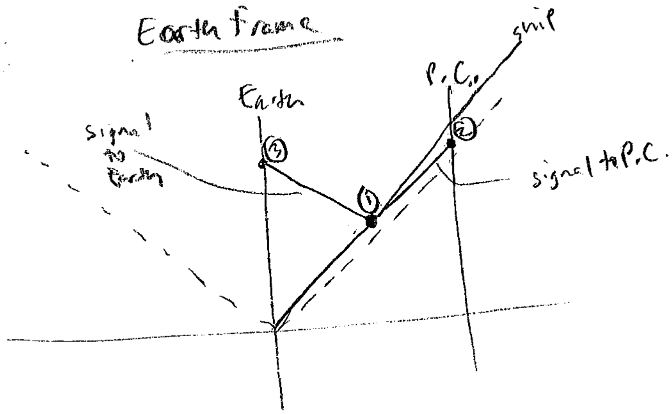
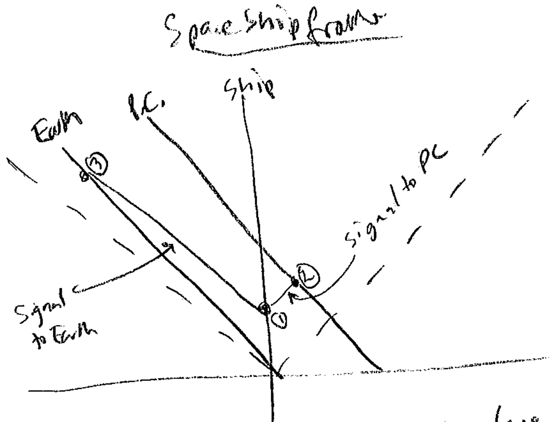
Lecture 40 "Worked Problem #3"

(Reminder: this problem got canceled.)

39-2. Suppose we are on our way to Proxima Centauri, which is 4.2 ly away (in the reference frame of the earth). We are in a space ship which is traveling with a speed of [02] 0.99687 c. When we are half-way there, we send a signal to both the earth and Proxima Centauri. The signals travel at the speed of light c. In the reference frame of the earth and Proxima Centauri, both signals travel 2.1 ly and thus arrive at their destinations at the same time. (a) In our reference frame, how long does it take for the signal to reach earth? Remember that in our reference frame, the distance between earth and Proxima Centauri is less than 4.2 ly because of length contraction. (b) In our reference frame, how long does it take for the signal to reach Proxima Centauri? (c) These two events (the signal reaching earth and the signal reaching Proxima Centauri) are simultaneous in the earth's reference frame. How much time elapses between these two events in *our* reference frame?

Answers: (a) 53.04; (b) 0.0831; (c) 52.96





Consider 3 events:

1. signals sent
2. signal reaches P.C.
3. signal reaches Earth

Note: I don't yet know exactly where events 1, 2, and 3 are.

Aha! Now it's clear why the times aren't the same in spaceship frame: P.C. is "catching up" to us; Earth is "running away" from us.

To solve problem, we just need to know where 1, 2, and 3 are in spaceship frame. Then it's easy.

39-2 cont

In Earth frame

① $x_1 = 2.1$ l.y.

$t_1 = \frac{x_1}{v} = \frac{2.1}{.99687} = 2.10659$ yrs

② $x_2 = 4.2$ ly

$t_2 = t_1 + \frac{2.1}{1} = 4.20659$ yrs

③ $x_3 = 0$

$t_3 = 4.20659$ yrs

Transform to ship frame

$\beta = .99687$

$\gamma = 12.64891$

$\beta\gamma = 12.60931$

use negative signs since we need to rotate left to get ship frame

① $$\begin{pmatrix} x \\ ct \end{pmatrix}_{ship} = \begin{pmatrix} 12.64891 & -12.60931 \\ -12.60931 & 12.64891 \end{pmatrix}\begin{pmatrix} 2.1 \\ 2.10659 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ .16652 \end{pmatrix}$$

② $$\begin{pmatrix} x \\ ct \end{pmatrix}_{ship} = \begin{pmatrix} \text{same matrix} \end{pmatrix}\begin{pmatrix} 4.2 \\ 4.20659 \end{pmatrix}$$

$$= \begin{pmatrix} .08322 \\ .24968 \end{pmatrix}$$

③ $$\begin{pmatrix} x \\ ct \end{pmatrix}_{ship} = \begin{pmatrix} \text{same matrix} \end{pmatrix}\begin{pmatrix} 0 \\ 4.20659 \end{pmatrix}$$

$$= \begin{pmatrix} -53.0423 \\ 53.2088 \end{pmatrix}$$

Now we know everything!

