# Physics 123 Class Schedule – Fall 2011

Note 1: In the reading assignments below, PpP refers to "Physics phor Phynatics". All other reading assignments refer to Serway & Jewitt.
Note 2: Labs are set up and taken down on Saturday mornings. If a lab is due on a Saturday, you might not be able to do it that day.

| | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| **September** | 29 Start of classes<br>Lecture 1<br>Intro, Pressure<br>Reading: syllab,14.1-14.2 | 30 | 31 Lecture 2; **HW 1**<br>Archimedes' Principle<br>Reading: 14.3-14.4 | 1 | 2 Lecture 3; **HW 2**<br>Fluid motion<br>Reading: 14.5-14.7 | 3 **Begin Lab 1**<br>(Pressure) |
| | 5 Lab 1 ongoing<br>**Labor Day Holiday** | 6 Lab 1 ongoing | 7 Lab 1 ongoing<br>Lecture 4; **HW 3**<br>Thermal expansion,<br>Ideal gas law<br>Reading: 19.1-19.5 | 8 Lab 1 ongoing | 9 Lab 1 ongoing<br>Lecture 5; **HW 4**<br>Kinetic Theory<br>Reading: 21.1, 21.5 (and 21.6 if your book has it) | 10 **Lab 1 due**<br>**Begin Lab 2**<br>(Specific Heat) |
| | 12 Add/drop deadline; Lab 2 ongoing<br>Lecture 6; **HW 5**<br>Calorimetry<br>Reading: 20.1-20 | 13 Lab 2 ongoing | 14 Lab 2 ongoing<br>Lecture 7; **HW 6**<br>Heat transfer<br>Reading: 20.7 | 15 Lab 2 ongoing | 16 Lab 2 ongoing<br>Lecture 8; **HW 7**<br>1$^{st}$ Law of Thermodyn.<br>Reading: 20.4-20.6 | 17 **Lab 2 due** |
| | 19 Lecture 9; **HW 8**<br>Molar Specific Heats<br>Reading: 21.2-21.4 | 20 | 21 Lecture 10; **HW 9**<br>Heat engines<br>Reading: 22.1, 22.5 | 22 | 23 Lecture 11; **HW 10**<br>Refrigerators & Carnot<br>Reading: 22.2-22.4 | 24 |
| | 26 Lecture 12; **HW 11**<br>Entropy<br>Reading: 22.6-22.7 | 27 | 28 Lecture 13; **HW 12**<br>What is entropy?<br>Reading: 22.8; handout | 29 | 30 Lecture 14; **HW 13**<br>Waves<br>Reading: 16.1-16.2 | 1 **Begin Exam 1:**<br>**Thermodynamics** |
| **October** | 3 Lecture 15; **HW 14**<br>Waves on a string<br>Reading: 16.3-16.6;<br>PpP 2.1-2.2 | 4 | 5 Lecture 16; **HW 15**<br>Complex exponentials<br>Reading: PpP 1.1-1.4 | 6 **End Exam 1** | 7 Lecture 17; **HW 16**<br>Reflection, Transmission,<br>Dispersion<br>Reading: PpP 3.1-3.5, 5.1 | 8 **Begin Lab 3**<br>(Dispersion) |
| | 10 Lab 3 ongoing<br>Lecture 18; **HW 17**<br>Sound waves<br>Reading: 17.1-17.3 | 11 Lab 3 ongoing | 12 Lab 3 ongoing<br>Lecture 19; **HW 18**<br>Doppler, Superposition<br>Reading: 17.4, 18.1 | 13 Lab 3 ongoing | 14 Lab 3 ongoing<br>Lecture 20; **HW 19**<br>Standing waves, Reson.<br>Reading: 18.2-18.6 | 15 **Lab 3 due**<br>**Begin Labs 4-5**<br>(Standing Waves) |
| | 17 Labs 4-5 ongoing<br>Lecture 21; **HW 20**<br>Beats, Uncertainty<br>Reading: 18.7; PpP 4.1 | 18 Labs 4-5 ongoing | 19 Labs 4-5 ongoing<br>Lecture 22; **HW 21**<br>Fourier transforms<br>Reading: PpP 6.1-6.5 | 20 Labs 4-5 ongoing | 21 Labs 4-5 ongoing<br>Lecture 23; **HW 22**<br>Fourier, cont.<br>Reading: PpP 6.6-6.7 | 22 **Labs 4-5 due**<br>**Begin Lab 6**<br>(Fourier Transf.)<br>**Proposal due** |
| | 24 Lab 6 ongoing<br>Lecture 24; **HW 23**<br>Music<br>Reading: PpP 7.1-7.3 | 25 Lab 6 ongoing | 26 **Lab 6 due**<br>Lecture 25; **HW 24**<br>Reflection, Refraction,<br>Dispersion<br>Reading: 35.1-35.5 | 27 **Begin Exam 2:**<br>**Waves** | 28 Lecture 26; **HW 25**<br>Huygens, TIR<br>Reading: 35.6-35.8 | 29 **Begin Lab 7**<br>(Brewster) |
| **November** | 31 Lab 7 ongoing<br>Lecture 27; **HW 26**<br>Polarization, Brewster<br>Reading: 38.6 | 1 Lab 7 ongoing<br>**End Exam 2** | 2 Lab 7 ongoing<br>Lecture 28; **HW 27**<br>Images from mirrors<br>Reading: 36.1-36.2 | 3 Lab 7 ongoing | 4 Lab 7 ongoing<br>Lecture 29; **HW 28**<br>Images from lenses<br>Reading: 36.3-36.4 | 5 **Lab 7 due**<br>**Begin Lab 8**<br>(Telescope) |
| | 7 Withdraw deadline; Lab 8 ongoing<br>Lecture 30; **HW 29**<br>Aberrations, camera, eye<br>Reading: 36.5-36.7 | 8 Lab 8 ongoing | 9 Lab 8 ongoing<br>Lecture 31; **HW 30**<br>Magnifier, telescope<br>Reading: 36.8, 36.10 | 10 Lab 8 ongoing | 11 Lab 8 ongoing<br>Lecture 32; **HW 31**<br>Interference from slits<br>Reading: 37.1-37.3 | 12 Lab 8 ongoing<br>**Begin Labs 9-10**<br>(Interferometer;<br>Diffraction)<br>**Prog. Report due** |
| | 14 Labs 8-10 ongoing<br>Lecture 33; **HW 32**<br>More interference<br>Reading: 37.4-37.6 (and 37.7 if your book has it) | 15 Labs 8-10 ongoing | 16 Labs 8-10 ongoing<br>Lecture 34; **HW 33**<br>Diffraction from wide slits<br>Reading: 38.1-38.2 | 17 Labs 8-10 ongoing | 18 Labs 8-10 ongoing<br>Lecture 35; **HW 34**<br>Resolving, gratings<br>Reading: 38.3-38.5 | 19 **Labs 8-9 due** |
| | 21 Lab 10 ongoing<br>Lecture 36; **HW 35**<br>Waves in 3-dimensions;<br>Modern optical devices<br>Read: PpP 8.1-3, 10.1-4 | 22 Discontin. deadln; Fri instr.<br>**Lab 10 due**<br>Lecture 37; **HW 36**<br>Intro to relativity<br>Reading: 39.1-39.3 | 23 **No classes** | 24 **Thanksgiving**<br>**Holiday** | 25 **Thanksgiving, cont.** | 26 |
| **December** | 28 **Begin Exam 3: Optics**<br>Lecture 38; **HW 37**<br>Special relativity<br>Reading: 39.4 | 29 | 30 Lecture 39; **HW 38**<br>Lorentz transformations<br>Reading: 39.5 | 1 | 2 Lecture 40; **HW 39**<br>Lorentz, cont.<br>Reading: 39.6 | 3 **End Exam 3**<br>T.C. closes at 4 pm |
| | 5 Lecture 41; **HW 40**<br>$E = mc^2$<br>Reading: 39.7-39.9 | 6 | 7 **Term Project due**<br>Lecture 42; **HW 41**<br>Project Show & Tell<br>Reading: none | 8 Last Day of Classes | 9 **Reading Day** | 10 **Reading Day**<br>**Late HW, extra-**<br>**credit papers due** |
| | 12 **Begin Final Exam** | 13 | 14 | 15 **End Final Exam** | 16 (Last day of finals) | 17 |

# Physics 123 – Fall 2011 – Section 2 "Physics Majors and Minors"
## Dr. John S. Colton

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> You can navigate there via www.physics.byu.edu → Courses → Class Web Pages → Physics 123 (Colton).

**Prerequisites:** Everyone should have had Physics 121 and differential & integral calculus. There is a "math review" posted to the course website, which you should look over at the start of the semester.

**Textbooks:** (both available in the bookstore)
- *Physics for Scientists and Engineers*, by Serway & Jewitt ($6^{th}$, $7^{th}$, or $8^{th}$ editions). You will need a textbook, or combination of textbooks, that covers chapters 14, 16–22, and 35–39. Inexpensive used versions are perfectly acceptable.
- *Physics phor Phynatics*, by Dallin Durfee. This book contains supplementary material specific to this section of 123. It is a very inexpensive book, and Dr. Durfee does not receive any royalties.

**Learning Outcomes**: This course focuses on these five fundamental areas of physics: fluids, thermodynamics, waves, optics, and special relativity. Specifically, after taking this course, you should be able to:
- Fluids: Solve problems and answer conceptual questions using the basics of fluid statics and dynamics, including Bernoulli's principle and Pascal's law.
- Thermodynamics: Answer conceptual questions and calculate changes in temperature, pressure, entropy and volume for quasistatic ideal gas processes and be able to determine work done and efficiency for gas engines, heat pumps, and refrigerators. Determine heat flow and temperatures in systems in steady state.
- Waves: Solve problems and answer conceptual questions involving waves, using concepts such as wave speed, wavelength, frequency, superposition, beats, and resonance. Solve wave interference problems. Calculate group and phases velocities, solve for Fourier coefficients of periodic functions, and frequencies of notes in the equal temperament musical scale.
- Optics: Find the location and magnification of images in single- and multiple-lens/mirror systems by calculation and by ray tracing, and be able to work general problems in optics using Snell's law and specular reflection.
- Special Relativity: Solve problems in special relativity involving length contraction, time dilation, transformations to different reference frames, and relativistic energy & momentum.

I also hope that as you learn more about the physical laws governing the universe you will learn to recognize physics principles at work in the world around you, and that your appreciation for the order, simplicity and complexity of God's creations will increase. I sincerely believe that one can come to know the Creator better by studying His creations. Along those lines, I have been struck by these three quotes:

Brigham Young:
> Man is organized and brought forth as the king of the earth, to understand, to criticize, examine, improve, manufacture, arrange and organize the crude matter and honor and glorify the work of God's hands. This is a wide field for the operation of man, that reaches into eternity; and it is good for mortals to search out the things of this earth.

Steve Turley (former BYU Physics Department chair):

My faith and scholarship also find a unity when I look beneath the surface in my discipline to discover the Lord's hand in all things (see D&C 59:21). It is His creations I study in physics. With thoughtful meditation, I have found striking parallels between His ways that I see in the scriptures and His ways that I see in the physical world. In the scriptures I see a God who delights in beauty and symmetry, who is a God of order, who develops things by gradual progression, and who establishes underlying principles that can be relied on to infer broad generalizations. I see His physical creations following the same pattern.

Dallin Durfee (former instructor of Physics 123):
In addition to learning physics, I hope [Physics 123] will broaden your interest in and understanding of, well… life, the universe, and everything! My understanding of science and math has affected all aspects of my life, from the way I manage my finances to my understanding and appreciation of the gospel. It has sharpened my reasoning skills and awakened a fascination of the universe we live in.

**Class Identification Number:** Each of you will receive a personal identification number for this course, called a "Class ID" (CID). The purpose of this number is to protect your privacy. If you did not receive your CID by e-mail, you can obtain it from the link on the class website. Include this number—and NOT your name—on all work you turn in.

**Where to turn things in:** Turn in assignments to the slot labeled "physics 123, section 2" in the boxes near room N375 ESC. Be sure to staple your assignments together with a real staple (not just a fold!) and write your CID number in at the top of each assignment. Assignments will be returned to the slots next to those boxes, sorted by the first two digits of your class ID. Because these "turn back" slots are open, other students will be able to see your work—so to maintain confidentiality, please do not write your name on your assignments.

**Student Email Addresses:** I will periodically send class information via email to your email address that is listed under Route-Y. If that is not a current address for you, please update it.

**Clickers:** We will use "i-clickers" in class. On the reverse side of your clicker is an alphanumeric ID code for your transmitter. You must go to the course website as soon as possible and register your transmitter ID in order to get credit for your in-class quizzes.

**Mathematica:** Some of the homework problems will require numerical calculations and plots. Mathematica is the recommended program for this, but you can use other similar programs if you prefer. At any rate, when a problem says, "Use a computer program such as Mathematica to make a plot," a hand-drawn plot is NOT sufficient. A computer printout must be turned in, preferably also with the code used to generate the plot.

Mathematica will be the major topic of Physics 230 if/when you take that course. In the meantime, for a basic, concise introduction which contains everything you should need to know for this course, see my Basic Commands of Mathematica document on the course website. (That document must be opened with Mathematica, not a word processor.) Mathematica is found on all departmental computers. You can gain access to these computers by following the instructions given here: http://www.physics.byu.edu/ComputerSupport/ComputerAccounts.aspx

**Grading:** If you hit these grade boundaries, you are guaranteed to get the grade shown. I may make the grading scale easier than this in the end, if it seems appropriate, but I will not make it harder. Because students are not graded relative to each other, it is to your advantage to learn collaboratively!

| A | 93% | B+ | 84% | C+ | 73% | D+ | 60% |
|---|-----|----|----|----|----|----|----|
| A- | 89% | B | 80% | C | 69% | D | 56% |
| | | B- | 77% | C- | 64% | D- | 50% |

Grades will be determined by the following weights:

- Clicker quizzes: 4%
- 3 Midterm Exams: 30%
- Final Exam: 19%

- Term Project: 8%
- Labs: 6%
- Homework: 33%

Your current grade can be viewed through the class web page. Please check your scores regularly to make sure they are recorded correctly.

**Clicker quizzes:** There will be two types of in-class clicker quiz questions: (1) graded "reading quizzes" which should not be very difficult if you have done the reading assignment, and (2) ungraded "thought questions" which will help me pinpoint misconceptions and encourage class discussion. For the graded questions, you will get 2 points each for a right answer and 1 point for a wrong answer (for participating). For the ungraded questions, you will get 1 point each as long as you attempt an answer.

All of the questions from a given class period constitute a single quiz which will be recorded in your grades. You will not be allowed to make up missed quizzes **for any reason** (tardy, excused absence, unexcused absence, registered late, forgot/lost clicker, etc.). However, so that you are not penalized unduly for missing class when circumstances necessitate, you will get <u>four free quizzes</u>: I will convert your four quizzes with the most missed points into perfect scores. I will bend the "no make-up quizzes" rule only if circumstances beyond your control have resulted in you missing more than four class periods.

**Midterm Exams:** Three midterm exams will be given in the Testing Center and will be available for the days indicated on the schedule. Exams will include worked problems similar to homework problems, as well as conceptual questions related to things we discussed in class such as thought questions, demonstrations, etc.

**Final Exam:** A comprehensive final exam will be given during the regularly scheduled time for our class, as indicated on the schedule.

**Term Project:** The term project is an opportunity for you to propose and conduct a simple experiment or to theoretically, mathematically, or computationally investigate an aspect of the course in more depth. Term project guidelines, as well as a list of possible projects and examples of projects done in prior semesters are available on the class web page. There are three parts to the term project: a proposal, a progress report, and a final report. Due-dates for each of the three parts are on the class schedule.

**Labs:** You will perform several short experiments. Most will be similar to the "walk-in labs" in Physics 121, and will be set up in room S415 ESC. Two of the labs will be computer simulations available through the class website. The availability and due-dates of the labs are listed on your schedule. Each lab has a worksheet with instructions and questions to be answered; the worksheets are located at the end of this syllabus packet. You are encouraged to work and discuss the labs in groups, but everyone must be present and participate, and all analysis must be your own work. Because you are given a week in which to do each lab, labs typically <u>may not be made up</u>.

**Homework:** This will be a very homework-intensive class, and homework scores will count as a substantial fraction of your overall course grade. The homework problems for this course are found later in this packet. Problems 1-1 through 1-7 belong to Homework 1, problems 2-1 through 2-8 belong to Homework 2, etc. Some problems require numeric answers which will be graded by the computer, others (labeled "Paper only") do not require you to enter your answers into the computer. A few problems contain both computer-graded and paper-only questions in different parts of the problem.

Be they computer-graded or paper-only problems, you must turn in your work for all homework problems, and your work must be legible with all steps clearly presented. Practice good problem solving skills: draw pictures of the problems, write and solve equations with symbols as much as possible before plugging in numbers, write neatly, and use plenty of space. Substitute units with your numbers into your algebra, and check to see that the units work out right on your final answer. Think about whether your final answer makes physical sense before submitting it.

You are strongly encouraged to work with other students to figure out the problems; however, don't copy others' work or allow others to copy your work. Any assignment handed in must be your own work. If you do get help on a homework problem, be sure to learn the strategy, concepts and steps used to solve the problem, and think about how they would apply to related situations.

Assignments are due on the dates marked on the schedule. Your work on paper is due any time before the building closes; your computer-graded answers must be submitted via the website by 11:59 pm. To allow for emergencies or adding the class late, you will get four free late assignments (chosen to maximize your points); after that, late work only counts for half credit. I will bend this rule only if circumstances beyond your control have prevented you from turning in more than four homework sets on time. No homework assignments will be dropped.

Each homework assignment will include a standard 5 points to be given at the TA's discretion, used to grade the legibility of your work. If the assignment is reasonably neat and complete, you will get the full 5 points. If it is messy, missing sections, not stapled, etc., then the TA will reduce your points accordingly.

**Computer-graded homework details**: The computer-graded problems use a custom-designed system created by BYU Physics Department faculty members. You may have used this system in Physics 121. This system offers several major advantages to students and professors:

- Students get instant feedback as to whether they did the problem correctly.
- Because the HW problems are not assigned directly from the textbook, students can purchase cheap older editions instead of all being forced to use the same, newest edition.
- Students get multiple tries to get the problems right. Specifically, I have arranged things so that you get two attempts at a problem for full credit; after that, you start losing points.
- Each student gets a slightly different—but closely related—problem to work; this makes copying off of other students more difficult. (Yes, sadly even at BYU this is sometimes a problem.)

*Data for the problems*: Each of you will do the problems using different numbers ("data"), resulting in different numerical answers. Blanks are left in the problem statements where you can write in your own data. Your data for the entire semester is available via the internet: once you have a CID, go to the class website, click on "Online Homework", and then click on "Homework Data Sheet". You can get your same personal data again anytime during the semester if you lose your original data sheet. Assume that the numbers given in the problem and in your data sheet are exact. If you are given 2.2 m/s, it means 2.2000000..., to as many digits as you wish to imagine.

*Answer ranges and precision*: At the end of the list of homework problems, there is information about the answers. You are given a range of possible values for each answer, along with the units in which you must submit your answer. For example, "400, 800 J" means that your answer will lie between 400 and 800 J, and that you must give your answer in Joules (not kJ, BTU, ergs, foot-pounds, or any other energy units). These numbers also indicate the accuracy to which you must calculate the answer. This is simply the number of digits shown—for example, "400, 800 J" means that the answer must be given to the nearest 1 J. As another example: "15.0, 60.0 N" means that the answer must be given to the nearest 0.1 N. In some cases, the accuracy is indicated via a plus/minus sign. For example, "32000, 39000 ±100 km" means the answer must

be given to the nearest 100 km. <u>You can always submit a more precise answer with no penalty</u>. Tip: When working a problem, do not round off any numbers until you get your final answer; rounding along the way can lead to compounded errors that cause the final answer to be outside the specified precision range. That is one reason I recommend that when possible you should write and solve your equations with symbols before plugging in numbers.

*How to submit answers*: After working the problems, you must submit your answers over the internet. Go to the class website, click on "Online Homework", and then click on the assignment number. Fill in the numerical answers as indicated. Do not put units on your answer, but make sure that the number you submit is given in the units specified by the answer range. If a very large or very small value needs to be written in scientific notation, as specified by the answer range, indicate the exponent of 10 with an "e". For example, $2.998 \times 10^8$ would be written 2.998e8, and $1.6 \times 10^{-19}$ would be written 1.6e-19. <u>Do not</u> put any spaces, commas, or "x"s in the number. <u>Do</u> put in negative signs where appropriate.

*Grading and viewing correct answers*: Your submission will be graded immediately: after submitting your answers, you should see a status window that shows you which problems you got right and which you got wrong. You can see the status report again at any time by going to the class website, clicking on "Online Homework", and selecting "Homework Status". In addition to your score, the computer will show you the correct answers for the problems you missed; that should help you figure out where you went wrong.

*Try again*: You have 3 tries to get the problem right before the 11:59 pm deadline. After each try, a <u>new set of data</u> will appear at the bottom of the homework status page (because you will have been given the answers for the old set of data). Use this new data for the next try. You only need to resubmit the parts that you missed in the previous try. Retries will also be graded immediately.

*Points per problem*: You will receive 5 points for each part of each problem done correctly on the first or second tries, 3 points for the third try, and no points thereafter.

*Special case: Multiple choice questions*: Some computer-graded problems are multiple choice. Each correct multiple choice answer is also worth 5 points. Multiple choice problems will have drop-down boxes for submitting your answers. There are <u>no retries</u> for multiple choice problems.

**Paper-only problems:** Problems, or parts of problems, that do not involve computer grading will be graded by the TA out of a maximum score to be set relative to the difficulty of the problem, typically 5-20 points.

**Late credit:** Any points from computer-graded or paper-only problems received after the deadline will be marked late. You will receive full credit for late points on the four assignments with the most late points. That is, you get <u>four free late assignments</u>, chosen to maximize your points. You will receive <u>half credit</u> for all other late points. You will get <u>no credit</u> for any HW turned in after the deadline marked on the schedule (the second reading day).

**Extra credit:** Some of the HW problems are marked as extra credit. These problems will be graded the same as regular problems, except you will not be penalized if you skip them. If you do them, they allow you to increase your score beyond the listed maximum for that assignment.

**Getting help:** There are multiple ways for you to get help solving homework problems.

<u>Other Students</u>. One of your first lines of defense should be the other students in the class. Introduce yourself to people you sit next to. Be proactive: call others to discuss the homework, form study groups to work on homework or review for exams, etc. It has been shown in several studies that personal contact with

classmates (and with faculty members) is one of the most important factors in a student's success in college. Students in this class in the past who have gotten to know their fellow students have formed friendships that have lasted well beyond Physics 123, and which have helped their studies in future courses as well.

Dr. Colton's Office Hours. You should take full advantage of my office hours, which are held directly after class in the Underground Lab. (The secret passageway to the Underground Lab is located on the ground floor of the ESC, on the north end of the building. There you'll find a door without a lock which opens to a long, descending staircase going down to the Underground Lab.) I recommend that you get as far as you can on the homework before class, and then come down to the UGL study area directly after class. You will find other students from the class to work with, and you will have ready access to me when you have questions that your classmates can't answer.

Course TA. The course TA will also hold regular office hours where you can get help on upcoming homework problems or find out why you missed points on past homework problems.

