light with a wavelength of about 630 nm. Looking at a picture in the newspaper, Bill notices that the picture is made up of tiny dots. If Bill's pupils are dilated to a diameter of 2 mm, and if the dots are spaced about 0.1 mm apart, how far does Bill need to hold the newspaper from his face if he doesn't want to be able to see the dots with his left eye (the one that can only see light with a wavelength of 450 nm)?
  - (b) If Gzxeiye goes to a good optometrist and gets glasses, what will the focal length of the lenses be?
  - (c) Gzxeiye doesn't have a swimming pool. In fact, she's allergic to water. But if she had one that was 2.5 meters deep, she could put a quarter at the bottom of the pool and look at it from above the water. If she did, how far below the surface would the image of the quarter be? Note that the index of refraction for water is 1.333.

Problem 3 continued...

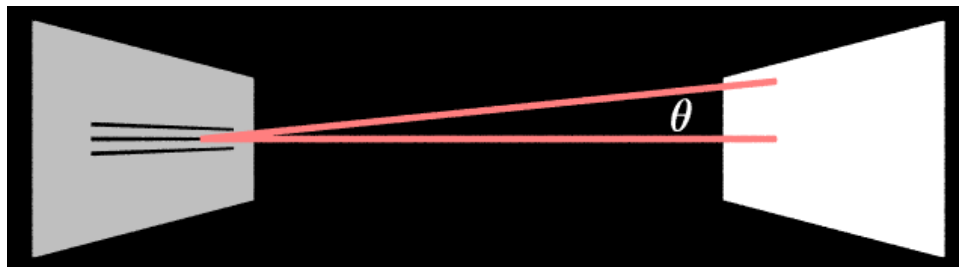
4. Two submarines are each at a depth of 100 meters and are separated by a distance of 200 meters. An undersea mountain is directly between the submarines. In order to send a message to the other submarine, one of the submarines is shooting a laser and bouncing it off of the surface of the water to hit the other submarine.
- (a) At what angle from the vertical (in degrees) should they aim the laser in order to hit the other submarine?
  - (b) If the submarines are far enough apart, none of the light will be transmitted out of the water. At a depth of 100 meters, what is the minimum distance that the submarines need to be separated by so that none of the laser light is transmitted into the air (where it can be detected by enemy aircraft)?
  - (c) Now imagine that the submarine is trying to communicate with an airplane and does not want to be detected by other submarines. If the laser has the correct polarization it is possible for all of the light to transmit out of the water with no reflection if they aim their laser the at the correct angle from the vertical. What is the correct angle?

The index of refraction for water is 1.333. Assume that the surface of the water is perfectly calm (i.e. there are no waves on the surface).

Problem 4 continued...



5. A laser beam with a wavelength  $\lambda$  is used to illuminate three extremely thin slits in a piece of aluminum foil, as shown in the figure below (not drawn to scale). The slits are spaced a distance  $d$  apart (such that the outer two slits are a distance  $2d$  from each other). The laser beam isn't exactly a plane wave. As a result, although the oscillating field has the same phase at the location of each of the three slits, the intensity of light at the middle slit is four times the intensity at the location of the other two slits. If I look on a screen a long distance from the foil at a point an angle  $\theta$  above the center of the interference pattern, what will the intensity of the light be? Give your answer in terms of  $\lambda$ ,  $d$ ,  $\theta$ , and the intensity at the center of the interference pattern  $I_{max}$ .



Problem 5 continued...

**Extra Credit.** This is only worth a total of 3 points, so be sure to work the other problems as well as you can before working on the extra credit problem. This problem is about a very strange type of lens, called a GRIN (gradient index) lens. Imagine that you have a very very thin piece of glass, of thickness  $d$ . It is a very strange piece of glass, however, which has an index of refraction that varies as you move radially away from the center of the glass: the index of the glass is equal to  $n_0$  at the center of the lens, and decreases as you move away from the center. If  $n(r)$  is just right, collimated light passing through the glass at normal incidence will focus to a point  $f$  away from the glass. What should the function  $n(r)$  be in order to make a lens which has a focal length  $f$ ? (Hint, a good way to do this is with Fermat's principle.) Feel free to make the thin lens and paraxial ray approximations, and note that, in the limit of small  $\epsilon$ ,

$$(1 + \epsilon)^a \approx 1 + a\epsilon.$$