Physics 123 Equations – Fall 2011 Semester

To know by heart (not an exhaustive list, I'm sure)

Fluids

pressure: $P = F/A$
density: $\rho = m/V$
specific gravity: $SG = \rho/\rho_{H20}$
stationary fluids: $P = P_0 + \rho gh$ (pressures at same depths are same)
Archimedes' Principle: buoyant force: $B = \rho_{fluid}\, V_{object}\, g$ (= weight of the "displaced fluid")
moving fluids, at positions 1 and 2 along path:

$VFR = A_1 v_1 = A_2 v_2$ (*Eqn of continuity*, from mass conservation, "garden hose")

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$ (*Bernoulli's Law*, from energy conservation)

Thermodynamics

$PV = nRT = Nk_BT$
Equipartition theorem: each d.o.f. has energy $k_BT/2$, for each molecule

transl. $KE_{ave} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$ (can use to solve for v_{rms})

How to find pressure exerted by balls (atoms) from change in momentum

$Q = mc\Delta T;\ Q = mL$

$W_{by\,gas} = \int PdV$ = area under P-V curve

First Law: $\Delta E_{int} = Q_{added} + W_{on};\ Q_{added} = \Delta E_{int} + W_{by}$
$\gamma = C_P/C_V$

	Monatomic	Diatomic (~ 300K)	Solids
#dof	3	5	6
E_{int}	3/2 nRT	5/2 nRT	6/2 nRT
ΔE_{int}	3/2 $nR\Delta T$	5/2 $nR\Delta T$	6/2 $nR\Delta T$
C_V	3/2 R	5/2 R	6/2 R
C_P	5/2 R	7/2 R	same as C_V
γ	5/3	7/5	n/a

Note: for monatomic, n = #moles of molecules = #moles of atoms. For diatomic, n = #moles of molecules = 2× #moles of atoms. For solids, n = #moles of atoms.

Adiabatic: $P_1V_1^{\gamma} = P_2V_2^{\gamma}$
How to find W, ΔE_{int}, and Q, for isobaric, isovolumetric, isothermal, adiabatic, and cycles
How to find when W, ΔE_{int}, and Q are positive or negative for any changes
Engine: $Q_h = |W| + Q_c;\ e = |W|/Q_h = (Q_h - Q_c)/Q_h$
Refrigerator: $COP_R = Q_c/|W|$
Heat pump: $COP_{HP} = Q_h/|W|$
Carnot: $e_{max} = (T_h - T_c)/T_h$; $COP_{R,max} = T_c/(T_h - T_c)$; $COP_{HP,max} = T_h/(T_h - T_c)$

$\Delta S = \int \frac{dQ}{T}$

How to find ΔS for isobaric, isovolumetric, isothermal, adiabatic, free expansion, and cycles
How to find expected random fluctuations for various situations

Waves & Sound

wave parameters: x, t, A, λ, f, v, k, ω, ϕ
$f = A\cos(kx - \omega t + \phi) \leftrightarrow Ae^{i(kx - \omega t + \phi)}$
speed of wave: $v = \lambda f$
$k = 2\pi/\lambda;\ \omega = 2\pi/T$
$v_{phase} = \omega/k$

$v_{group} = (d\omega/dk)|_{\text{kave}}$
Complex numbers: converting polar ↔ rectangular
How to add sinusoidal waves together
$I = P/A$
+ 10 to dB = ×10 to I; +3 to dB = ×2 to I
dBm: reference = 1 mW
Mach# = v/v_{sound}
$\Delta PL = m\lambda$ (constructive); $\Delta PL = (m + ½)\lambda$ (destructive)
How to find standing wave resonances
Result: o-o/c-c: $f_n = nf_1$; $n = 1, 2, 3, \ldots$ o-c: $f_n = nf_1$; $n = 1, 3, 5, \ldots$
f_1 = fundamental frequency, = v/(largest λ)
$f_{beat} = |f_1 - f_2|$