Tutorial Lab. A physics tutorial lab is provided in N304 and N362 ESC (it changes each semester; check the signs on the doors). Teaching assistants will be available roughly from 9 am to 9 pm every weekday, and for several hours on Saturday. The exact schedule can be found via a link on our course website. One cautionary note: the TAs in the tutorial lab will likely focus on the 123 section 1 homework problems, so they may not always be able to help with the section 2 problems.

**Homework Solutions:** My own handwritten homework solutions will be posted in the display cases around the corner from the homework turn-in boxes, close to room N362. Typically the most recent several HW assignments will be posted, so that you can see how I myself did the problems.

**Extra Credit:** In addition to extra-credit homework problems mentioned above, there are extra-credit papers you can write during the semester.

1. Book review. This is a book review of a physics-related book that you read during the semester, written in a style similar to book reviews that you find on amazon.com. A list of allowed books is included later in this syllabus packet; if you want to write a review of a book not on the official list, you must get my permission first. At a minimum you must include this information in your review: (1) title and author of the book, (2) a rating out of five stars, (3) a paragraph description of what the book was about, and (4) a paragraph with your personal assessment of the quality of the book. Your review will be graded out of 8 points based on the quality of the writing and helpfulness/completeness of the review, the maximum score being the equivalent of +8 points on one of your midterms. This extra credit item can be done twice during the semester. Send your review(s) to Dr. Colton via email.

2. Physics-related lecture. This is a brief report of a physics-related lecture you attended during the semester. At a minimum you must include this information in your report: (1) name of speaker, (2) time/place of lecture, and (3) some info about what kind of physics was discussed, (4) at least one thing you learned that you (hopefully) found interesting. This could be one of the weekly Physics Department colloquia (warning: these often—but not always—get very technical), an honors lecture, a university forum, a planetarium show, or any other physics-related science lecture that you can find. If you wonder if a certain lecture is appropriate, please ask me. Your report will be graded out of 2 points, the maximum score being the equivalent of +2 points on one of your midterms. This extra credit item can be done three times during the semester. (A planetarium show can only count for one of the three, though.) Send your report(s) to Dr. Colton via email.

**Final Thoughts from Dr. Colton:** In a BYU seminar for new faculty that I attended, experts on student learning taught that **most student learning is done *outside* of the classroom**. I expect this class to follow that same trend. For the most part, you learn how to solve physics by *solving physics problems*: figuring out what needs to be done by yourself or with help from others and doing it—not by *listening* to someone tell you how to solve physics problems during a lecture. This will therefore likely be a very homework-intensive class for you, with labs, extra credit assignments, and a term project in addition to the regular homework problems. The BYU Undergraduate Catalog states that "The expectation for undergraduate courses is three hours of work per week per credit hour for the average student who is appropriately prepared; much more time may be required to achieve excellence". To me, for this particular three credit hour class, that means **an average student should spend at least six hours per week on study and work outside of class**, in order to get an average grade. Many of you will spend many more hours than that. However, I hope that will not be an undue burden. We have a lot of cool things going on in this class, in my opinion, and if you are in the right major/minor, you should find them cool, too!

**Handbook for Physics Majors**: I encourage all physics majors and minors to take a good look at the *Handbook for Physics Majors*, available here: http://www.physics.byu.edu/Undergraduate/docs/handbook.pdf.

**BYU Policies:**

*Prevention of Sexual Harassment:* BYU's policy against sexual harassment extends to students. If you encounter sexual harassment or gender-based discrimination, please talk to your instructor, or contact the Equal Opportunity Office at 801-422-5895, or contact the Honor Code Office at 801-422-2847.

*Students with Disabilities:* BYU is committed to providing reasonable accommodation to qualified persons with disabilities. If you have any disability that may adversely affect your success in this course, please contact the University Accessibility Center at 801-422-2767, room 1520 WSC. Services deemed appropriate will be coordinated with the student and your instructor by that office.

*Children in the Classroom:* The serious study of physics requires uninterrupted concentration and focus in the classroom. Having small children in class is often a distraction that degrades the educational experience for the entire class. Please make other arrangements for child care rather than bringing children to class with you. If there are extenuating circumstances, please talk with your instructor in advance.

# Book Review Extra Credit Book List

Please see the main syllabus packet for instructions about the book review extra credit assignment. If you want to get credit for reading/writing a report for a book not on this list, you must get prior approval from Dr. Colton first.

*A Brief History of Time*, by Stephen Hawking

*A Briefer History of Time*, by Stephen Hawking

*A Short History of Nearly Everything*, by Bill Bryson

*Albert Einstein – A Biography*, by Alice Calaprice and Trevor Lipscombe

*Benjamin Franklin: An American Life*, by Walter Isaacson

*Beyond Star Trek: Physics from Alien Invasions to the End of Time*, by Lawrence Krauss

*Einstein: His Life and Universe*, by Walter Isaacson

*The First American: The Life and Times of Benjamin Franklin*, by H.W. Brands

*From Clockwork to Crapshoot: A History of Physics*, by Roger G. Newton

*Front Page Physics*, by Arthur Jack Meadows

*Galileo's Daughter: A Historical Memoir of Science, Faith, and Love*, by Dava Sobel

*Genius: The Life and Science of Richard Feynman*, by James Gleick

*How Math Explains the World: A Guide to the Power of Numbers, from Car Repair to Modern Physics*, by James D. Stein

*In Search of Schrödinger's Cat: Quantum Physics and Reality*, by John Gribbin

*Isaac Newton*, by James Gleick

*Lise Meitner: A Life in Physics*, by Ruth Lewin Sime

*Measured Tones*, by Ian Johnston

*Miss Leavitt's Stars: The Untold Story Of The Woman Who Discovered How To Measure The Universe*, by George Johnson

*Mr. Tompkins in Paperback/ Mr. Tompkins in Wonderland* (essentially the same book), by George Gamow

*Parallax: The Race to Measure the Cosmos*, by Alan Hirshfeld

*Physics for Future Presidents: The Science Behind the Headlines*, by Richard Muller

*Physics of the Impossible: A Scientific Exploration into the World of Phasers, Force Fields, Teleportation, and Time Travel*, by Michio Kaku

*Quantum: A Guide for the Perplexed*, by Jim Al-Khalili

*Six Easy Pieces*, by Richard P. Feynman

*Stephen Hawking: A Biography*, by Kristine Larsen

*Symmetry and the Beautiful Universe*, by Leon M. Lederman and Christopher T. Hill

*The Accelerating Universe: Infinite Expansion, the Cosmological Constant, and the Beauty of the Cosmos*, by Mario Livio

*The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*, by Brian Greene

*The Fabric of the Cosmos: Space, Time, and the Texture of Reality*, by Brian Greene

*The God Particle: If the Universe Is the Answer, What Is the Question?* by Leon Lederman

*The Making of the Atomic Bomb*, by Richard Rhodes

*The New Cosmic Onion: Quarks and the Nature of the Universe*, by Frank Close

*The Physics of Baseball*, by Robert K. Adair

*The Physics of Basketball*, by John Joseph Fontanella

*The Physics of NASCAR: How to Make Steel + Gas + Rubber = Speed*, by Diandra Leslie-Pelecky

*The Physics of Star Trek*, by Lawrence Krauss

*The Physics of Superheroes*, by James Kakalios

*The Quantum World: Quantum Physics for Everyone*, by Kenneth Ford

*The Universe and Dr. Einstein*, by Lincoln Barnett

*The Universe in a Nutshell*, by Stephen Hawking

*Thirty Years that Shook Physics: The Story of Quantum Theory*, by George Gamow

*Voodoo Science: The Road from Foolishness to Fraud*, by Robert Park

# How to Solve Physics Problems

Here's the "Colton method" for solving physics problems. It's not just the way *I* do problems, though; if you look at the worked problems in the book, you'll find they all follow this same sort of procedure.

**Picture** – Always draw a picture, often with one or more FBDs. Make sure you understand the situation described in the problem.

**Equations** – <u>Work forward</u>, not backward. That means look for equations that contain the *given* information, not equations that contain the *desired* information. What major concepts or "blueprint equations" will you use? Write down the general form of the equations that you plan to use. Only after you've written down the main equations should you start filling things in with the specific information given in the problem.

**Algebra** – Be careful to get the algebra right as you solve the equations for the relevant quantities. Use letters instead of numbers if at all possible. Even though you (often) won't have any numbers at this stage, solving the algebra gives you what I really consider to be the answer to the problem. And <u>write neatly</u>!

**Numbers** – After you have the answer in symbolic form, plug in numbers to obtain numerical results. <u>Use units</u> with the numbers, and make sure the units cancel out properly. Be careful with your calculator—punch in all calculations *twice* to double-check yourself.

**Think** – Does your final answer make sense? Does it have the right units? Is it close to what you were expecting? In not, figure out if/where you went wrong.

**Example problem:** Using a rocket pack, a lunar astronaut accelerates upward from the Moon's surface with a constant acceleration of 2.1 m/s$^2$. At a height of 65 m, a bolt comes loose. (The free-fall acceleration on the Moon's surface is about 1.67 m/s$^2$.) (a) How fast is the astronaut moving at that time? (b) How long after the bolt comes loose will it hit the Moon's surface? (c) How high will the astronaut be when the bolt hits?

**Colton solution:** (notice how I use the five steps given above)



When I first did part (c), I got 0 m. This didn't seem right (using the final step, "Think"), so I had to figure out what went wrong. I had used the wrong acceleration.

# Some things to remember before you begin Homework #1:

- Be sure to put your HW in the right box! If your HW is handed into the wrong box it will be counted late.

- Be sure to staple your assignments (with a REAL staple) or you will lose points.

- Work all numerical answers to the number of digits specified by the answer key (located at the end of the HW problems). Typically that is 3 significant figures, but sometimes it is more. For intermediate results, keep more sig figs than that so that you do not accumulate rounding errors.

- Use the system described on the previous page (you can call it the PEANuT system, if you like):
    - **Picture**
    - **Equations**
    - **Algebra**
    - **Numbers**
    - **Think**

- Don't be shy about asking for help from fellow classmates, the TA, or Dr. Colton.

- DO ALL OF YOUR HOMEWORK.  This is how you will learn the material, and this is the BEST way to prepare for exams. You will learn far more by completing—and understanding—the homework problems than you will learn from (for example) listening to Dr. Colton in class.

OK, now you can go to the next page and start your homework.

# Physics 123 Homework Problems, Fall 2011

## Section 2, John Colton

1-1. A [01] _____-kg ballet dancer stands on her toes during a performance with 26.5 cm$^2$ in contact with the floor. What is the pressure exerted by the floor over the area of contact (a) if the dancer is stationary, and (b) if the dancer is jumping upwards with an acceleration of 4.41 m/s$^2$?

1-2. What must be the contact area between a suction cup with [02] _____ atm inside and the ceiling in order to support a 127-lb student? Please note the handy conversion table inside the back cover of your textbook.

1-3. If a certain nuclear weapon explodes at ground level, the peak over-pressure (that is, the pressure increase above normal atmospheric pressure) is [03] _____ atm at a distance of 6.0 km. What force due to such an explosion will be exerted on the side of a house with dimensions 4.5 m × 22 m? Give the answer in tons (1 ton = 2000 lb).

1-4. Piston 1 in the figure has a diameter of [04] _____ in.; piston 2 has a diameter of 1.5 in. In the absence of friction, determine the force F necessary to support the 500-lb weight.
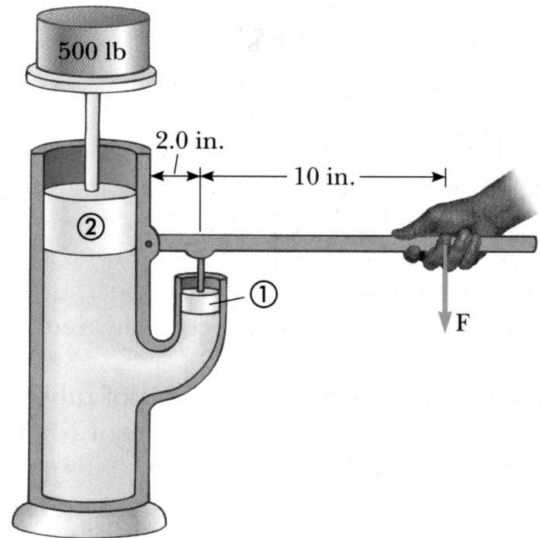
1-5. A U-tube of uniform cross-sectional area and open to the atmosphere is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in the figure, with $h_2 = $ [05] _____ cm, determine the value of $h_1$.
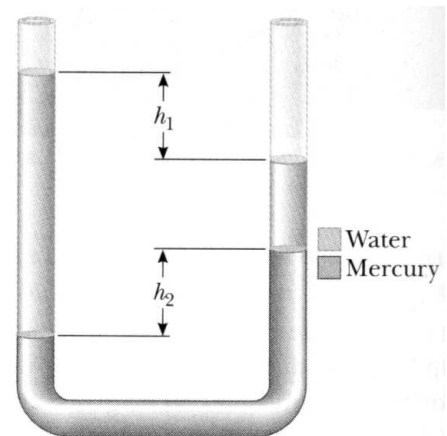
1-6. (Paper only.) Work the problems on the math review posted to the course website. Check your answers against the solutions, also posted to the website, and learn how to do any problems that you missed. Turn in a statement saying that you have done this.

1-7. (Extra credit.) The tank shown in the figure is filled with water to a depth of $h = $ [06] ——————— m. At the bottom of one of the side walls is a rectangular hatch 1.00 m high and 2.00 m wide. The hatch is hinged at its top. Determine the net force exerted by the atmosphere and water on the hatch. Hint: Since the pressure is not constant, you will have to integrate in order to get the force. If you divide the hatch into narrow horizontal stripes, $P \times \text{width} \times dy$ will be the force on each stripe (since force = pressure × area), where $P$ is the pressure that is changing with depth.
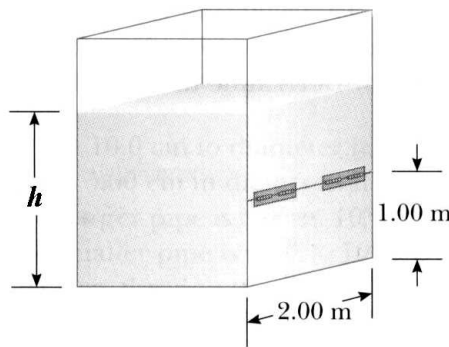


**Extra problems I recommend you work (not to be turned in):**

- Visit the Cartesian diver exhibit on the north-west side of the lobby of the Eyring Science Center. Play with the diver, and read the explanation on the wall. Why is the diver inside the bottle affected when you squeeze the outside of the bottle?

_____

2-1. A rectangular air mattress is 2.1 m long, 0.48 m wide, and [01] ——————— m thick. If it has a mass of 2.3 kg, what additional mass can it support in water?

2-2. A raft is made of solid wood and is 2.31 m long and 1.59 m wide. The raft is floating in a lake. A woman who weighs [02] ——————— lb steps onto the raft. How much further into the water does the raft sink? You do not need the thickness of the raft or the density of the wood to solve this problem.

2-3. A light spring of constant $k = 163$ N/m rests vertically on the bottom of a large beaker of water. A 5.29-kg block of wood (density=[03] _____ kg/m³) is connected to the spring and the mass-spring system is allowed to come to static equilibrium. (a) Draw a free-body diagram of the block. (b) What is the elongation $\Delta L$ of the spring?



(a)                    (b)
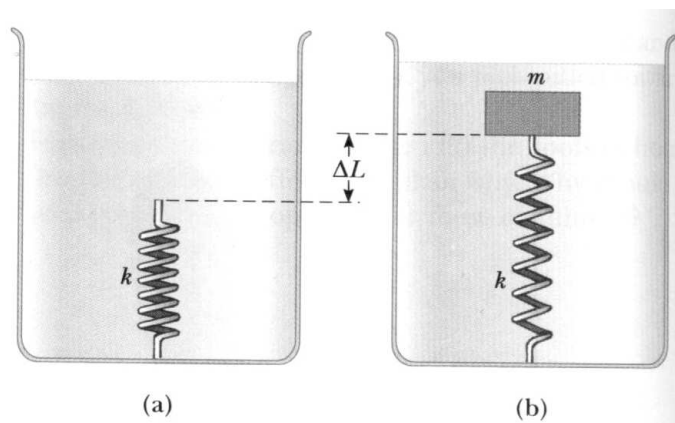
2-4. A 10.0-kg block of metal is suspended from a scale and immersed in water as in the figure. The dimensions of the block are 12.0 cm × 10.0 cm × [04] _____ cm. The 12.0-cm dimension is vertical, and the top of the block is 5.00 cm below the surface of the water. What are the forces exerted by the water on (a) the top and (b) the bottom of the block? (Take atmospheric pressure to be $1.0130 \times 10^5$ N/m².) (c) What is the buoyant force? Think about how your answers to (a) and (b) relate to your answer to (c). (d) What is the reading of the spring scale?
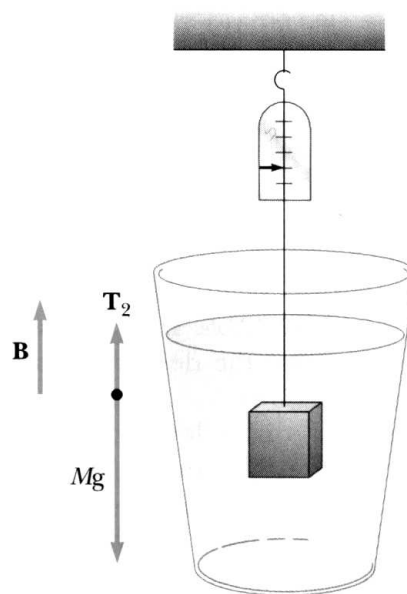


2-5. A geological sample weighs 10.3 lb in air and [05] _____ lb under water. What is its density in g/cm³?

2-6. An extremely precise scale is used to measure an iron weight. It is found that in a room with the air sucked out, the mass of the weight is precisely [06] _____ kg. If you add the air back into the room, by how many grams will the new measurement differ from the old? Use a positive answer to indicate the scale reading has increased, and a negative answer to indicate the scale reading has decreased. Use the densities of iron and air given in the book for 0°C and 1 atm.

2-7. (Paper only.) As is mentioned in the syllabus, you will periodically use Mathematica to plot functions or otherwise help you do homework problems. For problems such as the following you should turn in a printout which includes both your Mathematica code and the plots that Mathematica generated for you. (Alternate computer programs are acceptable if they have the same capability. If you want to use an alternate program, then tailor the following instructions accordingly. You still have to turn in hard-copy printouts, and if possible the code you used.)

(a) Find and gain access to a computer with Mathematica. There are instructions on how to do this in the syllabus, on page 3. Use Mathematica to open up Dr. Colton's *Basic Commands of Mathematica* document, posted to the class website. Read that up to and including the "How to plot a function" section.

(b) Define the following function: $f(x) = 3\sin(2x)$. Evaluate the function at $x = 1$, 2, and 3. Give your answers as numerical results to five decimal places.

(c) Plot the function from $x = 0$ to 10.

2-8. (Extra credit; paper only.) A lead weight is placed on one end of a cylindrical wooden log having cross-sectional area $A$, in a fluid with density $\rho$. Because of the weight, the log tips vertically out of the fluid, with the weight on the bottom. The combined mass of the log and the weight is $m$. Show that if the log is pushed down from its equilibrium position, it will undergo simple harmonic motion. What will the period of the motion be? Use the letter $g$ to represent the acceleration due to gravity. Hint: To show that the log will undergo SHM, show that the net force on the log equals a constant times the displacement from equilibrium, just like the force on a mass from a spring. Then, the period of oscillation is $2\pi \times \sqrt{m/constant}$, again just like a mass on a spring.

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3-1. A cowboy at a dude ranch fills a horse trough that is 1.53 m long, 61 cm wide, and 42 cm deep. He uses a 2.0-cm-diameter hose from which water emerges at [01] _____ m/s. How long does it take him to fill the trough?

3-2. Suppose the wind speed in a hurricane is [02] _____ mph (mi/h). (a) Find the difference in air pressure outside a home and inside a home (where the wind speed is zero). The density of air is $1.29 \text{ kg/m}^3$. (b) If a window is 61 cm wide and 108 cm high, find the net force on the window due to the pressure difference inside and outside the home.

3-3. What *gauge* pressure must a pump generate to get a jet of water to leave its nozzle with a speed of 5.2 m/s at a height of [03] —————— m above the pump? Assume that the area of the nozzle is *very small* compared to that of the pipe near the pump.

3-4. A U-tube open at both ends is partially filled with water, as in Figure (a). Oil ($\rho = 754$ kg/m$^3$) is then poured into the right arm and forms a column $L = $ [04] —————— cm high, as in Figure (b). (a) Determine the difference $h$ in the heights of the two liquid surfaces. (b) The right arm is then shielded from any air motion while air is blown across the top of the left arm until the surfaces of the two liquids are at the same height, as in Figure (c). Determine the speed of the air being blown across the left arm. Assume that the density of air is 1.29 kg/m$^3$.



(a)    (b)    (c)
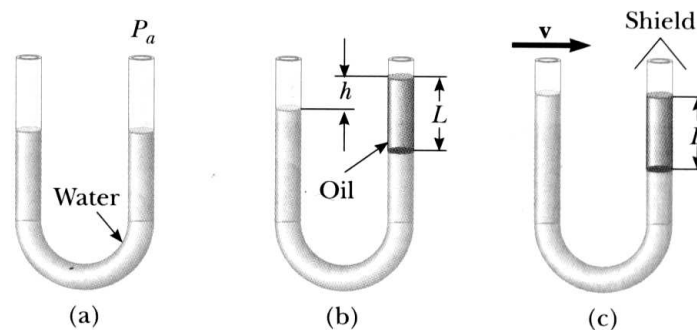
3-5. (Paper only.) Imagine that you had a cylindrically-shaped paper cup filled to a height $h$ with water sitting on a level table top. If you poked a small hole in the side of the cup, water would shoot out in an arc and hit the table. (a) If you want to maximize the distance that the water goes before hitting the table, how far from the bottom of the cup should you poke the hole? Hint: To maximize the distance, you will have to calculate a derivative and set it equal to zero. (b) If you place the hole at that location, how far will the water travel before hitting the table? Assume the hole is small enough that the height of the water in the cup doesn't change significantly over the time that you make the measurements, and assume that you can neglect viscosity.

3-6. (Paper only.) Now let's test it out: Find a paper or Styrofoam cup. A cylindrical one would be best, but those are hard to come by, so just get one as close to cylindrical as you can find. Punch a small hole at the correct height to maximize the distance that the water will go before hitting the table. The hole should be small so you can make your measurements before the height of the water in the cup changes appreciably, but not too small or viscosity will change your results. If you use a pencil to make your hole, you will probably do well, but you will have to watch what happens quickly before the water level in the cup drops. Now place your cup on the table and mark where you expect the water to hit the table. Put your finger over the hole, and fill the cup with water. Quickly remove your finger and note how close to your mark the water hits. (a) How close were you? (b) Now put tape over your hole and punch a new hole which is higher and try again. Did the water go farther or not as far? (c) Now do the same with a hole below the optimum height. Did the water go farther or not as far?

3-7. (Paper only.) This is a continuation of the introduction to Mathematica problem from the previous assignment. (a) Continue reading the *Basic Commands of Mathematica* document; up to and including the "How to differentiate a function" section.
(b) Define the following function: $f(x) = 3x^3 \sin(2x)$. Find the function that is the integral of $f(x)$.
(c) Define a new function, $g(x) = e^{-5x^2}$. Find the numerical value of the integral of this function, integrated from -0.5 to 0.5.
(d) Define a new function, $h(x) = \cos(x)\sqrt{1 + \pi x}$. Find the numerical value of the derivative of this function at $x = 3$.