Answer to (a) $= t_3 - t_1 = \boxed{53.0423 \text{ year}}$

Answer to (b) $= t_2 - t_1 = \boxed{0.08314 \text{ year}}$

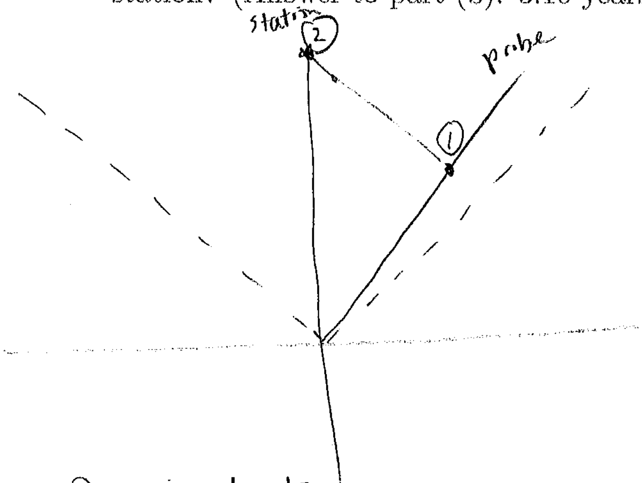
Answer to (c) $= t_3 - t_2 = \boxed{52.9591 \text{ year}}$

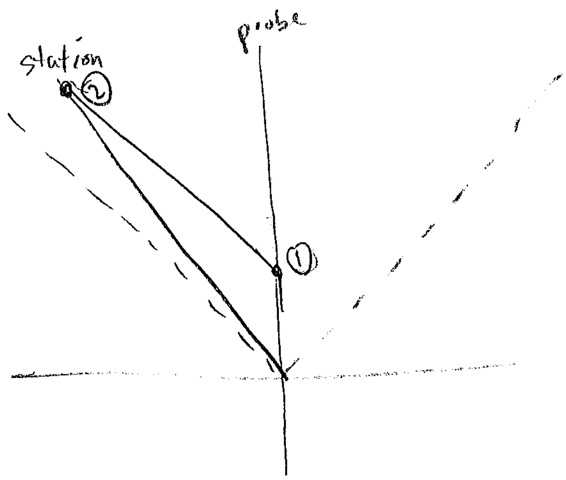
Lecture 40 "Worked Problem #4"

$\beta = .811$
$\gamma = 1.709$
$\gamma\beta = 1.386$

(Optional problem from HW 40.)

- A deep space probe is launched from the Earth and passes a deep space station on its way into the unknown. The probe travels at a constant velocity of $0.811c$ relative to the station. It has an on-board atomic clock connected to a computer which is programmed to send a microwave signal back to the station exactly one year later (as measured in the probe's frame of reference). (a) Draw two space-time diagrams: one from the station's frame of refernece, and one from the probe's frame of reference. In each diagram, include world lines for the station and the probe, and the microwave signal sent by the probe. Mark these two events on each diagram: probe sends signal, and station receives signal. (b) From the reference frame of someone on the space station, how much time elapses from the time the probe passes by, to when the microwave signal arrives back at the station? (Answer to part (b): 3.10 years.)





① = signal sent
② = signal received

Given: Probe frame
$x_1 = 0$
$t_1 = 1\,\text{yr}$

Transform from probe to station frame

① $\begin{pmatrix} x_1 \\ t_1 \end{pmatrix}_{\text{station}} = \begin{pmatrix} 1.709 & +1.386 \\ +1.386 & 1.709 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1.386 \\ 1.709 \end{pmatrix}$

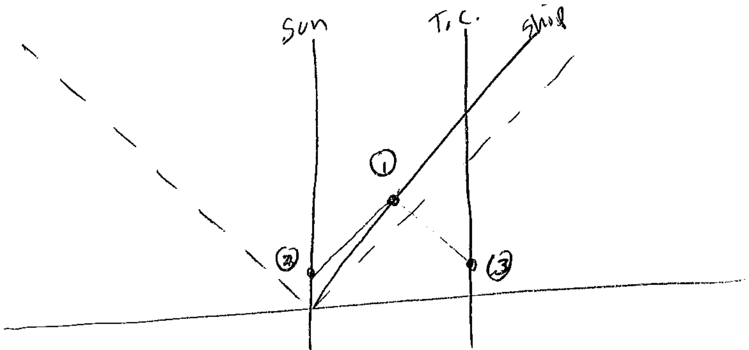
Event ① is connected to event ② by 45° line, so in station frame, $t_2 = 1.386 + 1.709 = \boxed{3.095 \text{ years}}$