Optics

Law of reflection: $\theta_{incident} = \theta_{reflected}$
Law of refraction (Snell's law): $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (θ measured from the perpendicular)
→ n = "index of refraction", speed of light = c/n
Condition for TIR: set $\theta_2 = 90°$ (n_2 = higher index)
$\lambda_{\text{material}} = \lambda_{\text{vacuum}}/n$
Difference between linear and circular polarization
Difference between *s*- and *p*-polarization
Thin lens equation: $1/f = 1/p + 1/q$
Difference between real and virtual images
magnification: $M = h_i/h_o$ (definition) $= -q/p$ (useful for calculation)
Sign conventions for lens/mirror equation
How to handle multiple lens/mirror problems
f/# = f/D
near point & far point
$m = \theta/\theta_0$
How to figure out the angular magnification of a magnifying glass
How to figure out the angular size of an ant, planet, or other object for which the small angle approximation is valid
Telescope: $m = f_o/f_e$
2 slit: $d\sin\theta_{\text{bright}} = m\lambda$; $d\sin\theta_{\text{dark}} = (m + ½)\lambda$
1 slit: $a\sin\theta_{\text{dark}} = m\lambda$
grating: $d\sin\theta_{\text{bright}} = m\lambda$
How to combine 2 slit with 1 slit formulas
How to derive intensity equation for any number of slits
When to use "Approx. 2": $\theta \approx \sin\theta \approx \tan\theta = y/L$
What the sinc function is
OPL = PL × n
ΔOPL + phase shifts = $m\lambda$ (constructive); = $(m + ½)\lambda$ (destructive)
How to write/interpret general 3D wave equation for arbitrary wave direction and arbitrary oscillation direction

Relativity

$\beta = v/c$; $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

$\Delta t' = \gamma\Delta t$
$\Delta L' = \Delta L/\gamma$
$\beta_{1-3} = \dfrac{\beta_{1-2} + \beta_{2-3}}{1 + \beta_{1-2}\beta_{2-3}}$
$p = \gamma mv$
$E = \gamma mc^2$
KE = $(\gamma - 1)\, mc^2$
$E_{rest} = mc^2$

cons. of momentum & energy

To be given on exam

These equations should be exactly same as the ones on the first page of exam as posted on the website, but with some details missing there. For example, on the exam I give the equation $l = \frac{1}{\sqrt{2}\pi d^2 n}$, but I don't say that it is for the mean free path like I do below. There are many such omissions on the exam itself, so be sure to look over the first page prior to taking the exam.

Fundamental constants
Materials parameters
Conversion factors
$(1 + x)^n \approx 1 + nx$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Surface area of sphere = $4\pi r^2$

Volume of sphere = $(4/3)\pi r^3$

Thermodynamics

Thermal expansion: $\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta T$; $\beta = 3\alpha$

Maxwell-Boltzmann: $f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_B T}$

$v_{most\ probable}$ = velocity where $f(v)$ is a maximum $= \sqrt{\frac{2k_B T}{m}}$

$$v_{avg} = \int_0^\infty v\, f(v)\, dv = \sqrt{\frac{8k_B T}{\pi m}}$$

$$v_{rms} = \sqrt{\int_0^\infty v^2\, f(v)\, dv} = \sqrt{\frac{3k_B T}{m}}$$

Mean free path: $l = \frac{1}{\sqrt{2}\pi d^2 n}$

Ave time between collisions: $\tau = l/v_{avg}$

Conduction: $P = \frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{L}$

$$P = \frac{\Delta Q}{\Delta t} = \frac{A\Delta T}{R};\ R = L/k$$

Radiation: $P = \frac{\Delta Q}{\Delta t} = e\sigma A T^4$

$e_{Otto} = 1 - \frac{1}{r^{\gamma - 1}}$; r = compression ratio = V_{max}/V_{min}

$S = k_B \ln W$

coin flips: *# microstates* = $\binom{N}{k} = \frac{N!}{k!(N-k)!}$; *# macrostates* = 2^N