**Extra problems I recommend you work (not to be turned in):**

- I find that I can blow 1000 cm$^3$ of air through a drinking straw in 2 s. The diameter of the straw is 5 mm. Find the velocity of the air through the straw. (Answer: 25.46 m/s.)

- A horizontal pipe 11.5 cm in diameter has a smooth reduction to a pipe 5.2 cm in diameter. If the pressure of the water in the larger pipe is 84.1 kPa and the pressure in the smaller pipe is 60.0 kPa, at what rate (kg/s) does water flow through the pipes? (Answer: 36.56 kg/s.)

4-1. Imagine that we want to invent a new temperature scale, called the BYU scale, where 0°B is the same as −40°C, and 100°B is the same as [01] _____°C. What would absolute zero be on the BYU scale?

4-2. The figure shows a circular steel casting with a gap. If the casting is heated, (a) does the width of the gap increase or decrease? (b) The gap width is 1.6000 cm when the temperature is 30°C. Determine the gap width when the temperature is [02] _____°C.
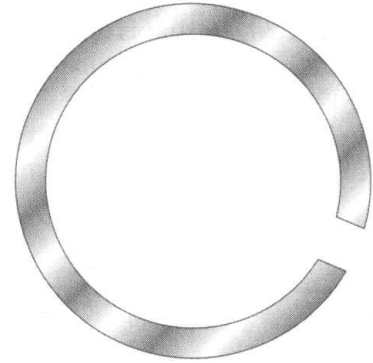
4-3. An underground gasoline tank at 54°F can hold [03] _____ gallons of gasoline. If the driver of a tanker truck fills the underground tank on a day when the temperature is 90°F, how many gallons, according to his measure on the truck, can he pour in? Assume that the temperature of the gasoline cools to 54°F upon entering the tank. Use the coefficient of volume expansion for gasoline given in the textbook.

4-4. A grandfather clock is controlled by a swinging brass pendulum that is 1.3 m long at a temperature of 20°C. (a) By how much does the length of the pendulum rod change when the temperature drops to [04] _____°C? (b) If a pendulum's period is given by $T = 2\pi\sqrt{L/g}$, where $L$ is its length, does the change in length of the rod cause the clock to run fast or slow? (c) Assuming the clock kept perfect time before the temperature drop, over the course of 24 hours how many seconds does the clock gain or lose? Give your answer as a positive number.

4-5. Inside the house where the temperature is 20°C, we measure the length of an aluminum rod with a micrometer made of steel. (A micrometer is a device which measures distances very accurately.) We find the rod to be 10.0000 cm long. If we repeat this measurement outside where the temperature is [05] _____°C, what result would we obtain? Caution: the size of the micrometer is also affected by the temperature, so we no longer obtain the true length of the rod when we measure it with the micrometer. We want to find the length of the cold rod according to the cold micrometer.

4-6. A tank having a volume of 100 liters contains helium gas at 150 atm. How many balloons can the tank blow up if each filled balloon is a sphere [06] —————— cm in diameter at an absolute pressure of 1.20 atm? Don't worry about the fact that when the pressure in the tank gets below 1.2 atm, the tank wouldn't be able to force the helium into any more balloons.

4-7. With specialized equipment, it is routine to achieve vacuums with pressures below $10^{-10}$ torr (1 torr = 1 mm of Hg = 133.3 Pa). However, special care must be taken in cleaning and baking the walls of the stainless steel chamber, or "outgassing" of contaminants will seriously increase the pressure (by orders of magnitude). If the pressure is $1.00 \times 10^{-10}$ torr and the temperature is [07] —————— °C, calculate the number of molecules in a volume of $1.00 \text{ m}^3$.

4-8. (Paper only.) This is a continuation of the introduction to Mathematica problem from the previous assignments. (a) Continue reading the *Basic Commands of Mathematica* document; up to and including the "How to find the maximum/minimum of a function" section.

(b) Define the following function: $f(x) = \sin(x)e^{-x}$. Plot $f(x)$ from $x = 0$ to 10.

(c) Find the location close to $x = 2$, where $f(x) = 0.1$.

(d) Find the location close to $x = 1$, where $f(x)$ has a maximum.

4-9. (Extra credit.) If you push on an object from all sides, it will compress a bit. The amount it compresses is measured by the bulk modulus $B$. If a pressure increase of $\Delta P$ reduces the volume of the object from $V$ to $V + \Delta V$ (where $\Delta V$ is negative because the object is getting smaller), the bulk modulus is defined as:

$$B = -\frac{\Delta P}{\Delta V/V}.$$

Imagine that you make a copper sphere and embed it in a block of some super material which has an extremely high bulk modulus and a linear thermal expansion coefficient of [08] —————— °C$^{-1}$. Assume that the sphere is in contact with the block at all points on its surface. Assume that the sphere is a perfect fit for the cavity in the block—it's a really snug fit, but the copper is not being compressed by the block. If you then heat the block and copper sphere by 20°C, with what pressure (in atm) will the copper push on the block? The bulk modulus and linear expansion coefficient of copper can be found in the textbook. Hint: Since $\alpha \Delta T << 1$, you can use the approximation that $\beta = 3\alpha$.

**Extra problems I recommend you work (not to be turned in):**

- The volume expansion coefficient for mercury is $1.82 \times 10^{-4}/°\text{C}$. So how can the mercury level in a mercury thermometer go from almost one edge of the tube to almost all the way to the other edge when the temperature changes by less than $100°\text{C}$?

- An air bubble has a volume of 1.50 cm$^3$ when it is released by a submarine 100 m below the surface of a lake. What is the volume of the bubble when it reaches the surface where the atmospheric pressure is 1.00 atm? Assume that the temperature and the number of air molecules in the bubble remains constant during the ascent. (Answer: 16.01 cm$^3$.)

- A tire is filled to 35 psi (gauge pressure) on an unusually hot day in autumn ($90°\text{F}$). What will be the pressure on an unusually cold morning in December ($-20°\text{F}$)? Hint: Don't forget to include the 14.7 psi of atmospheric pressure before computing the change. Then convert back to gauge pressure. Ignore any thermal contraction of the tire. (Answer: 25.05 psi gauge pressure.)

- The specifications on a particular scuba tank says that it should be filled to a pressure of 4350 psi (= 295.9 atm). It also claims that the volume of air that it holds is 90 cubic feet—but what they really mean is that the air that it holds at 4350 psi, if expanded at constant temperature until it was at atmospheric pressure, would fill that amount of volume. (a) What is the actual volume of the tank? (b) If the average mass of the molecules in the air is $4.81 \times 10^{-26}$ kg, how much does the mass of the tank change when it is pressurized from 1 atm to 295.9 atm at $25°\text{C}$? (Answers: 0.3042 cu ft, 3.006 kg.)

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5-1. In a 30.0-s interval, 492 hailstones strike a glass window with an area of 0.624 m$^2$ at an angle of [01] _____° to the window surface. Each hailstone has a mass of 5.00 g and a speed of 8.00 m/s. If the collisions are elastic, what are the average (a) force and (b) pressure on the window?

5-2. Twenty cars are moving in the same direction at different speeds on the highway. Their speeds are [02] _____, 42, 44, 45, 49, 51, 52, 57, 59, 62, 66, 66, 67, 67, 71, 72, 77, 79, 81, and [03] _____ mi/h. (a) What is their average (mean) speed? (b) What is their rms speed? Advice: use a computer program such as a spreadsheet or Mathematica; don't do the calculations with a hand calculator.

5-3. (a) How many atoms are required to fill a spherical helium balloon to a diameter of 30.0 cm at a temperature of [04] _____°C? Take the pressure to be 1.00 atm.

(b) What is the average kinetic energy of individual helium atoms?

(c) What is the root-mean-square speed of the atoms?

(d) What is the average speed of the atoms?

5-4. The *mean free path* $l$ is the average distance a molecule travels between collisions. As discussed in the 6th edition of the textbook (but omitted in later editions), it is related to the number of molecules per volume $n$, and the average diameter of the molecules $d$, in this way:

$$l = \frac{1}{\sqrt{2}\pi d^2 n}$$

(That equation is derived in the 6th edition by visualizing the cylinder that is swept out by the motion of a molecule, and comparing it to the average spacing between molecules. And some hand-waving.)

The mean free path also relates to the average time between collisions $\tau$, through the average velocity $v_{\text{avg}}$:

$$v_{\text{avg}} = \frac{l}{\tau}$$

For an ultra high vacuum situation similar to that described in the previous homework assignment, suppose there are [05] _____ molecules per cubic meter. The temperature is 300K. Determine (a) the mean free path and (b) the time between collisions for diatomic nitrogen molecules ($d \approx 10^{-10}$ m).

5-5. (Paper only.) Use a program such as Mathematica for this problem.

(a) Make a plot of the Maxwell-Boltzmann probability density function (which is the $N_v$ function given in the book, divided by $N$) for oxygen molecules at 500 K. Go up to high enough velocities that you can see the full shape of the curve.

(b) Verify that this function is properly normalized: that the integral from 0 to infinity equals 1.

(c) Use these statistical definitions to calculate $v_{\text{mp}}$, $v_{\text{avg}}$, and $v_{\text{rms}}$ for this situation:

$$v_{\text{mp}} = \text{the velocity where } f(v) \text{ is a maximum (i.e., where the derivative} = 0)$$

$$v_{\text{avg}} = \int_0^\infty v f(v) dv$$

$$v_{\text{rms}} = \sqrt{\int_0^\infty v^2 f(v) dv}$$

Verify that the equations given for those quantities in the textbook produce the same numerical results.

(d) If there are $10^{20}$ molecules in your distribution, how many will have speeds between 300 and 400 m/s? (This is the total number of molecules times how much area the probability density function has between 300 and 400 m/s.)

**Extra problems I recommend you work (not to be turned in):**

- Suppose that Moses consumed on average 2 liters of water per day during his lifetime of 120 yrs. If this water is now thoroughly mixed with the Earth's hydrosphere $(1.32 \times 10^{21}$ kg), how many of the same water molecules are found today in your 1-liter bottle of water? (Answer: $2.22 \times 10^9$.)

- The escape velocity for the Earth is 11.2 km/s. At what temperature will the most probable velocity in a gas of nitrogen molecules be greater than the Earth's escape velocity? (Answer: $2.113 \times 10^8$ K.)

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6-1. A 3000-lb car moving at [01] —————— mi/h quickly comes to rest without skidding the tires. The kinetic energy is converted into heat in each of the four 15-lb iron rotors. By how much will the temperature rise in the rotors?

6-2. Suppose your water heater is broken, so you plan to heat your bath water by converting potential energy to heat. You hoist buckets of water up really high, then tip them over so that the water falls down into the bathtub. If you want to increase the temperature of the water by [02] —————— °C, how high will you have to lift the buckets?

6-3. Most electrical outputs in newer homes can deliver a maximum power of about 1800 W. Using this much power, how long would it take to heat up a bathtub containing [03] —————— m$^3$ of water from 25°C to 40°C?

6-4. (Paper only.) Imagine an ideal aluminum calorimeter with a mass of 150 g (i.e., an aluminum cup that is thermally isolated from the rest of the world). The calorimeter contains 200 g of water in thermal equilibrium with the calorimeter at a temperature of 25.00°C. You then heat an 80 g piece of an unknown metal to a temperature of 100°C and put it into the water. The system comes to equilibrium some time later at a temperature of 27.32°C. (a) What is the specific heat of the metal? (b) From the table in your book, determine what the metal is.

6-5. A [04] _____-g block of ice is cooled to −78.3°C. It is added to 567 g of water in an 85-g copper calorimeter at a temperature of 25.3°C. Determine the final temperature. Remember that the ice must first warm to 0°C, melt, and then continue warming as water. The specific heat of ice is 2090 J/kg·°C.

6-6. What mass of steam that is initially at 121.6°C is needed to warm [05] _____ g of water and its 286-g aluminum container from 22.5°C to 48.5°C?

**Extra problems I recommend you work (not to be turned in):**

- An aluminum rod is 20 cm long at 20°C and has a mass of 350 g. If 15.5 kJ of energy is added to the rod by heat, what is the change in length of the rod? (Answer: 0.2362 mm.)

- A 0.42-kg iron horseshoe that is initially at 652°C is dropped into a bucket containing 19 kg of water at 22°C. By how much does the temperature of the water rise? Neglect any energy transfer to or from the surroundings. (Answer: 1.487°C.)

- A 20 kg iron shell from a tank goes off course and lands in a frozen lake. If the shell is moving at 300 m/s and is at a temperature of 40°C when it hits the 0°C ice, how much ice will melt? (Answer: 3.779 kg.)

_____

7-1. A Styrofoam box has a surface area of 0.832 m$^2$ and a wall thickness of 2.09 cm. The temperature of the inner surface is 4.8°C, and that outside is 25.5°C. If it takes [01] _____ h for 5.54 kg of ice to melt in the container, determine the thermal conductivity of the Styrofoam.

7-2. Suppose you have two solid bars, both with square cross-sections of 1 cm$^2$. They are both [02] _____ cm long, but one is made of copper and one of iron. You place the two side by side and braze them together, making a composite bar with a cross-section of 2 cm$^2$. If one end of this rod is placed in boiling water and the other end in ice water, how much power will be conducted through the rod when it reaches steady state?

7-3. A sheet of copper and a sheet of aluminum with equal thickness are placed together so that their flat surfaces are in contact. The copper is in thermal contact with a reservoir at [03] _____ °C, and the aluminum is in contact with a reservoir at 0°C. What is the temperature at the interface between the metals?

7-4. (Paper only.) The light from the sun reaches the Earth's orbit with an intensity of 1340 W/m$^2$. That means that a 1 m by 1 m perfectly absorbing square, oriented directly at the sun, would absorb 1340 J of heat every second. Assuming that the emissivity of the Earth is the same for all wavelengths of light, calculate the temperature of the Earth in steady state by balancing the heat gained from sunlight (which intersects the Earth's cross-section) with the heat lost from blackbody radiation (which radiates out from the entire surface area of the Earth).

You should get something close to—but a bit colder than—the actual average surface temperature of the Earth, believed to be about 15°C. The primary reason why the Earth is not as cold as your result is due to the fact that the emissivity of the Earth depends strongly on wavelength via the so-called "greenhouse effect". Because of the atmosphere, the Earth absorbs and emits visible radiation much better than infrared radiation. Since the sun is very hot, it emits a lot of visible light which is absorbed by the Earth. The colder Earth, however, emits mainly infrared light. The clouds are very reflective in the infrared, so the emissivity is small right where the Earth would be radiating most of its blackbody radiation otherwise.

7-5. (Paper only.) In your job as an intergalactic pizza deliverer, you accidentally deliver a pizza to the wrong location—so far off, in fact, that there aren't even any stars nearby. The pizza, initially at 340 K, cools through emitting blackbody radiation.

(a) How warm is the pizza after 1 sec? 1 min? 1 hr? 1 day? 1 month (30 days)? Specify any assumptions you make to solve the problem. Hint: Combine the radiation equation (left hand side is $dQ/dt$) with the differential of the specific heat equation (left hand side will be $dQ$). Then move all of the temperature quantities to the left hand side, all of the time quantities to the right hand side, and integrate both sides with definite integrals.

(b) Use a program such as Mathematica to plot the temperature as a function of time for the first month. Force the vertical scale to go from 0 to 340 K.

7-6. (Extra credit; paper only.) A cylindrical insulating bucket is filled with water at $0°C$. The air above the water has a temperature of $-12°C$. If the air remains at this temperature, how long will it take for a 1 cm layer of ice to form on the surface of the water? Hint: How much heat gets transferred through when the ice thickness is "$x$". You will have to figure out how to set things up so that you can integrate from $x = 0$ to $x = 0.01$. My answer was between 600 and 700 seconds.

**Extra problems I recommend you work (not to be turned in):**

- Water is being boiled in an open kettle that has a 0.52-cm-thick circular aluminum bottom with a radius of 12.0 cm. If the water boils away at a rate of 0.355 kg/min, what is the temperature of the lower surface of the bottom of the kettle? Assume that the top surface of the bottom of the kettle is at $100.0°C$. (Answer: $106.46°C$.)

- A typical 100 W incandescent light bulb has a filament which is at a temperature of 3000 K. Typically, of the 100 W that goes into the bulb, 97.4 W is conducted or convected away as heat, and only 2.6 W is radiated as light (and most of that is invisible infrared light—now you see why incandescent lights are so inefficient). (a) If you assume the emissivity of a tungsten filament to be about 0.4, what is the filament's surface area? (b) If the temperature were raised, one would expect that the losses due to conduction and convection would go up by about the same factor as the temperature increase, but that the radiation power would scale as $T^4$. Given those scaling factors, if you could increase the temperature of the filament by 50% to 4500 K, how much light power would now be radiated? (Assume the same 100 W total power.) Unfortunately, if the filament gets too hot, it will melt or vaporize. This is why almost all incandescent bulbs run at about the same temperature—as hot as possible without quickly destroying the tungsten filament. This is also the secret to halogen bulbs: the halogen gas in the bulb reduces the rate at which the tungsten evaporates from the filament, allowing it to operate at higher temperatures for more brightness and efficiency. (Answer: 9.009 W.)

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8-1. We have some gas in a cylinder like that in the figure. The diameter of the cylinder is 8.1 cm. The mass of the piston is [01] _____ kg. The atmospheric pressure is $9.4 \times 10^4$ Pa. (a) Find the pressure of the gas. (Both the weight of the piston and the pressure of the atmosphere on top of the piston contribute to the pressure of the gas inside the cylinder.) (b) If we heat up the gas so that the piston rises from a height of 12.3 cm to 15.6 cm (measured from the bottom of the cylinder), find the work done on the gas. Note that the pressure of the gas remains constant as it is heated up.
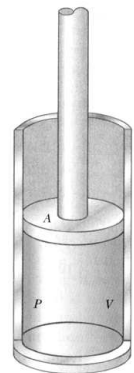
8-2. A gas expands from I to F along the three paths indicated in the figure. Calculate the work done on the gas along paths (a) IAF, (b) IF, and (c) IBF. $P_i = [02]$ _____ atm and $P_f = [03]$ _____ atm.



P(atm), I, A, $P_i$, B, $P_f$, F, 0, 1, 2, 3, 4, V(liters)
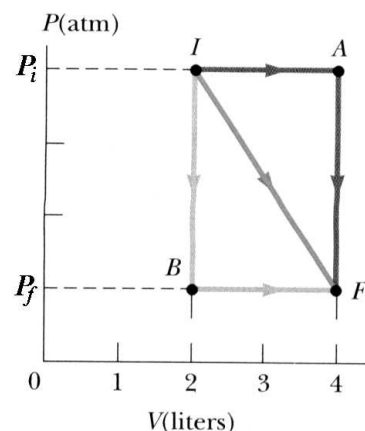
8-3. A monatomic ideal gas undergoes the thermodynamic process shown in the $PV$ diagram in the figure. Determine whether each of the values (a) $\Delta U$, (b) $Q$, (c) $W$ for the gas is positive, negative, or zero. (Note that $W$ is the work done *on* the gas.)



$2P_0$, $P_0$, $V_0$, $2V_0$
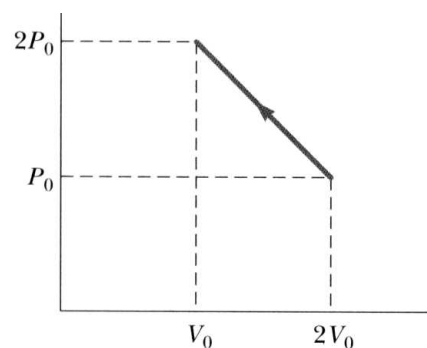
8-4. We have some gas in a container at high pressure. The volume of the container is [04] _____ cm$^3$. The pressure of the gas is $2.52 \times 10^5$ Pa. We allow the gas to expand at constant temperature until its pressure is equal to the atmospheric pressure, which at the time is $0.857 \times 10^5$ Pa. (a) Find the work done on the gas. (b) Find the change of internal energy of the gas. (c) Find the amount of heat we added to the gas to keep it at constant temperature.

8-5. (Paper only.) An ideal gas is initially at 1 atm with a volume of 0.3 m$^3$.
(a) The gas is then heated at constant volume until the pressure doubles. During this process 1200 J of heat flow into the gas. How much work does the gas do?
(b) What is the change in the internal energy of the gas as it is heated?
(c) Now the pressure of the gas is kept at 2 atm and the gas is heated while its volume increases to twice its initial volume. In the process, the internal energy of the gas increases by 1000 J. How much work does the gas do?
(d) How much heat flows into the gas during the expansion?
(e) Draw a P-V diagram of this sequence of processes. Label the initial state of the gas A, the state after the constant volume process B, and the state after the constant pressure process C.

**Extra problems I recommend you work (not to be turned in):**

- One mole of an ideal monatomic gas is at an initial temperature of 305 K. The gas undergoes an isovolumetric process, acquiring 728 J of energy by heat. It then undergoes an isobaric process, losing this same amount of energy by heat. What is the final temperature of the gas? (Answer: 328.3 K.)

- An ideal gas is contained inside a cylinder with a moving piston on the top. The piston has a mass $m$ which keeps the gas at a pressure $P_0$. The initial volume of the gas is $V_0$. For this whole problem give your answers in terms of $P_0$ and $V_0$. (a) The gas is heated until the volume has expanded to twice its initial volume. How much work is done by the gas during this process? (b) By what factor does the temperature increase during this expansion? (c) The piston is then locked in place and the gas is cooled back to its original temperature. What is the pressure of the gas after it is cooled? (d) How much work is done on the gas as it is cooled? (e) The cylinder is then placed in a bucket of water which keeps the temperature constant (at the original temperature), and the piston is released and allowed to slowly drop until the gas returns to its initial pressure $P_0$. How much work is done on the gas during this process? (f) Draw a P-V diagram of this sequence of processes. Label the initial state of the gas $A$, the state after expanding $B$, and the state after it is cooled $C$. (Answers to parts (a)–(e): $P_0V_0$, $\times 2$, $\frac{1}{2}P_0$, 0, $P_0V_0\ln 2$.)

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9-1. [01] _____ moles of a monatomic ideal gas have a volume of 1.00 m$^3$, and are initially at 354 K. (a) Heat is carefully removed from the gas as it is compressed to 0.50 m$^3$, causing the temperature to remain constant. How much work was done on the gas in the process? (b) Now the gas is expanded again to its original volume, but so quickly that no heat has time to enter the gas. This cools the gas to 223 K. How much work was done by the gas in this process?

9-2. We have a container of a hot ideal monatomic gas. The volume of the container is 25 liters. The temperature of the gas is [02] _____°C, and its pressure is $0.858 \times 10^5$ Pa. We allow the gas to cool down to room temperature, which at the time is 21°C. We do not allow the volume of the gas to change. (a) Find the final pressure of the gas. (b) Find the amount of heat that passed from the gas to its surroundings as it cooled (a positive number), by finding the change in internal energy and the work done on the gas, and using the First Law of Thermodynamics. (c) Find the amount of heat that passed from the gas to its surroundings as it cooled, by using $C_V$, the molar heat capacity for constant volume changes.