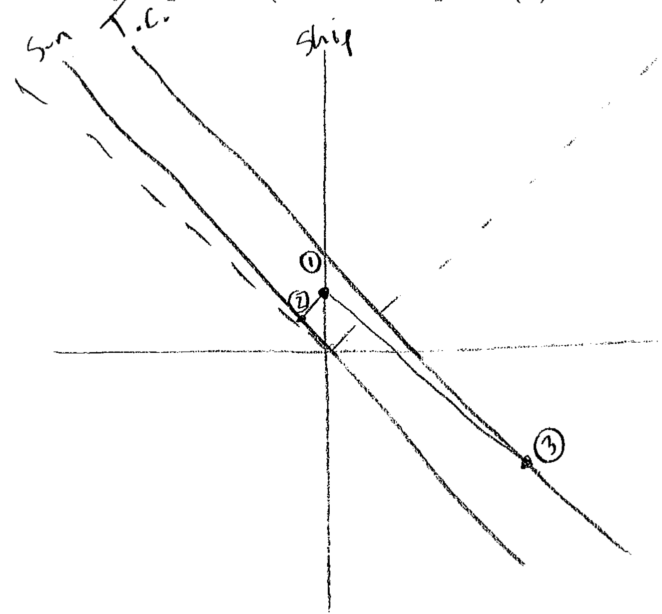
Lecture 40 "Worked Problem #5"

(Optional problem from HW 40.)

$\beta = .82$
$\gamma = 1.7471$
$\beta\gamma = 1.4327$

- Suppose our Sun is about to explode and we escape in a spaceship toward the star Tau Ceti, which is 12 light years away (not including Lorentz contraction). We travel at $v = 0.82c$. When we reach the midpoint of our journey, we see our Sun explode and, unfortunately, we see Tau Ceti explode as well (we observe the light arriving from each explosion). (a) Draw two space-time diagrams: one from the Sun's frame of refernece, and one from our frame of reference. In each diagram, include world lines for the Sun, Tau Ceti, our spaceship, and the light rays arriving from each explosion. Mark these three events on each diagram: Sun explodes, Tau Ceti explodes, and the instant we observe both explosions. (b) In the rest frame of the Sun (and Tau Ceti), did the explosions occur simultaneously? (c) In the spaceship frame of reference, did the explosions occur simultaneously? (d) In the spaceship frame of reference, how long before we saw the Sun explode did it actually explode? (e) In the spaceship frame of reference, how long before we saw Tau Ceta explode did it actually explode? (Answers to parts (d) and (e): 1.89 years, 19.08 years.)





① = light from explosions reaches us
② = Sun explodes
③ = T.C. explodes

Sun Frame

① $x_1 = 6$ l.y.,
$t_1 = \frac{x_1}{v} = \frac{6}{.82} = 7.317$ years

② $x_2 = 0$
$t_2 = t_1 - 6$ years (since light at 45° angle)
$= 1.317$ years

③ $x_3 = 12$ l.y.,
$t_3 =$ same as t_2 $= 1.317$ years

Ship Frame

① $\begin{pmatrix} x_1 \\ ct_1 \end{pmatrix}_{ship} = \begin{pmatrix} 1.7471 & -1.4327 \\ -1.4327 & 1.7471 \end{pmatrix} \begin{pmatrix} 6 \\ 7.317 \end{pmatrix} = \begin{pmatrix} 0 \\ 4.18803 \end{pmatrix}$

② $\begin{pmatrix} x_2 \\ ct_2 \end{pmatrix}_{ship} = \begin{pmatrix} '' & '' \\ & \end{pmatrix} \begin{pmatrix} 0 \\ 1.317 \end{pmatrix} = \begin{pmatrix} -1.8869 \\ 2.3011 \end{pmatrix}$

③ $\begin{pmatrix} x_3 \\ ct_3 \end{pmatrix}_{ship} = \begin{pmatrix} '' & '' \\ & \end{pmatrix} \begin{pmatrix} 12 \\ 1.317 \end{pmatrix} = \begin{pmatrix} 19.0788 \\ -14.8908 \end{pmatrix}$

Answer to d $= (t_1 - t_2)_{ship} = \boxed{1.8892 \text{ years}}$

Answer to e $= (t_1 - t_3)_{ship} = \boxed{19.079 \text{ years}}$