Waves & Sound

$P = \frac{1}{2} m\omega^2 A^2 v$

$r = \frac{v_2 - v_1}{v_2 + v_1}$; $t = \frac{2v_2}{v_2 + v_1}$

$R = |r|^2$; $T = 1 - R$

$v_{string} = \sqrt{T/\mu}$; $\mu = m/L$

$v_{rod} = \sqrt{Y/\rho}$; $Y = \frac{stress}{strain} = \frac{F/A}{\Delta L/L}$

$v_{sound} = \sqrt{B/\rho}$

$v_{sound} = 343 \frac{\text{m}}{\text{s}} \sqrt{\frac{T}{293\text{K}}}$

$\beta = 10 \log\left(\frac{I}{I_0}\right)$; $I = I_0 10^{\beta/10}$; $I_0 = 10^{-12}$ W/m^2

$\sin\theta = 1/\text{Mach\#}$

$f' = f \frac{v \pm v_o}{v \pm v_s}$

$\Delta x \Delta k \geq ½$; $\Delta x \Delta p \geq \hbar/2$

$\Delta t \Delta\omega \geq ½$; $\Delta t \Delta E \geq \hbar/2$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi n x}{L}\right)dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi n x}{L}\right)dx$$

half step: $f_2/f_1 = 2^{1/12}$

Optics

$\theta_{\text{Brewster}} = \tan^{-1}(\theta_1/\theta_2)$

$r = \frac{n_1 - n_2}{n_1 + n_2}$; $t = \frac{2n_1}{n_1 + n_2}$

$f = R/2$

Lensmaker's eqn: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (R_1 = pos, R_2 = neg if convex-convex)

Image from surface: $\frac{n_1}{p} + \frac{n_2}{p} = \frac{n_2 - n_1}{R}$ (p = pos if object in front, q = pos if image in back, R = pos if center of curvature in back)

$\phi = 2\pi\Delta PL/\lambda$

Approx. 1: $\Delta PL = d\sin\theta$

$E = E_0(e^{i\phi 1} + e^{i\phi 2} + \ldots)$

$I \sim |E|^2$

2 slit: $I = I_0 \cos^2\left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)$

1 slit: $I = I_0 \text{sinc}^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$

circular aperture: Rayleigh: $\theta_{\text{min.resolve}} = 1.22\lambda/D$

$R = \lambda_{ave}/\Delta\lambda = \#\text{slits} \times m$
Bragg: $2d\sin\theta_{\text{bright}} = m\lambda$

$$f' = f\sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$

Relativity

$$x_{\text{frame}\,2} = \gamma x_{\text{frame}\,1} \pm \gamma\beta(ct)_{\text{frame}\,1}$$

$$(ct)_{\text{frame}\,2} = \pm\gamma\beta x_{\text{frame}\,1} + \gamma(ct)_{\text{frame}\,1}$$

$$\begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame}\,2} = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame}\,1}$$

particles with mass: $E^2 = (pc)^2 + (mc^2)^2$
photons: $E = pc$
Compton shift: $1/p'_{photon} - 1/p_{photon} = 2/(m_{electron}c)$

Would be given on a problem if needed (not an exhaustive list)

Thermodynamics
$B = \Delta P/(\Delta V/V)$
Otto cycle, what it looks like: adiabatic, C_V, adiabatic, C_V
Diesel cycle, what it looks like: adiabatic, C_P, adiabatic, C_V

$$e_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\left(\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right)$$; r = compression ratio = V_{max}/V_{min}; r_c = cutoff ratio = V_{middle}/V_{min}

Waves & Sound
Musical intervals
- Second = 2 half-steps
- Minor third = 3 half-steps
- Major third = 4 half-steps
- Major fourth = 5 half-steps
- Major fifth = 7 half-steps
- Sixth = 9 half-steps
- Minor seventh = 10 half-steps
- Major seventh = 11 half-steps
- Octave = 12 half-steps