9-3. We have some air in a cylinder like that in the figure. Assume that air is an ideal diatomic gas, with $\gamma = 7/5$. The diameter of the cylinder is 5.3 cm. The mass of the piston is negligible so that the pressure inside the cylinder is maintained at atmospheric pressure which is 1.00 atm. The height of the piston is 9.7 cm, measured from the bottom of the cylinder. The temperature of the air is [03] _____°C. (a) We heat the gas so that the piston rises to a height of [04] _____ cm. The pressure of the air remains constant. (a) Find the final temperature of the air. (b) Find the amount of heat that was put into the air, by finding the change in internal energy and the work done on the gas, and using the First Law of Thermodynamics. (c) Find the amount of heat that was put into the air, by using $C_P$, the molar heat capacity for constant pressure changes.
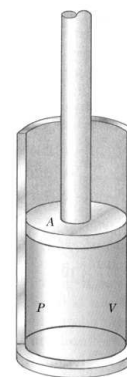
9-4. One mole of a *monatomic* ideal gas is compressed adiabatically from an initial pressure and volume of 2.00 atm and 10.0 L to a final volume of [05] _____ L.
(a) Using $W = -\int_{V_1}^{V_2} P \, dV$, find the work done on the gas. Be sure to include the sign if negative.
(b) Find the final pressure.
(c) Find the final temperature.
(d) Use the first law together with the knowledge of the initial and final temperatures to find the work done on the gas. Hint: Your answer should agree with part (a).
Now that you have learned the easy way to determine the work in an adiabatic process, you will never again have to integrate $PdV$ to get the work for an adiabatic change!

9-5. What are the number of degrees of freedom for

(a) helium at room temperature?

(b) oxygen at room temperature?

(c) water vapor at 200°C?

(d) hydrogen ($H_2$) at 6000 Kelvin? Hint: see Fig 21.7 (8th edition; might be a different figure number in other editions.).

9-6. (Paper only.) We've talked about degrees of freedom for molecules in gases, but how about for atoms in a solid? One view is that each atom in a solid should have 6 degrees of freedom: three from vibrational kinetic energy and three from vibrational potential energy. In other words, the total energy of an atom in a solid is $\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}kx^2 + \frac{1}{2}ky^2 + \frac{1}{2}kz^2$, where $k$ represents the "spring constant" of the restoring force holding each atom in place. If this view is correct, the molar heat capacity of all solids should be equal to $C = 6R/2 = 3R$. That is called the Dulong-Petit law.

(a) Let's test it out with real data. Make a list of the specific heats (units J/kg·°C) for the elements given in the table in your book. (The elements are on the left hand side of the specific heat table.) For each element, convert its specific heat $c$ into its molar heat capacity $C$ (J/mol·°C) by multiplying each specific heat by the appropriate molar mass (kg/mol). For each element, calculate the percent difference between the real value you obtained for $C$, and the value predicted by the Dulong-Petit law. Feel free to use a spreadsheet program to do all these calculations. You should find very good agreement for all but two of the elements. Wikipedia has this to say about the Dulong-Petit law: "Despite its simplicity, the Dulong-Petit law offers fairly good prediction for the specific heat capacity of solids with relatively simple crystal structure at high temperatures. It fails, however, at room temperatures for light atoms bonded strongly to each other [because there is not enough thermal energy to excite the higher frequency vibrational modes of the light elements]." Does that match what you found? Think about the atomic weights of the two elements that did not fit the law well.

(b) Explain why I just used the symbol $C$ to represent the molar heat capacity in the problem above instead of $C_V$ or $C_P$.

**Extra problems I recommend you work (not to be turned in):**

• Consider a gas composed of 3.5 moles of nitrogen molecules ($N_2$) at a temperature low enough that the vibration modes of the molecule are "frozen out". In other words, the molecules have 5 degrees of freedom: 3 translational and 2 rotational. (a) What is the molar specific heat at constant volume? (b) What is the molar specific heat at constant pressure? (c) If the gas is in a rigid container, how much will the temperature of the gas change if 75 J of heat are added to the gas? (d) If the gas is in a container kept at a constant pressure, how much will the temperature of the gas change if that same amount of heat is added to the gas? (e) In which case will the gas do more work as it is heated? (Answers to parts (a)–(d): $\frac{5}{2}R$, $\frac{7}{2}R$, $1.031°C$, $0.736°C$.)

• (a) Explain in your own words why the molar specific heat at constant pressure should always be higher than the molar specific heat at constant volume. (b) Explain why the change in internal energy ($\Delta E_{\text{int}}$) for a gas always equals $nC_V\Delta T$, even when it undergoes a process in which the volume changes.

• During the compression stroke of a certain gasoline engine, the pressure increases from 1.00 atm to 18.4 atm. Assuming that the process is adiabatic and that the gas is ideal, with $\gamma = 1.40$, (a) by what factor does the volume change and (b) by what factor does the absolute temperature change? If the compression starts with 0.0160 mol of gas at $27°C$, find the values of (c) $Q$, (d) $W$, and (e) $\Delta E_{\text{int}}$ that characterize the process. (Answers: decreases by a factor of 8.006, increases by a factor of 2.298, 0, $-129.6$ J, 129.6 J.)

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10-1. A heat engine performs [01] —————— J of work in each cycle and has an efficiency of 32.9%. For each cycle of operation, (a) how much energy is absorbed by heat and (b) how much energy is expelled by heat?

10-2. A nuclear power plant has an electrical power output of 1000 MW and operates with an efficiency of 33%. If the excess energy is carried away from the plant by a river with a flow rate of [02] —————— kg/s, what is the rise in temperature of the flowing water?

10-3. One mole of an ideal monatomic gas is taken through
the cycle shown in the figure, where
$P_1 = [03]$ ———————— atm and $P_2 = P_1/5$. The process
$A \rightarrow B$ is a reversible isothermal expansion. Calculate
(a) the energy added by heat to the gas, (b) the energy
expelled by heat from the gas, and (c) the net work
done by the gas, (d) the efficiency of the cycle. Hint:
the easiest way to get the net work is almost always by
subtracting $Q_c$ from $Q_h$, since the net work done by
the gas equals the net heat added (because $\Delta U = 0$ for
a cycle).
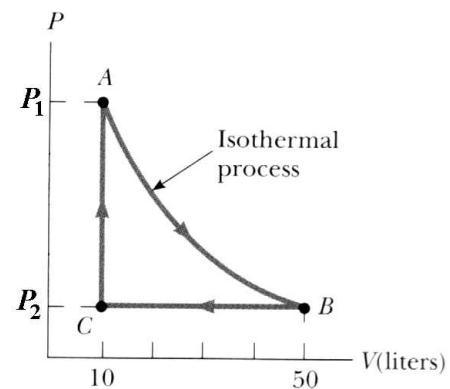


10-4. Suppose your gasoline car has a compression ratio of [04] ———————— to 1. The specs for
the car indicate that the engine produces 105 hp when being operated at 6000 rpm.
(a) Assuming that the air (or more properly, air-fuel mixture) is composed entirely of
diatomic molecules with 5 degrees of freedom at these temperatures, and assuming that
the actual cycle can be perfectly approximated as the ideal Otto cycle, find how much $Q_{in}$
per second is required to run the engine at that rpm. (b) If you can travel at 100 mph at
that rpm (watch out for cops!), how many miles per gallon will your car get? Gasoline
produces about 47000 kJ for each kg burned, and the density of gasoline is 0.75 $g/cm^3$.

10-5. (Paper only.) Show that the efficiency for an engine working in the Diesel cycle
represented ideally below is
$$e = 1 - \frac{1}{\gamma}\left(\frac{T_D - T_A}{T_C - T_B}\right).$$

Diesel cycle: Adiabatic compression AB heats the gas
until ignition at B when fuel is introduced (no spark
plug needed). A constant pressure expansion BC takes
place as combustion adds heat. Adiabatic expansion CD
accomplishes additional work before the exhaust is
exchanged for new air during what can be thought of as
a constant volume cooling DA.
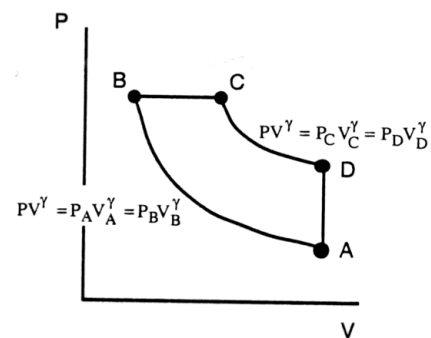
10-6. (Paper only.) Many people believe that a higher octane fuel means "more power". That's not quite correct; what higher octane means, is that the fuel does not self-ignite as easily as the fuel heats up during compression. Higher power engines often use higher compression ratios, because (as can be seen by doing the first optional problem below) a higher compression ratio will give you a higher efficiency. Therefore high power gas engines often require higher octane fuel to prevent the fuel from igniting before the spark plugs fire—hence the confusion. However, if the normal compression ratio is low enough that low octane fuel will not self-ignite, a higher octane fuel will provide absolutely no benefit. Some websites say that with 91 octane fuel, compression ratios up to about 11.5 can safely be used. Use this information to estimate the temperature at which an air-fuel mixture using 91 octane gasoline will spontaneously ignite. Assume an ambient air temperature of 25°C and a specific heat ratio $\gamma$ of 7/5.

**Extra problems I recommend you work (not to be turned in):**

- (a) In the Otto cycle, the ratio of maximum volume to minimum volume is called the "compression ratio" $r$. Use a program such as Mathematica to make a plot of the Otto cycle's efficiency vs. the compression ratio.
  (b) In the Diesel cycle, the ratio of maximum volume to minimum volume is called the compression ratio $r$, and the ratio of the intermediate volume to the minimum volume is called the "cut-off ratio" $r_c$. The equation you derived for efficiency of the Diesel cycle can be written as:
  $$e = 1 - \frac{1}{r^{\gamma-1}} \left( \frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)} \right)$$
  Use a program such as Mathematica to make plots of the Diesel cycle's efficiency vs. the compression ratio, for cut-off ratios of 1, 2, 3, and 4. Use $\gamma = 7/5$. Hint: for the first graph, you will actually have to use $r = 1.000001$, or something like that, because if you use $r =$ exactly 1, Mathematica will throw a divide by zero error.

- Prove that the two Diesel cycle efficiency equations given above are equivalent.

- An engine absorbs 1678 J from a hot reservoir and expels 958 J to a cold reservoir in each cycle. (a) What is the engine's efficiency? (b) How much work is done in each cycle? (c) What is the power output of the engine if each cycle lasts for 0.326 s? (Answers: 42.91%, 720 J, 2209 W.)

- Work out any of the cycle problems (given a cycle, find the efficiency) that are on the old exams posted to the class website.

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11-1. A refrigerator has a coefficient of performance equal to 5.21. Assuming that the refrigerator absorbs [01] _____ J of energy from a cold reservoir in each cycle, find (a) the work required in each cycle and (b) the energy expelled to the hot reservoir.

11-2. A refrigerator keeps its freezer compartment at $-10°C$. It is located in a room where the temperature is $20°C$. The coefficient of performance (heat pump in cooling mode) is [02] _____. How much work is required to freeze one 26-g ice cube? Assume that we put 26 g of water into the freezer. The initial temperature of the water is $20°C$. The final temperature of the ice cube is $-10°C$. The refrigerator removes the heat from the freezer compartment, maintaining its temperature at about $-10°C$.

11-3. Consider a heat pump which is used to cool down a home during the summer. Its coefficient of performance in cooling mode is [03] _____. On a particular hot day, the temperature outside the home is 90°F, and the temperature inside the home is maintained at 70°F. If the heat pump consumes 500 W of electrical power, at what rate does it remove heat from the home?

11-4. Suppose you want to keep the inside of your freezer at a temperature of $-5°C$ when your house is at [04] _____°C. (a) What is the maximum possible coefficient of performance for a refrigerator operating between those two temperatures? (b) If 350 J of heat leak from the environment into your freezer each second, what is the minimum theoretical power that your freezer will consume to keep the temperature inside the freezer at $-5°C$. (c) How much per year (365 days) would it cost you to operate such a freezer if you never open it up? Use 8 cents/(kilowatt·hour) as the price for electricity.

11-5. (Paper only.) A sample of a *monatomic* gas is taken through the Carnot cycle ABCDA. For your convenience, the cycle is drawn with the mathematical relationships of each part shown. Complete the table for the cycle.

|   | $P$ | $V$ | $T$ |
|---|---|---|---|
| A | 1400 kPa | 10.0 L | 720 K |
| B |  |  |  |
| C |  | 24.0 L |  |
| D |  | 15.0 L |  |
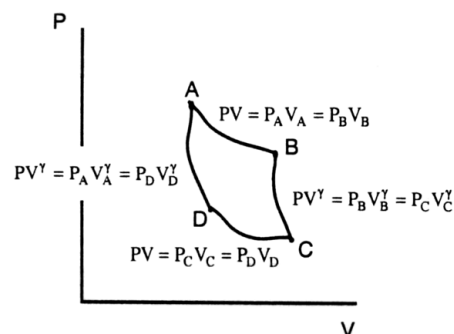


Avoid rounding intermediate steps so that errors do not accumulate. You may find it beneficial to solve for the unknowns in the order requested below.

First determine the number of moles from the data in row A.

(a) Find $P_D$.

(b) Find the value of $T_D$ and $T_C$, which are equal.

(c) Find $P_C$.

(d) Find $T_B$.

(e) Find $V_B$.

(f) You should then be able to find that $P_B = 875$ kPa (provided here as a check).

11-6. (Paper only.) For the parameters in previous problem, complete the table below.

|   | $Q$ | $W$ | $\Delta E_{int}$ |
|---|---|---|---|
| AB |  |  |  |
| BC |  |  |  |
| CD |  |  |  |
| DA |  |  |  |

Hint: Curves AB and CD are constant temperature, meaning that the internal energy is constant on the curves. Curves BC and DA are adiabatic, meaning that no heat flows into or out of the gas.

11-7. (Paper only.) (a) From the temperatures found in the problem before last, compute the theoretical maximum efficiency for this cycle. (b) From the heats and work found in the last problem, calculate the actual efficiency of the cycle using the definition of efficiency. It should match your answer to part (a).

**Extra problems I recommend you work (not to be turned in):**

- A reversible engine draws heat from a reservoir at $399°C$ and exhausts heat to a reservoir at $19°C$. (a) Find the efficiency of the engine. (b) Find the heat required to do 100 J of work with this engine. (Answers: $56.53\%$ , 176.9 J.)

- Would you save money if you were to somehow pipe the heat from your refrigerator's heat-exchanging coils (in the back of the refrigerator) to outside the house?

- Let's derive the efficiency for a general Carnot cycle. Take a look at the P-V diagram of the Carnot cycle as given in the figure. Efficiency is defined to be
  $e = W/Q_h = (Q_h - Q_c)/Q_h.$
  Unless otherwise noted, give all answers in terms of $n$, $T_h$, $T_c$, $V_A$, $V_B$, $V_C$, $V_D$, $\gamma$, and fundamental constants.
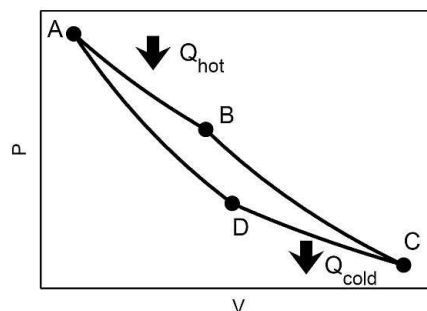


  (a) Find the heat that enters the gas during the adiabatic processes from B-C and from D-A. (In other words, what are $Q_{BC}$ and $Q_{DA}$?)
  (b) Find the change in the internal energy of the gas during the isothermal processes. (In other words what are $\Delta E_{AB}$ and $\Delta E_{CD}$?)
  (c) How much work is done on the gas during each isothermal process? (In other words, what are $W_{AB}$ and $W_{CD}$?)
  (d) Use your results above to find $Q_h$ and $Q_c$.
  (e) Use the adiabatic transitions to find a relationship between $(V_B/V_A)$ and $(V_C/V_D)$.
  (f) Use what you found above to write the Carnot efficiency in terms of just $T_h$ and $T_c$.

12-1. We have 2.451 moles of air in some container at 25.2°C. Assume that air is an ideal diatomic gas. We put [01] ————— J of heat into the air. (a) Find the change of entropy of the air if we hold the volume constant. (b) Find the change of entropy of the air if we hold the pressure constant.

12-2. (a) A container holds 1 mol of an ideal *monatomic* gas. A piston allows the gas to expand gradually at constant temperature until the volume is [02] ————— times larger. What is the change in entropy for the gas?
(b) What is the change in entropy for the gas if the same increase in volume is accomplished by a reversible adiabatic expansion followed by heating to the original temperature?
(c) What is the change in entropy for the gas if the same increase in volume is accomplished by suddenly removing a partition, which allows the gas to expand freely into vacuum?

12-3. We drop a [03] —————-g ice cube (0°C) into 1000 g of water (20°C). Find the total change of entropy of the ice and water when a common temperature has been reached. Caution: calculate the common temperature to the nearest 0.01°C.

12-4. Heat is added to 4 moles of a diatomic ideal gas at 300K, increasing its temperature to 400K in a constant pressure process. The heat is coming in from a reservoir kept at a constant temperature of [04] ————— K. What was the change in entropy of the universe during this process? (Hint: find the change in entropy for the gas and for the reservoir separately, then add them together. You can assume that the heat lost by the reservoir is equal to the heat added to the gas.)

12-5. (Paper only.) The goal of this problem is to figure out an equation for the change in entropy of an ideal gas for an arbitrary state change from state A to state C. Since entropy is a state variable, the entropy change of an *arbitrary* process from A to C will be the same as an entropy change of a *specific* process going from A to C. So, let's consider a specific process made up of two sections: a constant volume change from A to B (B having the same volume as A, and the same pressure as C) followed by a constant pressure change from B to C. The gas has $n$ moles of molecules and a molar heat capacity at constant volume of $C_V$.

(a) Draw a P-V diagram of the situation just described: pick two arbitrary points A and C on the diagram, locate the appropriate point B, and draw arrows indicating the two parts of the overall state change.

(b) How much will the entropy change if the gas undergoes a constant volume change during which the temperature changes from $T_A$ to $T_B$?

(c) How much will the entropy of the gas change if it undergoes a constant pressure change during which the temperature changes from $T_B$ to $T_C$?

(d) Use the ideal gas law to find a relation between the ratio of the temperatures before and after the isobaric process $(T_B/T_C)$ and the ratio of the volumes before and after the process $(V_B/V_C)$.

(e) Use what you have found in parts (b) through (d) to derive the general formula for the entropy change for any process (even irreversible ones) in an ideal gas:

$$\Delta S = nC_V \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i}$$

**Extra problems I recommend you work (not to be turned in):**

- One mole of a diatomic ideal gas (5 degrees of freedom), initially having pressure $P$ and volume $V$, expands so that the pressure increases by a factor of 1.8 and the volume increases by a factor of 2.2. Determine the entropy change of the gas in the process. (Answer: 35.16 J/K.)

- Prove that $\Delta S$ of the universe will always increase for calorimetry-type situations if the two objects start off at different temperatures. Hint: Add together the change in entropy for each object. Also, you may find what Wikipedia calls the "First mean value theorem for integration" to be helpful.

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13-1. Assume that our classroom has a volume of [01] _____ m$^3$ which is filled with air at 1.00 atm and 25°C.

(a) Calculate the probability that all of the air molecules will be found in the forward half of the room. Represent this remote possibility as 1 part in $10^x$, where $x$ is some large number. Give the value of $x$. NOTE: $2^N = 10^{N \log 2}$.

(b) How much *more* entropy is present when the air is distributed throughout the room rather than confined to the front half only?

13-2. (Paper only.) This problem involves flipping a fair coin and counting how many times you get heads, H, and how many times you get tails, T. You may want to refer to the similar example problem in the textbook where they describe choosing red and green marbles from a bag. The "microstates" are the specific ordered lists of heads and tails that you get ("HHTTHTHH" would be one possible microstate for a collection of 8 flips); the "macrostates" are the overall number of heads (or tails) that you get. (The above microstate would belong to the "5 heads", or "5H" macrostate.) Hopefully it's obvious that each macrostate will likely be associated with many different microstates. The probability of a given macrostate occurring is proportional to how many microstates are associated with it. Specifically, the probability of a particular macrostate is the number of microstates associated with it, divided by the total number of microstates. That may sounds complicated, but should make much more intuitive sense as you start doing the problem below.

(a) Suppose you flip the coin once. List the 2 possible microstates. For each of the 2 possible macrostates (0H and 1H), list how many microstates are associated with it. (Don't worry, this is not supposed to be complicated yet.)

(b) Suppose you flip the coin twice. List the 4 possible microstates. For each of the 3 possible macrostates (0H, 1H, and 2H), list how many microstates are associated with it.

(c) Repeat for three flips. There are 8 possible microstates and 4 possible macrostates (0H, 1H, 2H, and 3H).

(d) Repeat for four flips. OK, that should be enough. Think about this question: what's the probability of getting exactly 3 heads if you flip a coin four times? The answer is 4/16. Hopefully you can see why, from your list.

(e) Fill in the first four rows of this chart. Leave the table entries blank if not applicable.

|          | 0H | 1H | 2H | 3H | 4H | 5H |
|----------|----|----|----|----|----|----|
| 1 flip   |    |    |    |    |    |    |
| 2 flips  |    |    |    |    |    |    |
| 3 flips  |    |    |    |    |    |    |
| 4 flips  |    |    |    |    |    |    |
| 5 flips  |    |    |    |    |    |    |

Do you see the pattern? Fill in the fifth row based on the pattern.

Hopefully you have recognized Pascal's triangle. Each entry in the next row can be obtained by adding together two entries from the previous row. If you don't recall

learning about Pascal's triangle, Google it. Among other things, it gives you the coefficients to the expansion of $(x + y)^n$. Who would have thought that FOIL was related to flipping coins?

(f) Two important facts about Pascal's triangle that you might not have run across before are: (1) the numbers in the $n$th row add up to $2^n$. (For our situation, that's the total number of microstates. Hopefully it's clear to you why they must add up to $2^n$.) (2) The $k$th number in the $n$th row is given by the "choose" formula, the left hand side of this equation being read as "n choose k":

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

($k$ is the column label, which starts at 0 and goes to $n$.) This is essentially what mathematicians call the "Binomial Theorem". If you haven't seen that before, you should verify the formula for a few entries in your table before proceeding.

Use those facts to easily answer this question: If you toss the coin 100 times, what is the probability you will get *exactly* 50 heads and 50 tails? Give your answer as an exact expression as well as a numerical percentage.

13-3. (Paper only.) If you toss a fair coin, you should expect to get heads half the time, right? Well, hopefully the previous problem has convinced you that with large numbers of flips, getting heads *exactly* half the time is actually a pretty rare event. But you should expect to get heads *close* to half the time. How close is close? Understanding that is the point of this problem. You will analyze that by looking at the fluctuations around the expected value of 50%. You are welcome to work in groups for this, just make sure you are a full participant and that you understand everything that's going on.

(a) Toss a coin 50 times. After each toss write down how many total heads you have gotten, along with the fraction of total tosses which have resulted in heads. I recommend you keep track of this in a spreadsheet program as you go along. Here's some sample data I made up, just to show you what I mean:

| Number of toss, $N$ | Cumulative number of heads | Fraction of heads |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 1 | 0.5 |
| 3 | 2 | 0.667 |
| 4 | 3 | 0.75 |
| 5 | 3 | 0.6 |
| 6 | 4 | 0.667 |

(b) Now calculate the difference between your "Fraction of heads" and the expected value of 0.5. (Again, I'd use a spreadsheet program for all of this.) That's the statistical fluctuation in your results. Using statistical techniques similar to the previous problem, it can be shown that most of the time the absolute value of the difference from the expected value will be less than $1/\sqrt{N}$. To show that that is indeed the case, plot your difference as a function of $N$, the number of tosses. On the same graph plot these two functions: $f_1(N) = 1/\sqrt{N}$ and $f_2(N) = -1/\sqrt{N}$. Your graph should nearly always stay in between the $f_1$ and $f_2$ curves.

This type of thing becomes important time and time again in experimental physics. One situation that immediately springs to mind is in detecting light. In my lab we have detectors which can measure batches of individual photons. However, there are always statistical fluctuations present in the numbers of photons we detect, that are just like the fluctuations we saw above. Therefore, if we expect to see 1,000,000 photons each second, what we will actually see are photon numbers ranging from 1,000,000 + 1,000 down to 1,000,000 − 1,000 (because one thousand is the square root of one million). For very low light levels, this so-called "shot noise" becomes the dominant source of noise in most optical experiments.

13-4. (Paper only.) The graph represents 100 measurements that were performed between 0 and 1 s, on a voltage source putting out a voltage around 3 V. Each data point represents the average of the voltages measured during the previous 0.01 s. The standard deviation, a measurement of the voltage fluctuations of the graph, is 0.112 V. What would be the standard deviation if the voltage sensor averaged data for 1 s? 10 s? 0.001 s?
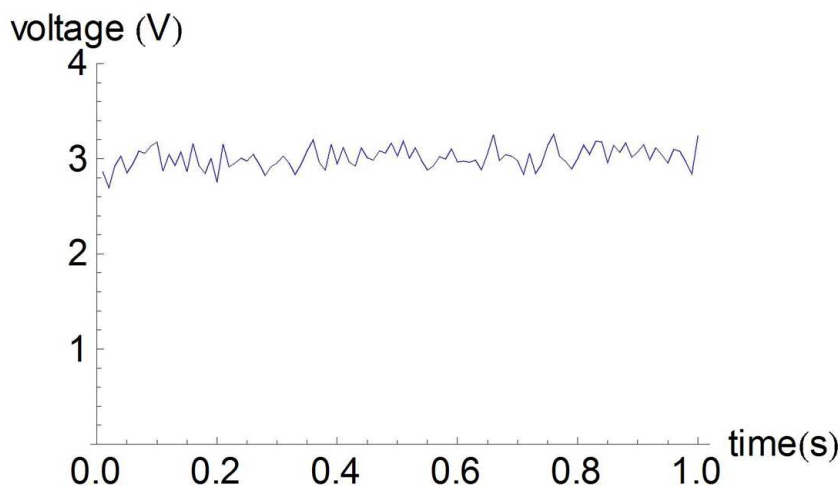
13-5. (Paper only.) Imagine that you are doing exit polls to determine the winner of an election between two candidates. It is a very close election, with each candidate receiving very close to half the votes. You are very careful to poll a balanced cross-section of voters. You may assume that the fluctuations in the poll results will be approximately $1/\sqrt{N}$. (a) If you poll 100 people, how close can the election be (in percentage points) if you want to be reasonably certain that you predict the right winner based on your poll? (b) What if you poll 10,000 people?

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14-1. (a) An AM radio station transmits at [01] _____ kHz. What is the wavelength of these radio waves? Radio waves travel at the speed of light, $3.00 \times 10^8$ m/s. (b) Repeat for an FM radio station which transmits at [02] _____ MHz.

14-2. At position $x = 0$, a water wave varies in time as shown in the figure. (The curve is at the 10-cm mark at both edges of the figure.) If the wave moves in the positive $x$ direction with a speed of [03] _____ cm/s, write the equation for the wave in the form,

$$y(x, t) = A\sin(kx - \omega t - \phi).$$

Give the values of (a) $A$, (b) $\omega$, (c) $k$, and (d) $\phi$. (Give the value of $\phi$ between 0 and $2\pi$ rad.) HINT: When taking an inverse sine to find $\phi$, you must be careful to use the right quadrant. Your calculator by default will use the 1st and 4th quadrants. Check your final answer to make sure that it actually fits the curve everywhere. To change quadrants, use $\sin^{-1} x \to \pi - \sin^{-1} x$.
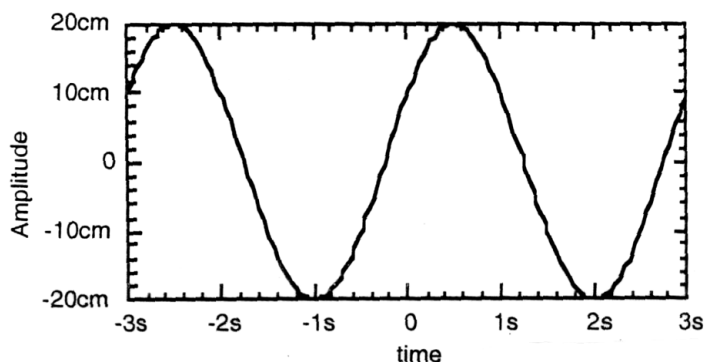
14-3. Suppose you are watching sinusoidal waves travel across a swimming pool. When you look at the water right in front of you, you see it go up and down ten times in [04] ————— s. At the peaks of the wave, the water is [05] ————— cm below the edge of the pool. At the lowest points of the wave the water is 6.0 cm below the edge of the pool. At one particular moment in time you notice that although the water right in front of you is at its maximum height, at a distance [06] ————— m away the water is at its minimum height. (This is the closest minimum to you.) (a) What is the frequency $f$ for this wave? (b) What is $\omega$ for this wave (rad/s)? (c) What is $\lambda$ for this wave? (d) What is $k$ for this wave (rad/m)? (e) What is the amplitude $A$ of this wave? (f) What is the speed of water waves in this pool?

14-4. (Paper only.) A particular transverse traveling wave has the form,
$y(x,t) = A\sin(kx - \omega t - \phi)$, where $A = 1$ cm, $k = 0.15$ cm$^{-1}$, $\omega = 7$ s$^{-1}$, and $\phi = 1$ rad.
(a) What is the amplitude of the wave?
(b) What is the wavelength?
(c) What is the period?
(d) What is the direction of the velocity?
(e) What is the magnitude of the velocity?
(f) Use a computer program such as Mathematica to plot the shape of the wave, i.e., $y(x)$, at time $t = 0$, and also at a time one fifth of a period later, on the same graph. Label the two plots. The wave at $t = 0.2$ period should be offset in the direction corresponding to your answer in (d).
(g) Verify that the peaks of the wave at $t = 0.2$ period have shifted by the amount predicted by your answer to (e). (One method would be to combine Mathematica's FindRoot command with its derivative command, in order to find out where a specific peak is.)

14-5. (Paper only.) Consider a transverse traveling wave of the form:

$$y(x,t) = \frac{1}{(x - 10t)^4 + 1}$$

(You may assume that the numbers have the appropriate units associated with them to make $x$, $y$, and $t$ be in standard SI units.)

(a) Is the wave moving in the $+x$ or $-x$ direction?

(b) Write an equation for a wave which is identical to this wave, but which is moving in the opposite direction.

(c) What is the wave's velocity?

(d) What is the transverse velocity of a section of the medium located at $x = 0$, at $t = 0.05$ s?

**Extra problems I recommend you work (not to be turned in):**

• As we will study in a future unit, light is a wave. Lasers can generate waves which are almost perfectly sinusoidal. The wavelength of light from a certain laser pointer is 620 nm. The speed of light is $2.9979 \times 10^8$ m/s. Find the (a) wavenumber, (b) frequency, (c) period, and (d) angular frequency of the light from this laser. (Answers: $1.013 \times 10^7$ rad/m, $4.835 \times 10^{14}$ oscillations/sec, $2.068 \times 10^{-15}$ s, $3.038 \times 10^{15}$ rad/sec.)

———————————————————————————————————

15-1. A phone cord is 4.89 m long. The cord has a mass of 0.212 kg. A transverse wave pulse is produced by plucking one end of the taut cord. That pulse makes four round trips (down and back) along the cord in [01] —————— s. What is the tension in the cord?

15-2. Imagine a clothesline stretched across your yard. It has a mass of 0.113 kg and a length of 6 m. When you flick the line, the pulse you generate travels down the line at a speed of [02] —————— m/s. When the pulse gets to the end, it is completely absorbed without reflection by the flexible pole it is tied to. If you stand near the other end of the line and wiggle it sinusoidally for one minute with an amplitude of 10 cm at a frequency of 3 Hz, how much energy will the flexible pole absorb?

15-3. (Paper only.) Two triangular shaped pulses are traveling
    down a string, as shown in the figure. The figure
    represents the state of the string at time $t = 0$. The pulse
    on the left is traveling to the right, and the pulse on the
    right is traveling to the left, as indicated by the arrows.
    The speed of waves on the string is 1 m/s. Draw the shape
    of the string at the following times: $t = 2$ s, $t = 2.5$ s,
    $t = 3.5$ s, and $t = 5$ s.
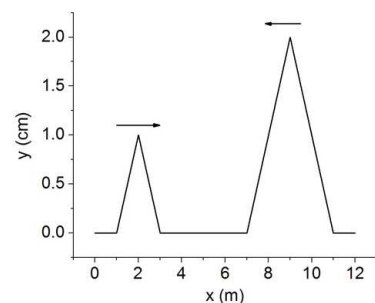


15-4. (Paper only.) Imagine your slinky stretched to a length $L$ and fixed at both ends.
    (a) Write the slinky's tension $T$ and linear mass density $\mu$ in terms of the mass $m$, spring
    constant $k$, and length $L$. Assume that the stretched length of the slinky is long enough
    compared to the length when it is not stretched that the unstretched length is negligible.
    What is the wave speed for transverse waves on a slinky in terms of $m$, $k$, and $L$?
    (b) Have someone hold one end of your slinky (or attach it to something like a doorknob).
    Take the other end and stretch the slinky until it is about five feet long. Now strike one
    end of the slinky to make a transverse pulse and watch as the pulse travels to the other
    end and then reflects back. Time how long it takes for the pulse to go out and back
    10 times, and use this to calculate the wave speed for transverse waves on the slinky.
    (c) Now predict what the wave speed would be if the slinky were stretched to about
    10 feet.
    (d) Stretch the slinky until it is about 10 feet long and measure the wave speed the same
    way you did before. Compare your answer to your prediction in (c).

15-5. (Paper only.) (a) If a transverse pulse travels down your slinky and reflects off of the end
    which is being held fixed by a friend, will the reflected pulse look the same as the
    incoming pulse, or will it be inverted?
    (b) Test our your prediction by having someone hold one end of your slinky (or attach it
    to something like a door knob) while you take the other end and pull it back until the
    slinky is stretched about 10 feet (don't stretch it too far or it won't slink back together
    again and the slinky will be ruined). Quickly strike the top of the slinky with your hand
    to make a transverse pulse. Watch carefully as the pulse reflects off of the fixed end. Did
    it match your prediction?

(c) Now hold one end of your slinky up high and let the other end dangle downward (don't let it touch the floor). If you whack the end of the slinky to make a transverse pulse, what do you think will happen to the pulse when it reaches the bottom? Will it reflect? Will the reflection be inverted? I want an honest educated guess; you won't lose points if your prediction is incorrect.

(d) Try it and see what happens. Did the dangling end of the slinky act as a free end, fixed end, or something else?

**Extra problems I recommend you work (not to be turned in):**

- (a) If you hold one end of a rope up high and let the other end dangle downward without touching the floor, how will the wave speed change as a function of the distance from the bottom of the rope? Hint: Pick a point on the rope a distance $x$ up from the bottom of the rope, and draw a free-body diagram for that point. There's some weight (but not all the weight) pulling down and some tension pulling up. That should give you tension as a function of distance. You already know how the wave speed depends on tension. (b) Use your answer to predict the time it would take for a transverse pulse to travel from the bottom of the rope to the top. Hint: Doing this requires some calculus. You should have found the speed $dx/dt$ as a function of $x$. The best way to solve this equation is to bring all of the $x$ quantities to the left hand side, all of the $t$ quantities to the right hand side, and integrate both sides of the equation.

- You are abducted by aliens and placed in a holding cell on an unknown planet. Due to your diligent study of the Starfleet Planetary Guide, you know that if you could determine $g$, the gravitational acceleration on the planet, you would be able to figure out where you are. So you pull a thread from your uniform which is 1.55 m long and which weighs 0.500 grams. You tie the end to your shoe, which weighs 0.21 kg. You then hold the top of the string with the shoe hanging at the bottom, and you pluck the string near the top. The pulse takes 0.112 seconds to travel down to the shoe. (a) What is the value of $g$ predicted by the wave speed? (b) To double-check your results, you now start the shoe oscillating back and forth. You time 5 periods in 68 s. What is the value of $g$ predicted by the motion of the pendulum? Ignore the length of the shoe. (Answers: 0.29 m/s$^2$, 0.33 m/s$^2$.)

- A light string of mass 15.2 g and length $L = 3.23$ m has its ends tied to two walls that are separated by the distance $D = 2.41$ m. Two objects, each of mass $M = 2.03$ kg, are suspended from the string as in the figure. If a wave pulse is sent from point A, how long does it take to travel to point B? (Answer: 33 ms.)
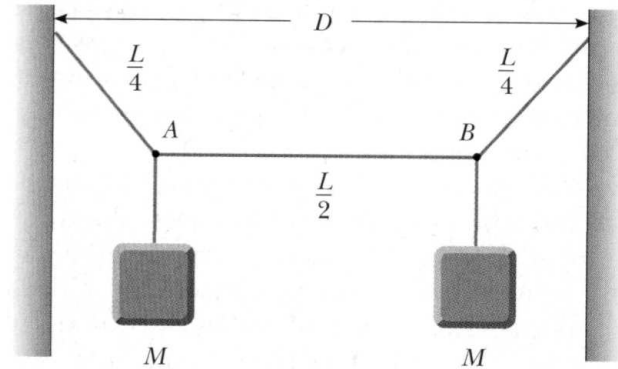


- (a) Consider the function $y = Ae^{(x-vt)^2/a^2}$ (where $A$, and $a$ are constants, and $v$ is the speed of waves on the string). Plug this into the linear wave equation and show that it is a solution. (b) Show that $y = A\sin(bxt)$ is not a solution to the wave equation (where $A$ and $b$ are constants). (c) By plugging things into the wave equation, show that if $y_A(x,t)$ and $y_B(x,t)$ are solutions to the wave equation, $y_A + 2.13y_B$ is also a solution.

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16-1. Parts (a) and (b): Write the complex number $\tilde{z} = a + bi$ (where a = [01] _____and b = [02] _____) as a real number times the exponential of an imaginary number. In other words, if I write $\tilde{z}$ as $Ae^{i\phi}$, what are the real numbers (a) $A$ and (b) $\phi$?
Parts (c) and (d): Write the complex number $\tilde{z} = Ae^{i\phi}$, (where $A$ = [03] _____and $\phi$ = [04] _____ rad) as a real number plus an imaginary number. In other words, if I write $\tilde{z}$ as $a + ib$, what are the real numbers (c) $a$ and (d) $b$?

16-2. (Paper only.) Note: many students have calculators that can do the following types of complex number problems automatically. However, I don't want you to use your calculator's complex number functions for these problems—instead, do them by hand (addition and subtraction can be done in rectangular form; multiplication and division should be done by converting to polar form).
(a) If $z_1 = 2 + 3i$ and $z_2 = 3 - 5i$, what is $z_1 + z_2$ (in rectangular form)? What is $z_1 \times z_2$ (in polar form)?
(b) If $z_1 = 1 - i$ and $z_2 = 3 + 4i$, what is $z_1 - z_2$ (in rectangular form)? What is $z_1 \div z_2$ (in polar form)?

16-3. (Paper only; no partial credit.) Pick two random cosine functions of the form $A\cos(\omega t + \phi)$. They should have different amplitudes and different phases, but the same frequency. (a) Use a computer program such as Mathematica to plot the sum of the two random functions. You should find that their sum is a cosine function with the same frequency, but with a still-different amplitude and phase. (b) Add the functions together using the complex exponential technique discussed in class, and obtain the amplitude and phase of the sum. Plot the cosine function with that amplitude and phase, and show that it really is the same as the combined function you plotted in part (a). Your grade will be based entirely on whether your plotted function in (b) is an exact match for your plotted function in (a). Remember to turn in your Mathematica code that you used to generate the plots.

16-4. (Paper only.) Use Euler's formula to prove that $\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ and that $\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$. Hint: First note that $e^{i(a+b)} = e^{ia} \cdot e^{ib}$. Then apply Euler's formula to each of the exponentials. Finally, note that the real part of the stuff on the left side of the equation must be equal to the real stuff on the right side, and the imaginary stuff on the left must equal the imaginary stuff on the right. This lets you separate your equation into two equations which will lead to the two equations you are trying to prove.

16-5. (Paper only.) (a) The equation of motion for a simple harmonic oscillator is:

$$\frac{d^2 x}{dt^2} = -\frac{k}{m}x.$$

That simply comes from Newton's $2nd$ Law, $\Sigma F = ma$, where I've plugged in the spring force, reversed the left and right hand sides, and divided by $m$. To solve equations like that, physicists often guess what the solutions are, then plug their guess into the equation to see what results. In this case, you should know that this equation produces simple harmonic motion, so guess a solution of the form $x(t) = A\cos(\omega t)$. Plug that $x(t)$ into the equation, take the derivatives, and show your guess solves the equation if $\omega$ has a particular dependence on $k$ and $m$. You should get a very familiar result. (Note: this guess doesn't describe *all* solutions, since, for example, there could be a phase shift in the cosine function.)

(b) As given in *Physics phor Phynatics*, the equation of motion for a *damped* harmonic oscillator is:

$$\frac{d^2x}{dt^2} = -\frac{\gamma}{m}\frac{dx}{dt} - \frac{k}{m}x.$$

The difference between this equation and the last is the damping term, a non-conservative force that is proportional to the velocity and measured by the "damping constant" $\gamma$. Guess a solution of the form $x(t) = Ae^{-t/\tau}\cos(\omega t)$. That is a decaying cosine function; $\tau$ is the characteristic time it takes for the decay to occur. Plug your guess into the equation, take derivatives, and show that although it's a pain, you can figure out what $\tau$ and $\omega$ must be (in terms of $k$, $m$, and $\gamma$). Hint: You will get an equation with various sine and cosine terms in it. The sine terms on the left side of the equation must be equal to the sine terms on the right side; same for the cosine terms. This lets you separate your equation into two equations which will let you solve for $\tau$ and $\omega$. Another hint: If you look closely, you should be able to see that your answer for $\omega$ is the same as your answer to part (a), times a factor of the form $\sqrt{1 - \text{stuff}}$.

(c) Now guess a solution of the form $x(t) = Ae^{-t/\tau}e^{i\omega t}$, realizing that the real solution will only be the real part of that. This is the same solution you guessed in part (b), only written in complex form. Plug your guess into the equation, take derivatives, and show that you get the exact same results for $\tau$ and $\omega$ as in part (b)—but that the algebra is *far* easier! Hint: If you write $x(t)$ as $Ae^{t(-1/\tau+i\omega)}$, the time derivatives are almost trivial. You will get an equation with various real and imaginary terms. The real terms on the left side of the equation must be equal to the real terms on the right side; same for the imaginary terms. This leads to the exact same two equations as in part (b).

**Extra problems I recommend you work (not to be turned in):**

- Use the techniques/principles of complex numbers to write the following as simple phase-shifted cosine waves (i.e. find the amplitude and phase of the resultant cosine wave). (a) $5\cos(4t) + 5\sin(4t)$. (b) $3\cos(5t) + 10\sin(5t + 0.4)$. (Answers: 7.07, $-45°$; 6.96, $-7.61°$.)

- Suppose you pick a lot of different values for $\phi$ and plot the real and imaginary components of $5e^{i\phi}$ for each one. (The real component is the $x$-coordinate and the imaginary component is the $y$-coordinate, of course.) What geometric shape will be defined by your plot?

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17-1. A beam of light crossing a boundary between two media at normal incidence (i.e. perpendicular to the boundary) shares many features with waves on strings reflecting and transmitting from boundaries. Among those features is the dependence of reflection and transmission coefficients on wave speed. Suppose you have a light ray going from air into glass. Light travels at $2.9979 \times 10^8$ m/s in the air and at [01] _____ m/s in the glass. What percent of the incident light power will reflect off of the surface of the glass?

17-2. Imagine that you have a copper wire with a round cross section, 0.411 mm in diameter. You splice the end of that wire to another wire with the same cross section, but which is made of an unknown metal with density of [02] _____ kg/m$^3$. (Copper has a density of 8920 kg/m$^3$.) You then pull on the joined wires until they are under a tension of $T = $ [03] _____ N. (a) What is the ratio of the wave velocity on the copper wire to that on the unknown wire (i.e., what is $v_{\text{copper}}/v_{\text{unknown}}$)? (b) What is the ratio of the wave numbers for the two wires ($k_{\text{copper}}/k_{\text{unknown}}$) for a sine wave with an angular frequency 500 rad/s? (c) If I send a sine wave down the copper wire, what fraction of the power in the incident sine wave is transmitted to the unknown wire?

17-3. If you splice a copper wire with a round cross section, [04] _____ mm in diameter, to an iron wire with a different diameter, what should the diameter of the iron wire be if you don't want waves to reflect at the junction when the wire is pulled tight? Copper has a density of 8920 kg/m$^3$, and iron has a density of 7860 kg/m$^3$.

17-4. (Paper only.) Imagine that I have a string which I can use to transmit waves. I make various measurements of the speed of sine waves and determine that they travel at a velocity given by $v_{\text{sine}} = 0.637 \, \omega^2$, where the number 0.637 has the appropriate units to make $v$ and $\omega$ be SI quantities. (a) What are the units of the 0.637 number? (b) Find the dispersion relation $\omega(k)$ for the string. If I have a pulse with an average $k$ of 1.42 radians per meter, what will be (c) the pulse's phase velocity and (d) its group velocity? (e) At what speed would I expect the center of that pulse to travel down the string?

17-5. (Paper only.) Consider the following two functions, which are just sums of sinusoidal waves having different amplitudes.

$$y_1(x, t) = \sum_{n=40}^{60} e^{-n^2/1000} \cos\left(2\pi n(x - t)\right)$$

$$y_2(x, t) = \sum_{n=40}^{60} e^{-n^2/1000} \cos\left(2\pi n(x - n^{0.25}t)\right)$$

Notice that the components making up the first function all have the same velocity ($v = 1$), whereas the components making up the second function have velocities which depend on their frequencies.

(a) Estimate the phase velocity and group velocity of function $y_1$. Using a computer program such as Mathematica, plot the function at times $t = 0$, 0.1, and 0.5. Adjust the x-axis range so that it stays centered on the peak, and has a width of exactly 1. Force the y-axis scale to go from $-2$ to 2. (In Mathematica that's done via the PlotRange option for the Plot command.) With what velocity is the peak moving? (That will the peak position on the $t = 0.5$ graph, divided by 0.5.) Does it match your prediction? Does the peak maintain its shape? Does it spread out?

(b) Repeat, for function $y_2$. Evaluate the group and phase velocities at the average $k$, which corresponds to $n = 50$. In *Physics Phor Phynatics* the average $k$ is referred to as $\bar{k}$. How does the measured speed of the peak compared to your predicted group and phase velocities? Does it match your prediction? Does the peak maintain its shape? Does it spread out?

**Extra problems I recommend you work (not to be turned in):**

- Show that in the limit as $\mu_2 \to 0$ or $\mu_2 \to \infty$, our equations for the transmitted and reflected amplitudes and powers are consistent with what we deduced earlier for a string with a fixed or a free end.

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18-1. A family ice show is held at an enclosed arena. The skaters perform to music with level 81.7 dB. This is too loud for your baby who yells at [01] _____ dB. (a) What total sound intensity engulfs you? (b) What is the combined sound level?

18-2. A stereo speaker (considered a small source) emits sound waves with a power output of [02] _____ W. (a) Find the intensity 10.5 m from the source. (Assume that the sound is emitted uniformly in all directions from the speaker.) (b) Find the intensity level, in decibels, at this distance. (c) At what distance would you experience the sound at the threshold of pain, 120 dB?

18-3. A firework explodes [03] _____ m directly above you. You record the explosion with a microphone and you find that the average intensity of the sound was [04] _____ dB and that the sound lasted for 2.32 ms. How much was the total sound energy released by the explosion (in Joules)? Assume that the sound waves were spherical.

18-4. (Paper only.) As given in Serway chapter 17, the speed of all mechanical waves follows an expression of the general form:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}},$$

For the speed of sound in air, the elastic property is the bulk modulus $B$ and the inertial property is the density $\rho$. For longitudinal sound waves in a solid rod the inertial property is still the density, but the elastic property is $Y$, "Young's modulus". Young's modulus was discussed back in chapter 12, but in case you missed it, Young's modulus of a material is defined by the following equation:

$$Y = \frac{\text{stress}}{\text{strain}},$$

where the stress is the force per cross-sectional area that's being applied, and the strain is the fractional change in length $\Delta L/L$, that occurs in response.

(a) Consider a solid rod. Derive the "spring constant" of the rod $k$, in terms of its Young's modulus $Y$, cross-sectional area $A$, and length $L$. Do this simply by comparing the definition of Young's modulus to Hooke's law.

(b) The velocity equation for a solid rod works well for slinkies, too, if you use your answer to part (a) to write Young's modulus in terms of the spring constant of the slinky. Do that, and derive an expression for the longitudinal wave speed of a slinky in terms of $A$, $L$, $k$, and the slinky's mass $m$.

(c) Specifically, how does the speed of compression waves change with $L$?

(d) Stretch your slinky until it is 5 feet long (hook it to something or have a friend hold the other end), and then whack the end to make a compression wave. Measure the time that it takes for the pulse to travel back and forth 10 times and calculate the speed of compression waves on your slinky.

(e) Now do the same thing with the slinky stretched to 10 feet. Does the speed of compression waves vary with length as you predicted?

(f) Compare the speed of longitudinal waves to the speed of transverse waves that you measured and predicted in an earlier assignment.

**Extra problems I recommend you work (not to be turned in):**

- The intensity level of an orchestra is 80.5 dB. A single violin achieves a level of 68.2 dB. How does the intensity of the sound of the full orchestra compare with that of the violin's sound? Find the ratio of the intensities. (Answer: 16.98.)

- Two small speakers emit spherical sound waves of different frequencies. Speaker $A$ has an output of 1.51 mW, and speaker $B$ has an output of 2.09 mW. Determine the sound level (in decibels) at point $C$ (see figure) if (a) only speaker $A$ emits sound, (b) only speaker $B$ emits sound, (c) both speakers emit sound. Assume that the two waves are incoherent so that intensities add. (Answers: 66.82 dB, 69.20 dB, 71.18 dB.)
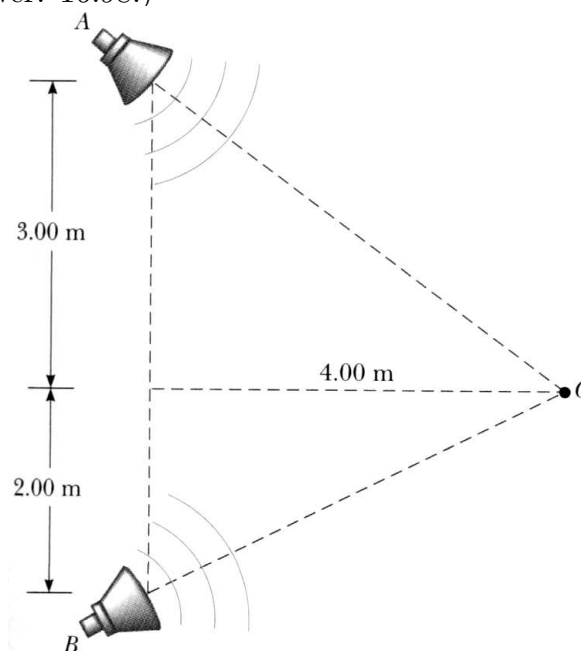
- I am sitting 2.37 m from a speaker listening to some music. How close to the speaker should I sit if I want the music to be 14.3 dB louder? (Answer: 0.46 m.)

19-1.  A bat flying at [01] —————— m/s emits a chirp at 40.95 kHz. If this sound pulse is reflected by a wall, what is the frequency of the echo received by the bat? (*Hint:* This is exactly the same as the situation where the source and observer are both moving towards each other.) Use 345 m/s for the speed of sound.

19-2.  A train is moving close to and parallel to a highway with a constant speed of 22 m/s. A car is traveling in the same direction as the train with a speed of [02] —————— m/s. The car horn sounds at a frequency of 519 Hz, and the train whistle sounds at a frequency of 327 Hz. (a) When the car is well behind the train, what frequency does an occupant of the car observe for the train whistle? (b) When the car is well in front of the train, what frequency does a train passenger observe for the car horn after the car passes? Assume that the speed of sound is 343 m/s. There is no wind.

19-3.  One day as I stepped out onto the street, I was very nearly hit by a car. Fortunately, the car honked its horn and I jumped out of the way. But I noticed that the frequency of the horn as the car approached me was a factor [03] —————— higher than its frequency after the car passed me and was moving away from me. Calculate the velocity of the car. The speed of sound is 343 m/s.

19-4.  When we analyze the light coming from a distant galaxy, we find a particular absorption line with a wavelength of [04] —————— nm. This same absorption line in light from the sun has a wavelength of 625 nm. (a) Is the galaxy moving towards us or away from us? (b) Calculate the magnitude of the velocity of the galaxy relative to us. Note that for light waves, the Doppler shift is given by

$$f' = f\sqrt{\frac{c \pm v}{c \mp v}}$$

where $c$ is the speed of light and $v$ is the relative velocity of the source and observer. Use the upper signs when the source and observer are moving towards each other and the lower sign when the source and observer are moving away from each other.

19-5.  You notice a supersonic jet flying horizontally overhead before the sonic boom arrives. As the jet recedes from view, you judge that its position makes a [05] ——————-degree angle with the horizon when you finally hear the sonic boom. What is the Mach number (i.e., the speed of the plane divided by the speed of sound)?

19-6. A jet airplane flies with a speed of 1120 mph (mi/h) at an altitude of [06] —————— ft. It passes directly over my head. How soon after it passes directly above me will I hear the sonic boom? The speed of sound is 343 m/s. (Caution: I am *not* asking how much time it took for the sound to travel from the airplane when it was directly overhead. The sonic boom originates from the airplane sometime *before* it reached the point directly overhead.)

19-7. A pair of speakers separated by [07] —————— m are driven by the same oscillator at a frequency of 690 Hz. An observer, originally positioned at one of the speakers, begins to walk along a line perpendicular to the line joining the two speakers. (a) How far must the observer walk before reaching a relative maximum in intensity? (b) How far will the observer be from the speaker when the first relative minimum is detected in the intensity? (Take the speed of sound to be 345 m/s.)

**Extra problems I recommend you work (not to be turned in):**

- On a very quiet morning, you drop a tuning fork vibrating at 512 Hz from a tall bridge. How long until you will hear a frequency of 475 Hz? Take the speed of sound in air to be 343 m/s and the acceleration of gravity to be 9.80 m/s$^2$. Hint: Don't forget to include the time it takes for the sound to return to the point of release. (Answer: 2.832 s.)

- Two speakers emitting 651 Hz are separated by a distance of 2.0 m. A student is positioned directly in front of the first speaker and along a path 90° from the line that joins the two speakers. As she walks directly toward the first speaker she notices minima in the sound level. (a) How many minima does she experience if she begins from far away? (b) How far is she from the speaker for the first minimum she encounters? Use 343 m/s for the speed of sound. (Answers: 4, 7.46 m)

20-1. In the arrangement shown in the figure, an object of mass $m = [01]$ ―――――― kg hangs from a cord around a light pulley. The length of the cord between point P and the pulley is $L = 2.0$ m. When the vibrator is set to a frequency of 150 Hz, a standing wave with six loops is formed. What must be the linear mass density of the cord?
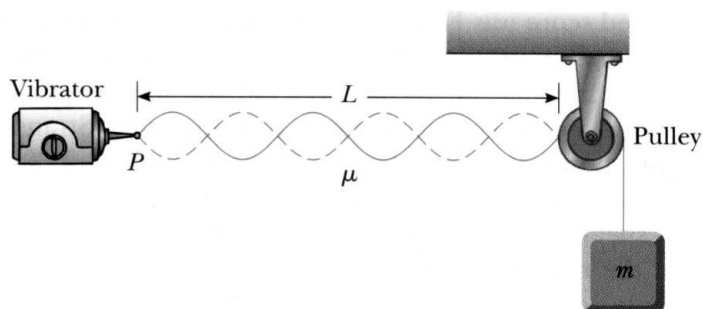


20-2. A steel piano string is $[02]$ ―――――― inches long. The diameter of the string is 0.0421 inches. When struck, this string produces the musical note B which has the pitch of 123 Hz. (a) Find the tension of this string. Give the answer in pounds. The density of steel is 7.86 g/cm$^3$. (b) There are 228 strings in a piano. (Some notes use more than one string.) If we assume that the tension in every string is the same, find the total force (in tons, 1 ton = 2000 lb) exerted on the frame of the piano. (Add together the tensions of all of the strings.) Piano frames are made of steel so that they can withstand this kind of force.

20-3. A pipe open at both ends has a fundamental frequency of $[03]$ ―――――― Hz when the temperature is 0°C. (a) What is the length of the pipe? (b) What is the fundamental frequency at a temperature of 30°C? Assume that the displacement antinodes occur exactly at the ends of the pipe. Neglect thermal expansion of the pipe.

20-4. A pipe is open at both ends. Its length is 22.81 cm. If I excite standing waves in the pipe by blowing air over one of its ends, I hear a pitch of $[04]$ ―――――― Hz. (a) Find the distance between the antinodes at the two ends of the pipe. This is not the same as the actual length of the pipe, since the antinodes are not exactly at the ends of the pipe. The speed of sound is 343 m/s. (b) How far past the physical ends of the pipe do the antinodes extend? Assume that it is the same amount for each antinode. (c) If I close one end of the pipe, what pitch will I hear? The node at the closed end is exactly at the position of the closed end. The antinode at the open end extends past the physical end of the pipe by the same amount found in part (b).

20-5. (Paper only.) The high E string on Dr. Durfee's guitar has a linear mass density of $3.93 \times 10^{-4}$ kg/m (he calculated it from information found at the string manufacturer's web site) and a length of 25.5 inches (64.77 cm). The frequency of the fundamental mode of this string is 330 Hz. (This mode is also known as the "first harmonic".) (a) What is the tension in the string? (b) When the string is plucked, a whole bunch of modes are excited. If the string is then touched right in the middle, all of the modes will be damped out except the ones that have a node at that point. After touching the string in the middle, what is the frequency of the lowest frequency mode which is still ringing? (c) What if the string is touched at a point 2/3 of the way from one end to the other?

20-6. (Paper only.) Transverse standing waves on a slinky: The picture represents a transverse wave on a slinky oscillating in the fundamental mode. The distance along the slinky is in the $x$-direction; because the wave is transverse, the displacement from equilibrium is in the $y$-direction.
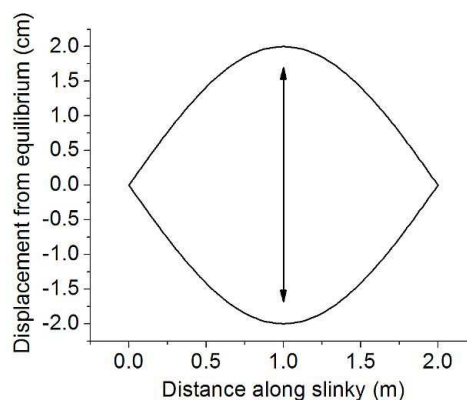


(a) Make a sketch of the displacement vectors of this mode, at the time where the displacement is maximum, for a number of evenly-spaced positions along the slinky. Make a similar sketch for the second, third, and fourth harmonics: the displacement as a function of position, and displacement vectors at a number of evenly-spaced positions.
(b) Stretch your slinky to a fixed length feet of $5 - 10$ feet. Either measure the wave velocity for this length, or else go back to your notes from the problem in HW 15 where you measured the velocity and predicted the dependence as a function of length.
(c) Use the measured velocity along with the known wavelengths of the first three harmonics to predict the frequency of oscillation of the first three harmonics.
(d) Test it out: verify your sketches in part (a), and measure the frequencies of the first three harmonics. Do this by measuring about 10 oscillations and dividing the time for the ten oscillations by 10 to get the period for a single oscillation with greater accuracy. Then use the period to find the frequency. Compare with your predicted values from part (c).

20-7. (Extra credit; paper only.) Dr. Durfee recounts the following: One evening I took a tall glass from my cupboard and measured the frequency of the fundamental mode of the air in the glass. I did this by whacking the bottom of the glass while my wife held a microphone above the glass. The microphone was connected to my computer, and I measured the frequency using "Spectrum Lab", a free program which you can download from the class web page. The inside of the glass is a cylinder which is 7 cm in diameter and 14.3 cm tall. The lowest frequency I measured when I whacked the glass was 570.8 Hz.

(a) If the glass were very narrow (i.e. if the diameter were much smaller than the height) then the waves would propagate almost as if they were one-dimensional. Otherwise you need to use a three-dimensional wave theory to get precise results. Assuming that the waves in the glass are one dimensional, calculate the velocity of sound using the height of the glass and the frequency of the fundamental mode. (Note: Although this assumption is not really very good in this case, you should still get an answer which is within 5% of the "expected" answer of 343 m/s. This is due to the fact that the wideness of the glass introduces two errors which partially cancel each other. First, the fact that the waves propagate in the glass three-dimensionally means that the wavenumber $k$ is really the sum of three components $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$, resulting in a wavelength for the fundamental mode which is shorter than it would be if the diameter of the glass were smaller. Second, the wide mouth of the glass introduces an "edge effect". Because the mouth is so wide, the oscillating wave pokes out of the cup and the pressure just outside the cup is not fixed at atmospheric pressure. This effectively increases the length of the cup, thereby increasing the wavelength of the fundamental mode.)

(b) I then filled the glass with water and found the frequency of the first harmonic to be 3168 Hz. Use this frequency to calculate the speed of sound in water. (Note: This answer won't turn out as nicely. You still have the shorter wavelength due to the three-dimensional nature of the oscillation. But the "edge effect" is gone because there is no water outside of the glass: because the air above the glass has such a different wave speed than the water in the glass, the oscillation doesn't penetrate out of the glass very far. Still, your answer should be within 25% of the expected value of 1480 m/s.)

(c) Then I put milk into the glass and measured a frequency of 3100 Hz. What is the speed of sound in milk?

(d) I then shook up the milk and found that the frequency of the first harmonic was cut in half. However, over the course of a few seconds the frequency drifted back up. This is because microscopic air bubbles in the milk decrease the density of the milk (air is less dense than milk) and decrease the bulk modulus (air is more compressible than milk). Which changed by a bigger factor, the density or the bulk modulus?

(e) (Optional.) This is a fun thing that you ought to try. You can do it with water and hear the pitch go down after you shake it. But water releases its air bubbles rather fast. It is easier in milk because the fat in milk increases the viscosity and holds onto the bubbles longer. And it works a lot better if you add ice cream. Make yourself a milkshake in the blender, pour it into a rigid cup (one made of glass works best), and then hit the bottom of the glass. You should hear a very deep, low frequency "thunk".

**Extra problems I recommend you work (not to be turned in):**

- Suppose we excite a two-loop standing wave in a rope using a tension of 1.5 N. What tension should we apply to the rope if we want to excite a one-loop standing wave with the same frequency? (Answer: 6 N.)

- Two sine waves traveling down a string interfere to create a standing wave. The displacement of the string is given by $y(x,t) = (3.21 \text{ cm}) \sin(0.342\frac{\text{rad}}{\text{m}} \cdot x) \cos(32.2\frac{\text{rad}}{\text{s}} \cdot t)$. (a) What is the amplitude of each of the two interfering waves? (b) What is the wavelength of each of the two interfering waves? (c) What is the speed at which the two interfering waves are traveling? (Answers: 1.605 cm, 18.37 m, 94.15 m/s.)

- (a) Find the speed of sound in helium gas at 38.5°C. (b) If an organ pipe produces a tone (pitch or fundamental frequency of the pipe) with frequency 484 Hz in air at room temperature, find the frequency of its tone in helium gas at 38.5°C. The speed of sound in air at room temperature is 343 m/s. (Answers: 1038 m/s, 1465 Hz.)

- Two speakers emit sound waves with frequency 584 Hz. They are driven by the same oscillator so that they are in phase with each other. We place the speakers so that they are a few meters apart and facing each other. Along the line joining the two speakers, the sound waves from the two speakers are traveling in opposite directions. This creates a standing wave between the two speakers. How far apart are the antinodes in that standing wave? Neglect the effect of reflection of waves from the speakers. The speed of sound is 343 m/s. (Answer: 29.4 cm.)

- Download and install Spectrum Lab on a computer with a microphone. Measure resonances in a glass, yourself. Compare your measured values to Dr. Durfee's.

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21-1. Two identical mandolin strings under 205.6 N of tension are sounding tones with fundamental frequencies of 523 Hz. The peg of one string slips slightly, and the tension in it drops to [01] _____ N. How many beats per second are heard?

21-2. While attempting to tune the note C at 523.0 Hz, a piano tuner hears 2 beats/s between a reference oscillator and the string. (a) When she tightens the string slightly, the beats frequency she hears rises smoothly to [02] _____ beats/s. What is the frequency of the string now? (b) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

21-3. A speaker at the front of a room and an identical speaker at the rear of the room are being driven at 456 Hz by the same sound source. A student walks at a uniform rate of [03] _____ m/s away from one speaker and toward the other. How many beats does the student hear per second? (Take the speed of sound to be 345 m/s.)

21-4. We place a speaker near the top of a drinking glass. The speaker emits sound waves with a frequency of [04] _____ kHz. The glass is 14.1 cm deep. As I pour water into the glass, I find that at certain levels the sound is enhanced due to the excitation of standing sound waves in the air inside the glass. Find the minimum depth of water at which this occurs (distance from surface of water to bottom of glass). The standing sound wave has a node at the surface of the water and an antinode at the top of the glass. Assume that the antinode is exactly at the top of the glass. The speed of sound in air is 343 m/s.

21-5. Suppose that your shower stall is [05] _____ m tall. (a) As you sing in the shower, how many frequencies in the range 300–1500 Hz will resonate? Ignore side-to-side sound waves and take the shower stall to be closed at both ends. Use 343 m/s for the speed of sound. (b) What is the lowest resonant frequency in that range? (c) What is its harmonic number? (d) What is the highest resonant frequency in that range? (e) What is its harmonic number?

21-6. (Paper only.) Longitudinal standing waves on a
slinky: The following picture represents a
longitudinal wave on a slinky oscillating in the
fundamental mode. The distance along the slinky is
in the $x$-direction; because the wave is longitudinal,
the displacement from equilibrium is also in the
$x$-direction.
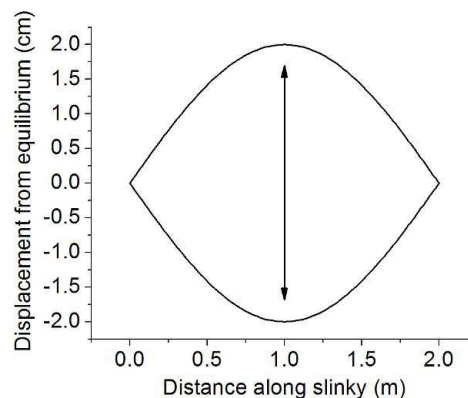
(a) Make a sketch of the displacement vectors of this
mode, at the time where the displacement is maximum, for a number of evenly-spaced
positions along the slinky. Make a similar sketch for the second, third, and fourth
harmonics: the displacement as a function of position, and displacement vectors at a
number of evenly-spaced positions. Be careful with directions. Keep in mind that the
*longitudinal* displacement vectors point in the same direction as the slinky's length.

(b) Stretch your slinky to the same length as the last problem. Either measure the wave
speed of longitudinal waves, or else refer to your notes from HW 18 as to how the
longitudinal wave speed of a slinky compares to the transverse wave speed.

(c) Use the measured velocity along with the known wavelengths of the first three
harmonics to predict the frequency of oscillation of the first three harmonics.

(d) Test it out: verify your sketches in part (a), and measure the frequencies of the first
three harmonics. (The third one may be tricky; if too difficult, just do the first two
harmonics.) Compare with your predicted values from part (c).

**Extra problems I recommend you work (not to be turned in):**

- The wavelength of one sound wave is 0.81 m. The wavelength of a second sound wave is
  a little bit longer. When the two sound waves are superimposed on each other, we hear a
  2.3 Hz beat. Find the difference in their wavelengths. The speed of sound is 343 m/s.
  (Answer: 4.4 mm.)

- Download the program "Spectrum Lab" from the class web page, if you haven't already. Install it and run it. Click on "View/Windows" and then on "Spectrum Lab Components." In the top left-hand corner of the window that opens is a box called "Signal Generator." Make sure that the switch below it points to the right. Make sure the "Mono" box on the far right hand side is green and set for "DAC". (Click it once if it's set to "(off)".) Now click on "View/Windows" and open the "Test Signal Generator". You can use this new window to make different tones. (a) Turn it on and generate two sine waves (with no AM or FM) at 440 and 441 Hz. Listen to the beats. (b) Play around with the program on your own. Try combining many different frequencies. What happens when you combine frequencies that are closer? Farther away? That are multiples of each other? Can you simulate the beat effect by using a single frequency but with amplitude modulation on? Etc.

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22-1. (Paper only) The function $f(x)$, graphed below, is defined as follows: it is zero most of the time, but equal to 1 when $-2 < x < -1$ and when $1 < x < 2$. It repeats with a period of $L = 10$.

(a) Find the Fourier coefficients of $f$. If any of the terms have obvious values, state the reason why. Hint: integrate from -5 to 5 instead of from 0 to 10.

(b) Write $f(x)$ as an infinite series, and use a program such as Mathematica to plot the series for the first 1 term, the first 10 terms, and the first 100 terms (in three separate graphs). The Sum command in Mathematica should make this easy. You can easily check your answer to (a): if correct, your graph of the first 100 terms should look nearly exactly like the graph below.
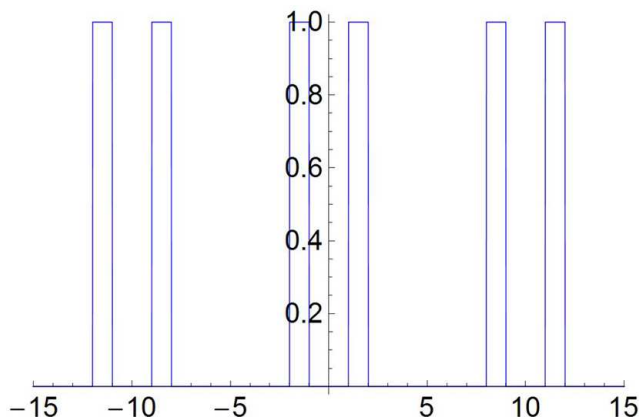
22-2. (Paper only) At a particular time, a wave has the shape shown in the figure ($y = 0.5x$ from $0 \le x \le 2$, repeated). You might get this type of wave, albeit not repeated, by (for example) plucking a guitar string very close to the right end.
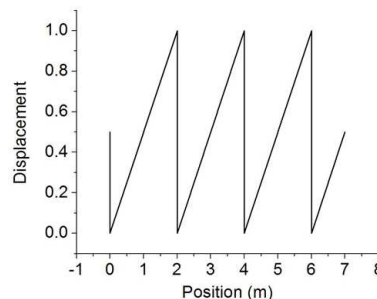


(a) Calculate the Fourier coefficients, and prove that this wave can be represented by:

$$y(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi x)$$

(b) Give a brief argument through symmetry as to why all of the cosine terms except the DC offset (if you want to call that a cosine term) were zero. If you want to save yourself a little bit of work, you can do this part before part (a).

(c) Verify that that function reproduces the above graph, by using a program such as Mathematica to plot the function with the summation having 1, 10, and 100 terms. (The Sum command in Mathematica should make this easy.)

22-3. (Paper only.) The power that comes out of the electrical outlets in your home is AC or "alternating current" power, characterized by a voltage that oscillates sinusoidally. Conversely, most electronic devices require a constant voltage (known as DC or "direct current" power). One way to generate DC power from AC power is to use a device called a diode to "kill" the negative part of the sine wave, resulting in a wave like the one shown below. Then the wave is filtered to keep only the DC component (i.e. the constant term of your Fourier series). The wave shown below is known as a "half-wave rectified" wave ("rectified" because it is only positive, and "half" because only half of the wave is left). If I wanted to make a 5 V DC power supply this way, what should the amplitude of the oscillating sine wave be before it is rectified?
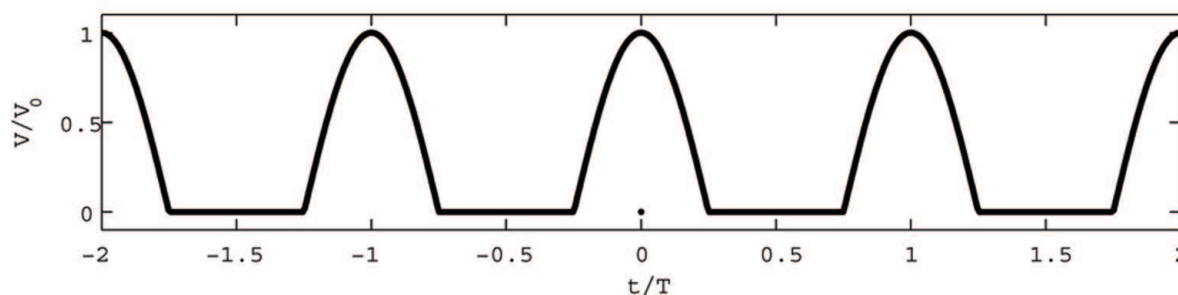
**Extra problems I recommend you work (not to be turned in):**

- Work out any of the Fourier series problems that are on the old exams posted to the class website.

- We discussed the equation for the time-averaged power carried by a sine wave traveling down a string: $P = \frac{1}{2}\mu\omega^2 A^2 v$. For a more complicated wave, the power is just the sum of the powers carried by each of the sine waves that make it up. Show that the square wave whose Fourier coefficients were solved in the text (see PpP equation 6.18) carries infinite time-averaged power. (This is one reason that you can't ever make a true square wave on a string, you can only make an approximate one.)

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23-1. (Paper only.) A string of length $L$ is fixed at both ends. At time $t = 0$ the string is stretched into the shape shown in the figure.
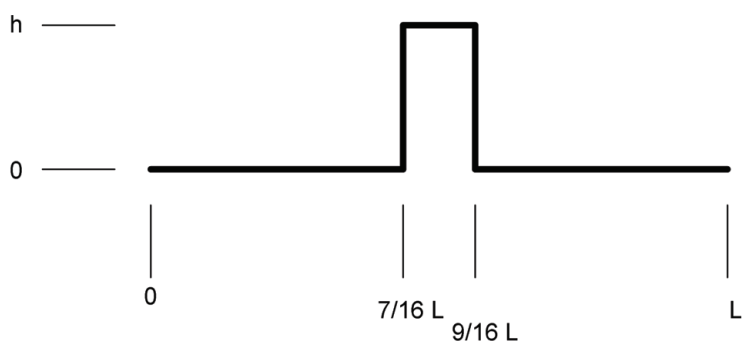


Mathematically, the shape of the string is given by the equation:

$$y(x) = \begin{cases} 0, & \text{if } 0 < x < \frac{7}{16}L \\ h, & \text{if } \frac{7}{16}L < x < \frac{9}{16}L \\ 0, & \text{if } x > \frac{9}{16}L \end{cases}$$

We want to use a Fourier transform to write $y(x)$ as a sum of the harmonic modes of the string. (These are sine functions only.)

(a) If we want to perform a Fourier transform to write $y(x)$ in terms of the harmonic modes of the string, what should be the size and shape of the basic repeating unit? (Hint: if you just repeat the given shape, the periodic function will be even and you will have cosine terms only.) In the integral(s) that you have to do below, it's probably easiest to integrate from $-\text{period}/2$ to $\text{period}/2$.

(b) Calculate the constant term of the Fourier series. If it equals zero due to a symmetry argument, state the argument.

(c) Calculate the sine coefficients. If they equal zero due to a symmetry argument, state the argument.

(d) Calculate the cosine coefficients. If they equal zero due to a symmetry argument, state the argument.

(e) Write $y(x)$ as a sum of harmonic modes on the string (similar to PpP equation 6.23, but with the parameters you calculated above inserted for $A_n$).

(f) Verify that the function you obtained reproduces the above graph, by using a program such as Mathematica to plot the function with the summation having 1, 10, and 100 terms. Use $h = L = 1$, and force the y-axis to display from $-0.2$ to $1.2$.

23-2. (Paper only.) Several of the BYU physics department faculty members have Ti:sapph lasers (pronounced "tye-saff"; short for "titanium-doped sapphire", the active medium) which produce pulses of light as short as about 25 femtoseconds long (1 fs $= 10^{-15}$ seconds). Use the uncertainty principle to estimate the minimum bandwidth that the laser gain medium needs to have to produce such pulses. The bandwidth is the maximum frequency of light that it will amplify minus the minimum frequency it will amplify, labeled $\Delta f$.

23-3. (Paper only.) Find a piano. A grand piano would be best. Push down the sustain pedal, then sing a note into the strings. What do you hear after you stop singing? The piano is doing a Fourier transform of your voice. Each string only resonates with certain frequencies. Those strings which have a harmonic at the same frequency as one of the frequencies present in the note you sang will absorb sound and begin to oscillate, largely reproducing the sound of your voice.

24-1. (Paper only.) I found a few websites that say the lowest note a piccolo can play is $d^2$, the D that is an octave and a note above middle C. The highest note a piccolo can play is $c^5$, the note that is four octaves above middle C. (a) What is the frequency ratio of these two notes (using an equal temperament scale)? (b) What is the wavelength of the $d^2$ note? Assume the speed of sound is 343 m/s. (c) Does that wavelength make sense with this homework problem I found in Serway, that states: "The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as in a pipe open at both ends." Why/why not? (d) When the piccolo is playing its highest note, what is the distance between adjacent antinodes?

24-2. (Paper only.) While you are practicing the piano, a car races past outside. You notice that as the car approached, the engine made a noise which was exactly in tune with a middle A (440.0 Hz). After the car passed, the pitch dropped down two half steps to a G. How fast was the car going? Assume that the piano is tuned to an equal temperament scale and that the speed of sound is 343 m/s.

24-3. (Paper only.) Guitar players often tune their instrument using harmonics. Imagine that I use an electronic tuner to tune my low E string to 164.814 Hz, precisely the correct frequency for an equal temperament scale referenced to a frequency for middle A of 440.000 Hz. Now I tune my next string, the A string which is 5 half-steps higher in frequency, using harmonics. Since 5 half-steps is a musical fourth, which corresponds to a frequency ratio of 4/3, I can do this by lightly touching the E string 1/4 of the way from the end of the string and lightly touching the A string 1/3 of the way from the end of the string. (a) Why is that? (b) I then play both strings and adjust the A string to make the beat frequency as low as possible. If I tune the beats completely away, how far off (in Hz) will the frequency of the A string be from the ideal frequency for this string according to the equal temperament scale?

24-4. (Paper only.) Find a piano. A grand piano would be best. Push down the sustain pedal, then clap your hands near the strings. Listen carefully to see which is the highest note excited by your clap. It may take several tries before you are convinced you have identified the correct note. Determine the frequency of that note. Now use the uncertainty principle to estimate the duration of your clap.

24-5. (Paper only.) Find a piano. (a) Gently push down the C below middle C lightly enough that no sound is made. Keep holding it down. Now strike middle C. Let go of the higher note while still holding down the lower one. Can you still hear middle C? That's because the second harmonic of the low C has nearly the same resonance frequency as the fundamental of the note that is one octave higher. This means that the low C strings can absorb energy at this frequency and begin to oscillate. (b) Now try holding down the low C and playing different notes to see which ones share harmonics with it. Find at least three such notes, and explain how/why they were able to excite the low C strings.

24-6. (Paper only.) Find a piano. Push lightly on one of the low notes such that it doesn't make a sound. While holding down that key, loudly strike the note which is one octave higher. You should now hear the lower strings oscillating at the fundamental frequency of the higher strings. (You observed that in the previous problem.) Now lightly strike and hold the higher note again. Chances are if you listen carefully, you will be able to hear beats. (a) Why? (b) How many beats did you hear, and what specifically does that tell you? (If you don't hear any beats, try a different set of notes.)

**Extra problems I recommend you work (not to be turned in):**

- The lowest string on a 6-string guitar is usually tuned to an E at a frequency of 82 Hz (not including the decimal places). (a) If this E is referenced to an equal temperament scale for which middle A is exactly 440 Hz, give the frequency of this E to four decimal places. (b) Sometimes guitar players will loosen the tension in this string to drop it down two half-steps to a D to hit lower notes in a particular song. This is known as "drop D tuning". What is the frequency of the D just below the E you found above? (to four decimal places) (c) By how much do you need to reduce the tension in the string to go from the E down to the D? (Answers: 82.4069 Hz, 73.4162 Hz, reduce by 20.6%.)

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25-1. A ray of light passes through a pane of glass which is 1.0 cm thick. The index of refraction of the glass is 1.53. The angle between the normal to the surface of the pane and the ray in the air as it enters the pane is [01] ————°. (a) Find the angle between the normal to the surface of the pane and the ray *inside* the glass. (b) Find the angle between the normal to the surface of the pane and the ray in the air after it exits the pane.

25-2. Light passes through a glass prism, as shown in the figure. The cross-section of the prism is an equilateral triangle. (a) Find the incident angle, if we want the light ray *inside* the prism to be parallel to the base of the prism. Use [02] _____ for the index of refraction of glass. Remember that the incident angle is measured with respect to a line normal to the surface of the prism. You may use $n = 1$ for the index of refraction of air. (b) The index of refraction of glass for blue light is 1.528. Using the incident angle from part (a), find the angle at which blue light exits the prism. This is *not* the angle of deviation $\delta$ shown in the figure. We want the angle between the light and a line normal to the surface from which the light exits. (c) Repeat part (b) for red light, for which the index of refraction is 1.511. Caution: In parts (b) and (c), the light ray inside the prism is no longer parallel to the base of the prism.
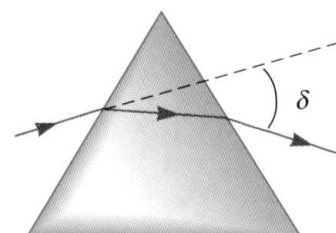
25-3. (Paper only.) An optics researcher sets up two mirrors as shown by the black lines in the figure below. She shines a laser towards the mirrors along the path marked by the gray arrows. Find the angle between the incoming and outgoing beams of light, $\phi$, in terms of the angle between the mirrors, $\theta$. Perhaps surprisingly, the angle $\phi$ does not depend on how the two mirrors are oriented relative to the incident light ray. Hint: There are likely many ways to do this problem. The way I chose, was to call the incident angle $x$ (relative to the surface, not relative to the normal). Then I worked out all of the other angles in the picture in terms of $x$. In the end, when I solved for $\phi$, all of the $x$'s canceled out.
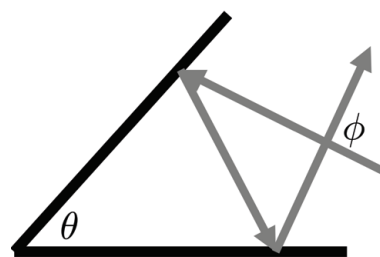
25-4. (Paper only.) When the sun heats a hot desert, the air near the ground heats up and becomes *less* dense than the air above it, such that the density of the air increases with the distance from the ground. Explain why this creates a mirage.

25-5. (Extra credit; paper only.) (From Peatross and
Ware, *Physics of Light and Optics*.) Ole Roemer
made the first successful measurement of the speed
of light in 1676 by observing the orbital period of Io,
a moon of Jupiter with a period of 42.5 hours. When
Earth is
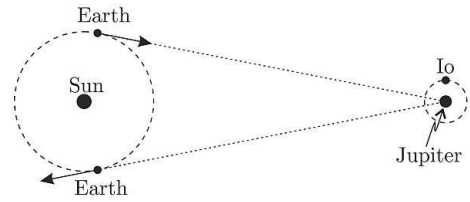


moving toward Jupiter, the period is measured to be shorter than 42.5 hours because
light indicating the end of the moon's orbit travels less distance than light indicating the
beginning. When Earth is moving away from Jupiter, the situation is reversed, and the
period is measured to be longer than 42.5 hours.

(a) If you were to measure the time for 40 observed orbits of Io when Earth is moving
directly toward Jupiter and then several months later measure the time for 40 observed
orbits when Earth is moving directly away from Jupiter, what would you expect the
difference between these two measurements be? Take the Earth's orbital radius to be
$1.5 \times 10^{11}$ m. To simplify the geometry, just assume that the Earth moves directly
toward or away from Jupiter over the entire 40 orbits. (See the figure.) Hint: Find the
Earth's orbital speed from its orbital radius (given) and period (you should know!). You
will need to determine how far the Earth moves closer to Jupiter in the $40 \times 42.5$ hrs
observation period when it's moving straight towards Jupiter (imagine straight-line
motion during this time).

(b) Roemer did the experiment described in part (a), and experimentally measured a
22 minute difference. What speed of light would one deduce from that value?

**Extra problems I recommend you work (not to be turned in):**

- Some rooms (such as many sealing rooms in LDS temples) have two parallel mirrors
  facing each other, so that when people look in the mirrors they see images of themselves
  "forever". Estimate sizes and positions of the mirrors in a typical configuration. Based
  on the angle that you need to use in order to look past the side of your head, about how
  many images, at most, will you actually be able to see?

- The speed of sound in air is 343 m/s. In water it is 1480 m/s. If a directed sound wave in air strikes the surface of a lake at an angle of 23° from the normal, at what angle from the normal will the transmitted sound wave travel in the water? (You would likely need an array of speakers, properly phased relative to each other, to produce such a sound wave. As we have discussed, sound waves typically travel outwards in all three dimensions rather than going in a specific direction like a laser beam.)

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26-1. You are a fish in deep, dark water with index 1.33. As you look up from [01] ―――――― m below the smooth surface, you see a bright circle through which light enters from the outside world.
    (a) Why is that?
    (b) What is the *radius* of the circle?

26-2. A rectangular block of clear plastic is sitting on the ground. A beam of light traveling horizontally enters one face of the plastic at an angle $\theta$ from the normal. The transmitted beam then strikes the front face of the block, as shown in the figure below. What is the maximum angle $\theta$ which will result in total internal reflection off of the top surface? The index of refraction for the plastic is [02] ―――――.
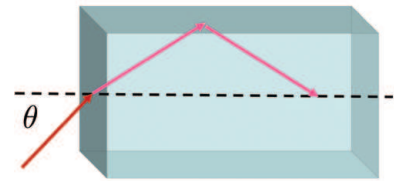
26-3. (Paper only.) *Fermat's Principle of Least Time.* Fermat realized that if you imagine all possible paths that light rays could take between the source and the destination, the actual path is the one that takes the least amount of time. It's kind of like this situation: suppose you are a lifeguard at position A below and must rescue a drowning swimmer at position B? What path should you take to get there the quickest? Let's prove that the fastest time is given by the path predicted by Snell's Law. Consider a light ray traveling from point A to point B, from a lower index of refraction ($n_1$) to a higher index of refraction ($n_2$). Find the time it takes for a hypothetical ray to travel from A to B when it enters the $n_2$ medium an arbitrary distance $x$ from the left, in terms of the $n$'s and $c$. Then, to find the $x$ that produces the minimum time, take the derivative and set it equal to zero. Show that this gives you Snell's Law.
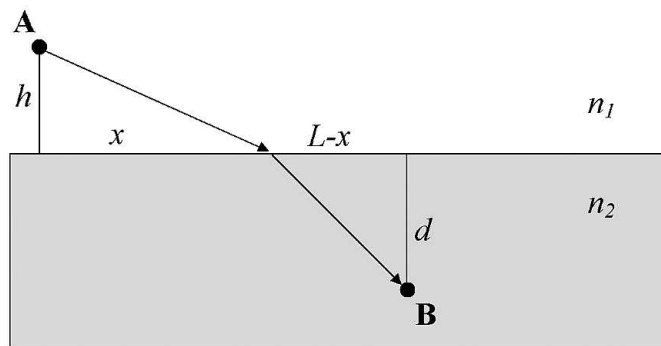


26-4. (Paper only.) A swimmer is a distance $h$ under smooth water, a distance $L$ from shore, and is named Jane.

(a) Is it possible for light from the swimmer to reach the eyes of her boyfriend on the shore (i.e., can he see her?), or will this be prevented by total internal reflection? Explain why or why not. (Yes, you have been given all of the information you need.)

(b) In light of your answer to this problem, why is it often difficult to see someone who is swimming under water?

(c) If the boyfriend cruelly shines a powerful laser at the water, will he always be able to hit her with the light (assuming he has excellent aim), or will this be prevented somehow? Explain why or why not.

(d) What if she wants to hit *him* with a laser?

26-5. (Extra credit; paper only.) *Huygen's Principle.* Huygen's principle says that each point of an advancing wave front can be considered as a point source of new circular (spherical, really) waves. The new waves add up to propagate the wave front. It's often used to describe/explain diffraction through a slits, but it can be also used to describe/explain refraction.

In the Wikipedia article on Huygen's principle, this picture is provided in order to graphically illustrate refraction. Stare at the picture until you can visualize that the green lines tangent to the circles (the parallel lines at the bottom of the picture) connect "matching" wavefronts. That is, if you label the wavefronts 1, 2, 3, etc., from the top of the picture on down, the first green line is tangent to all of the waves originating from wavefront #11, the second green line would be tangent to the waves from wavefront #12 (if those were drawn in), and so forth.
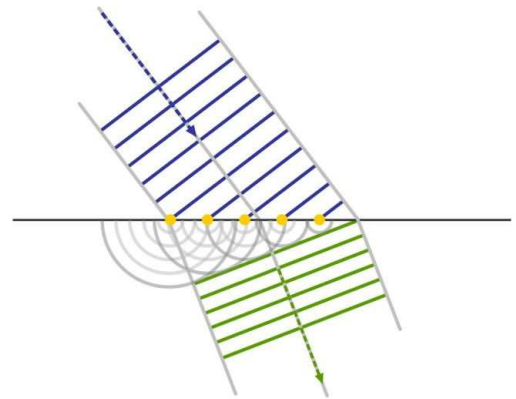


I want you to produce the same sort of picture, being as precise as you can with rulers/compasses/etc, to show that the graphical prediction of refracted angle from the Huygens' principle picture matches the numerical prediction from Snell's Law for at least one incident angle (you pick the angle). Draw an interface between an $n = 1$ and an $n = 2$ material. Draw the wavefronts of a wave hitting the interface at an angle. Just like the Wikipedia picture, treat each point where the wavefronts strike the interface as the source of circular waves propagating into the $n = 2$ material. Key: the wavelength of the circular waves (distance between wavefronts) must be exactly half the wavelength of the incident light because $\lambda$ is reduced in the material by a factor of $n$. Draw many circular waves going into the $n = 2$ material from at least four point sources and connect the matching wavefronts by drawing tangent lines like the green lines in the Wikipedia picture. Then measure the incident angle and the refracted angle and (hopefully!) prove that the refracted angle constructed this way is just the same as what Snell's law would predict.

**Extra problems I recommend you work (not to be turned in):**

- A particular optical fiber is made from a glass core which
  is 2 microns in diameter with an index of refraction of 1.7,
  covered by a thin "cladding" (an outer shell made out of a
  different material) with an index of refraction of 1.5.
  Calculate the radius of the smallest cylinder you could
  wrap the fiber around without destroying total internal
  reflection at the core/cladding interface and allowing light
  to leak out of the fiber core and into the cladding.
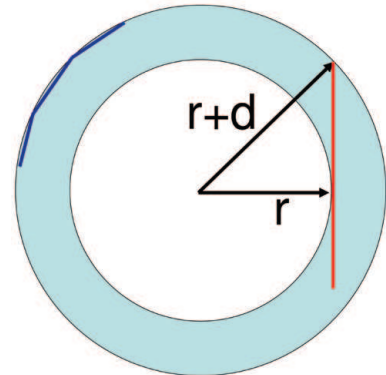
  

  Hint: If the fiber/cladding combo (gray-blue, diameter $d$) were wrapped around a white
  cylinder of radius $r$, it would look something like the figure shown. The blue reflections
  on the upper-left would have no problem doing TIR. The red line on the right, however,
  represents a worst-case scenario that might run into trouble because of its steeper angle.
  (Answer: 15.0 $\mu$m.)

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27-1. You wish to attenuate a polarized laser beam by inserting two polarizers. The second
polarizer is oriented to match the original polarization of the beam to ensure that the
final polarization remains unchanged. The first polarizer is then rotated through various
angles to control the intensity. Through what angle should the first polarizer be rotated
to reduce the final intensity by a factor of [01] _____? Assume perfect polarizers
with no losses.

27-2. Light from incandescent light bulbs is unpolarized. I shine light from a light bulb with an
intensity of [02] _____ W/m$^2$ onto a perfect linear polarizer. (a) What is the
intensity of the light which passes through the polarizer? (b) If a second polarizer is
placed after the first one, with its transmission axis rotated $35°$ from the transmission
axis of the first polarizer, what will be the intensity of the light after passing through the
second polarizer? (c) If I add a third polarizer after the second one, with its transmission
axis rotated $90°$ from the transmission axis of the first polarizer (i.e., an extra $55°$ from
the second polarizer), what will be the intensity of the light after passing through the
third polarizer? (d) If I now remove the second polarizer, what will be the intensity of
the light exiting the third polarizer?

27-3. You are looking across the surface of a very large, very calm lake. You are wearing polarized sunglasses, and you notice that your sunglasses nearly completely cut out the glare from the sun reflecting off of the lake. (a) Do your polarizing sunglasses block vertically or horizontally polarized light? (b) How far above the horizon is the sun in the sky? The index of refraction for water in the lake is [03] ——————.

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28-1. An object is placed [01] —————— cm in front of a concave mirror with a focal length equal to 3.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 2.00 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays. (If you don't want to use different colors, at least make the difference apparent somehow: dashed vs. non-dashed, pen vs. pencil, etc. The same goes for all of the problems that similarly say "different colors" in this assignment and the next assignment.)

28-2. An object is placed [02] —————— cm in front of a concave mirror with a focal length equal to 3.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.00 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

28-3. An object is placed [03] —————— cm in front of a convex mirror with a focal length equal to $-2.0$ cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 5.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

28-4. A concave mirror has a focal length of $f = 40.0$ cm.
(a) Find the position of the object that gives an image that is upright and [04] —————— times larger.
(b) Draw a ray diagram using different colors for the real and virtual rays.
(c) Repeat part (a) for a convex mirror with focal length $f = -40.0$ cm.
(d) Draw a ray diagram for part (c).

28-5. (Paper only.) At the north end of the foyer in the Eyring Science Center, there is a demonstration called "The Illusive Dollar". A large concave mirror produces an image of a dollar bill. The bill is 1.73 m from the mirror. (a) Is the image real or virtual? (b) What is the magnification of the dollar bill? Compare the image to an actual dollar bill. (c) Calculate the mirror's focal length (d) How far should you put a nickel from the mirror if you want to make a real image which is twice as big as the nickel, and will that image be upright or inverted? (e) Test out your predictions to part (c).

**Extra problems I recommend you work (not to be turned in):**

- (Paper only.) If I place an object in front of a concave mirror, under what conditions will a real image be created? Under what conditions will a virtual image be created? Under what conditions will the image be inverted? What about a convex mirror?

- (Paper only) Let's derive the focal length of a mirror. The curved line in the figure below is a spherical mirror. The dotted line runs from the center of curvature and has a length $R$. The red horizontal line at the top represents a beam of light traveling parallel to the principle axis. It makes an angle $\theta$ with respect to the normal of the mirror. (a) In terms of $\theta$ and $R$, what is $\alpha$? (b) What is $\beta$? (c) Find $f$ in the limit that $\theta$ is very small (i.e., such that the light represents a paraxial ray).
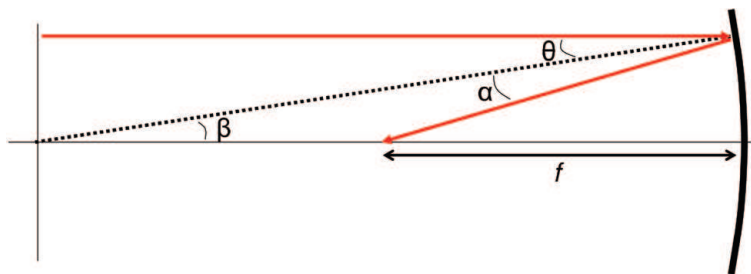


29-1. You are trapped in the wilderness and must spear fish in order to survive. While looking into the water from directly above, a fish appears to be [01] ———————— cm below the surface.

(a) How far below the surface is it in actuality?

(b) Draw a ray diagram, using different colors for the real and virtual rays.

29-2. A fortune teller gazes into her crystal ball and sees a scene of sorrow and tragedy. The scene appears to be inside the ball 4.31 cm from the front surface of the ball. But, of course she is really seeing the *image* of the scene of sorrow and tragedy. The actual scene of sorrow and tragedy is embedded in the ball at a different location. (a) How far from the front surface of the ball is the actual scene of sorrow and tragedy? The ball, [02] —————— cm in diameter, is made of quartz with an index of refraction of 1.54. (b) Is the image of the scene real or virtual?

29-3. An object is placed [03] —————— cm in front of a converging lens with a focal length equal to 2.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

29-4. An object is placed [04] —————— cm in front of a converging lens with a focal length equal to 6.0 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

29-5. An object is placed [05] —————— cm in front of a diverging lens with a focal length equal to $-5.0$ cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 5.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

29-6. An object is placed [06] —————— cm to the left of lens 1 (converging, $f = +20$ cm). Lens 1 is placed [07] —————— cm to the left of lens 2 (converging, $f = +10$ cm). (a) How far (in magnitude) from lens 2 will the final image be formed? (b) Will the image be to the left or the right of lens 2? (c) Will the image be real or virtual? You do not have to provide any ray diagrams for this problem.

29-7. A lens of focal length $f_1 = 10.0$ cm is placed a distance $x = $[08] _____ cm before a lens of focal length $f_2 = -10.0$ cm. An object is positioned 15.0 cm before the positive lens.

(a) At what position relative to the location of the *negative* lens does the final image occur? (Enter a negative number if on the left.)

(b) Calculate the overall magnification $M_{\text{overall}} = h_{\text{final image}}/h_{\text{object}}$.
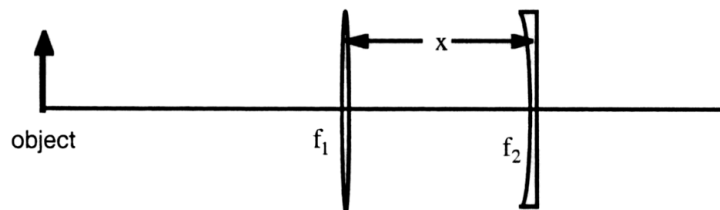


29-8. (Extra credit.) The lens and mirror in the figure have focal lengths of [09] _____ cm and $-50.0$ cm, respectively. An object is placed 1.00 m to the left of the lens, as shown. (a) Find the distance between the lens and the final image which is formed by light that has gone through the lens twice. (b) Is the image upright or inverted? (c) Determine the magnitude of the overall magnification.
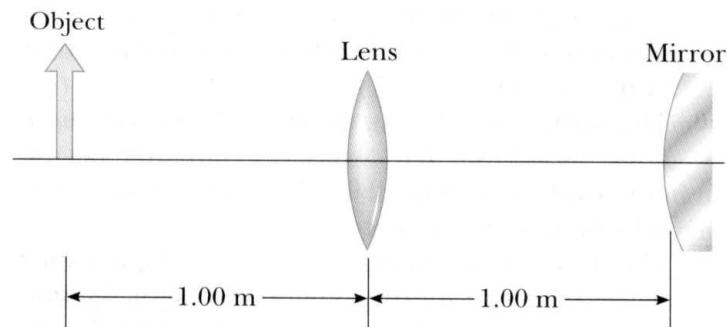


**Extra problems I recommend you work (not to be turned in):**

• (Paper only.) If an object is a distance $p$ in front of a converging lens ($p$ could be negative if the "object" is an image formed from another lens), under what conditions will a real image be created? Under what conditions will a virtual image be created? Under what conditions will the image be inverted? What about a diverging lens?

- A cube which is 1 cm in length is placed 15 cm from a lens with a focal length of 2 cm. Draw the 3D image of what the cube will look like. Hint: Figure out where the images of the front and back surfaces will form, then connect them.

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30-1. A near-sighted woman cannot focus on any object farther away than a distance $x = $ [01] ——————— cm. Find the focal length of the lens which will correct her vision. (If an object is very far away, the lens should produce an image a distance $x$ in front of her eyes so that she can focus on it.) Neglect the distance between the lens and the eye.

30-2. A farsighted man cannot focus on anything closer than [02] ——————— m away from him. If he wants to be able to hold his book at 25 cm, find the focal length of the lens which will correct his vision. Neglect the distance between the lens and the eye.

30-3. Consider a camera with film in it. The focal length of the lens is [03] ——————— mm. (a) If we want to take a picture of some distant object, where should we put the film? (Consider the distance to the object to be infinite.) (b) If we next want to take a picture of an object a distance $x = $ [04] ——————— m away, by how much should we change the distance between the film and the lens? (c) Suppose we didn't change the position of the film, but left it in the position for taking a picture of a distant object, as in part (a). A "point" of light a distance $x$ away would not be "focused" properly on the film but instead would produce a "dot" on the film. Find the diameter of the dot. The diameter of the lens is 1.0 cm. (d) If we cover up part of the lens so that light can only enter through a hole 3 mm in diameter, find the diameter of the dot in part (c).

30-4. You shine red light on a penny. You place a lens exactly 1 meter from the penny, and a red image of the penny forms at a distance of [05] ——————— cm from the lens, on the opposite side of the lens. You then shine blue light on the penny. How far from the lens will the blue image form? The index of refraction for this particular glass is 1.500 for the red light and 1.530 for the blue light.

30-5. (Extra credit.) Let's take a look at spherical aberration. Imagine a plano-convex lens (meaning that one side is flat and one side is convex) made of a glass with an index of refraction of [06] ——————. The magnitude of the radius of curvature of the curved side is 30 cm. Two rays of light strike the flat side of the lens, both traveling parallel to the principle axis, as shown in the figure below. Suppose one beam hits the lens a distance of 0.5 cm from the principle axis, and the other a distance of 10 cm from the principle axis. (Note: the lower ray on the figure is NOT drawn at the right height.) After being bent by the lens, the two rays both cross the principle axis. If the lens were free of aberrations, they would cross the principle axis at the same point. But, in fact, they don't. What is the distance $\Delta x$ between the points where the two rays cross the principle axis? All you need is Snell's Law, and some geometry/trigonometry.
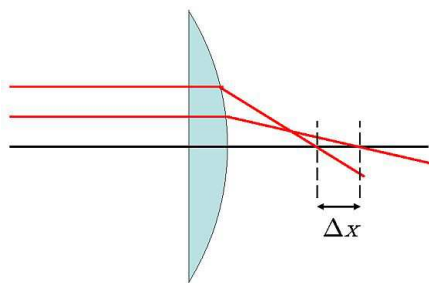


Hint: Here's a sketch for one of the rays, with the lens's radius of curvature expanded to a full circle, which should help you think about how to calculate the right distances.
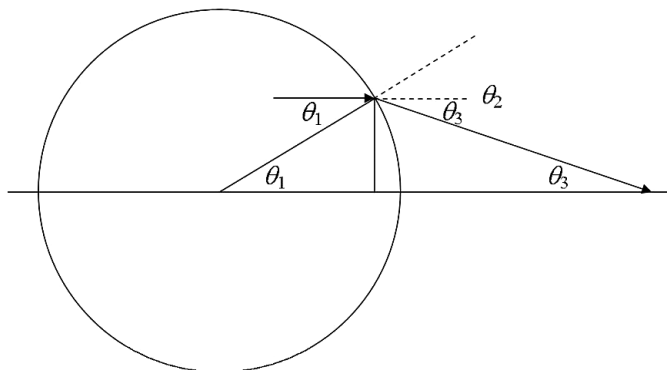


Final note: If you were to reverse the lens, so that the rays strike the curved side first, you would find $\Delta x$ to be smaller. That gives rise to the first half of this optics rule about positioning these common "plano-convex" lenses: "Parallel rays to curved, diverging rays to flat."

**Extra problems I recommend you work (not to be turned in):**

- You want to take a picture of an ant. You place your camera such that the film is 250 mm from the ant. The lens has a focal length of 50 mm. (a) Show that there are two possible positions for the lens which will produce a focused image of the ant on the film. Find $p$ and $q$ for both cases. (b) What is the magnification of the image for the case where $p > q$? (c) What is $M$ for the case where $q > p$? (Answers: 69.1 mm, 180.9 mm; $-0.38$, $-2.62$.)

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31-1. (a) I hold a flat mirror [01] ————— cm in front of my face. There is a freckle on my face 1 mm in diameter. Find the angular size of the freckle on the image of my face as viewed by my eye. (b) Repeat for a concave mirror which has a focal length of 39 cm. (c) What is the angular magnification of the concave mirror, as compared to the flat mirror?

31-2. Imagine that you are using a lens with a focal length of [02] ————— cm as a magnifying glass to look at (not cook!) an ant which is sitting on your finger. You put your eye up to the lens and adjust the position of the ant until the image of the ant is 25 cm from you. (a) What is the lateral magnification $M$ of the image? (b) What is the angular magnification $m$?

31-3. A hobby telescope has an objective lens with a focal length of [03] ————— cm and an eyepiece with a focal length of 8.2 mm. We view the planet Jupiter with this telescope. We do this at the time of year when we are closest to Jupiter. Data about the solar system can be found in the front inside cover of the textbook. (a) Find the diameter of the image of Jupiter produced by the objective lens. (b) Find the angular size of Jupiter as viewed through the eyepiece. (c) How far should I place a marble (1-cm diameter) from my eye to obtain the same angular size as in part (b)?

31-4. This is a common trick for expanding (or reducing) laser beams: two converging lenses are set up such that the first lens's right-hand focus is at the same point as the second lens's left-hand focus. (The beam is traveling left to right.) That forces the laser to emerge collimated from the second lens, but with a different beam diameter. (a) Draw a ray diagram for this situation, to show that if the laser beam is collimated going into the first lens, it really does emerge collimated from the second. (b) If the laser beam diameter before the first lens is [04] _____ mm $f_1 = 50$ mm, and $f_2 = $ [05] _____ mm, what is the laser beam diameter after the laser emerges from the second lens?

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32-1. Two loud speakers are 2.63 m apart. I am standing [01] _____ m from one of them and 3.58 m from the other. The two speakers are driven by a single oscillator. If the frequency of the oscillator is swept from 100 Hz to 1000 Hz, find the lowest frequency at which I will hear an enhancement of the sound intensity due to constructive interference of the waves from the two speakers. Use 343 m/s for the speed of sound. (DO NOT use the double-slit equation, $d \sin \theta = m\lambda$. This equation is only valid for observations far from the two slits, compared to the distance between the two slits.)

32-2. The figure shows an interference pattern from two slits. If the slits are 0.17 mm apart and the observed picture is seen on a screen [02] _____ m from the slits, find the wavelength of the light used. Assume that the photograph in the figure is life-size. To obtain an accurate value for the distance between fringes, measure the distance between the topmost and bottommost fringes and divide by 4. Note: Sometimes printers do not faithfully reproduce the actual size of a photograph. The box displayed below should be 5.0 cm wide. If not, then scale the size of the photograph accordingly.

| 5.0 cm |
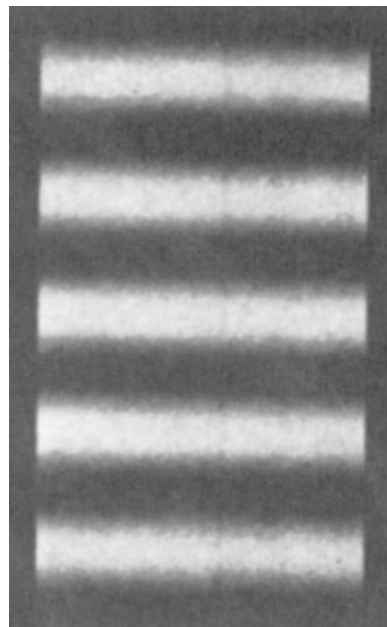


32-3. Two narrow slits separated by 0.85 mm are illuminated with [03] _____-nm light. The peak intensity on a screen 2.80 m away is 0.1 W/cm². What is the intensity at a distance 2.50 mm from the center of the central peak?

32-4. (Paper only.) (a) Coherent monochromatic light with a wavelength $\lambda$ passes through three parallel slits spaced evenly from each other with a distance $d$. Use phasor addition/complex numbers to show that the intensity in the interference pattern at an angle $\theta$ is given by:

$$I(\theta) = I_0 \left( 1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right) \right)^2$$

where $I_0$ is a constant. Hints: At a given point in the pattern on the screen, the amplitude of the oscillating electric field will be a sum of the electric fields from each slit. Those will only vary by their phase, which phase difference arises from a difference in path length: $\phi = (\Delta PL/\lambda) \times 2\pi$. Using complex numbers, you can easily include phase shifts by terms such as $E_0 e^{i\phi}$. Define your phase shifts relative to the middle slit, and remember that $\cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}$. The overall intensity is proportional to the magnitude of the electric field, squared.

(b) Using a program such as Mathematica, plot the intensity pattern for angles between $-30°$ and $30°$, with $I_0 = 1$ and $d/\lambda = 5$. Tip: if you use Mathematica, you cannot use a capital I as the function name; that's a reserved symbol for the imaginary constant.

(c) You should observe that this pattern has two different types of "bright" fringes. The higher intensity maxima are known as primary maxima, and the lower intensity ones are known as secondary maxima. For the general case, what is the ratio of the intensity in a primary maximum to the intensity in a secondary maximum? Assume that $\lambda < d$. Hint: The answer is the same for the two types of maxima in the graph of $f(x) = (1 + 2\cos x)^2$.

**Extra problems I recommend you work (not to be turned in):**

- We pass a laser beam through a double slit. On a screen 15 m away, we observe a series of bright lines which are 3.4 mm apart. The wavelength of the laser light is 633 nm. What is the distance between the two slits? (Answer: 2.79 mm.)

- Two coherent sources of light are both shining down onto a piece of paper. If I block source "A" the electric field at a given point on the paper is given by the equation $E_B = 1.5E_0 \sin(\omega t + 64°)$, where $E_0$ is a constant. If I block source "B", the electric field at the same point on the paper is given by the equation $E_A = 2.5E_0 \sin(\omega t - 14°)$. When both sources are unblocked, the electric field is given by $E_{both} = A \times E_0 \sin(\omega t + \phi)$. Find (a) $A$ and (b) $\phi$ (in degrees). (Answers: 3.17, 13.56°.)

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33-1. A pool of water is covered with a film of oil which is [01] ———————— nm thick. For what wavelength of visible light (in air) will the reflected light constructively interfere? The index of refraction of the oil is 1.65. Visible light (in air) has wavelengths between 430 nm (blue) and 770 nm (red) in air. Assume that the incident light is normal to the surface of the oil.

33-2. Solar cells are often coated with a transparent thin film of silicon monoxide (SiO, $n = 1.85$) to minimize reflective losses from the surface. Suppose that a silicon solar cell ($n = 3.5$) is coated with a thin film of SiO for this purpose. If the thickness of the coating is [02] ———————— nm, find the maximum wavelength of ultraviolet light (in air) for which the reflection will be maximized instead of minimized. (The wavelength of ultraviolet light is shorter than that of visible light.)

33-3. A wedged air space is created between two plates of glass, and a sample of hair is inserted at one edge. The plates are illuminated from above with [03] ————————-nm light and *reflections* from the air wedge are observed.
(a) Will the fringe near where the plates touch be bright or dark?
(b) The left end is dark. If a total of 24 *other* dark fringes are observed (including the one in part (a) if it is dark), what is the diameter of the hair?
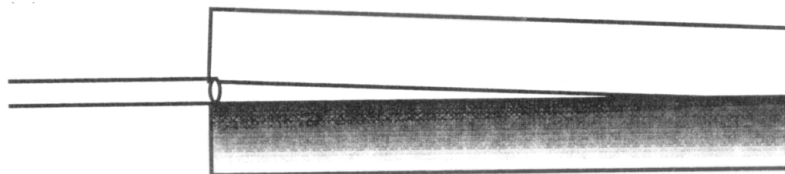


33-4. In a Michelson interferometer, 731 fringes cross the field of view when one of the mirrors is very precisely moved through [04] ———————— mm. What was the wavelength of the light?

34-1. The second-order bright fringe in a single-slit diffraction pattern is [01] _____ mm from the center of the central maximum. The screen is 81 cm from a slit of width 0.78 mm. Assuming that the incident light is monochromatic, calculate the light's wavelength. Hint: Although this isn't quite true for the "sinc" function, you should assume that the second order maximum is located exactly halfway in between the second and third order minima.

34-2. Babinet's principle says that the diffraction pattern from an opaque object blocking the light will produce the same diffraction pattern as a slit of the same size/shape allowing the light to pass. Only the overall intensity of the pattern will be different. This can be seen by noting that the resulting light field from an opaque object is just the field made by the laser beam by itself, minus the field that would be produced by a slit in the shape of the opaque object: $E = E_{\text{laser beam}} - E_{\text{slit}}$. In places where the undisturbed beam would not have reached, $E_{\text{laser beam}} = 0$. Therefore $E = -E_{\text{slit}}$ over most of the diffraction pattern. Of course your eye can't see the sign of the electric field (you only see the intensity), so you see essentially the same pattern that would be present if you used a slit rather than an opaque object.

Suppose you are working in a forensics lab, and you have a human hair whose diameter you need to measure. You decide to use Babinet's principle: you shine a HeNe laser ($\lambda = 633$ nm) at the hair and see a single-slit diffraction pattern. You look at this pattern on a screen which is 1.6 m away from the hair. The width of the central peak on the screen turns out to be [02] _____ mm, measured from the dark spot just to the left of the peak to the dark spot just to the right of the peak. What is the diameter of the hair?

34-3. (Paper only.) In the three-slit problem from a previous assignment, you found the total electric field in the diffraction pattern at the screen by adding up the phase shifts from three separate slits. This gave rise to the formula for the three slit pattern. The same technique can be used to find the formula from a single slit having finite width. According to Huygen's principle, a single slit behaves like an infinite number of point sources, spaced infinitely closely together. Thus, instead of adding three separate phase factors, like you did with three infinitely narrow slits, to solve the single finite width slit problem you must add an infinite number of phase factors: you must integrate.

(a) In the figure below, the slit goes from $y = -a/2$ to $y = +a/2$. Determine the phase shift of the light coming from an arbitrary $y$, relative to the light coming from $y = 0$ (i.e., the difference in phase of the two dashed lines). Show that this phase shift is equal to $\frac{2\pi}{\lambda} y \sin\theta$. ($\theta$ is the angle from the slit to the screen, which is essentially the same for the two dashed lines because the slit is actually much smaller than indicated in the figure.)
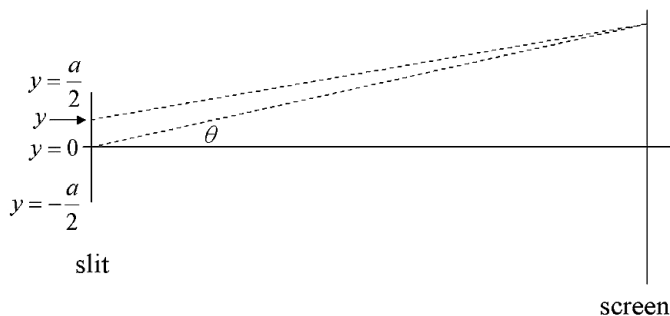


(b) Add up an infinite number of phase shifts (i.e., integrate), for points ranging from $y = -a/2$ to $y = a/2$. Hint: Here's a hand getting the integral set up. (1) Start off like you did for the three slit problem: $E_{tot}$ is a sum of $E_0 e^{i\phi}$ terms, with the various phase shifts depending on the $\Delta PL$ in the same fashion as that problem. (2) Convert the sum into an integral like this:

$$E_{tot} = \text{constant} \times \int_{-a/2}^{a/2} e^{i\phi} dy$$

where $\phi$ is the function of $y$ that you determined for part (a).

I broke a calculus rule by arbitrarily inserting a $dy$ before I integrated, but I did this because I knew I would somehow have to integrate $y$ from $-a/2$ to $a/2$. In a more complete analysis like maybe some of you will do in Phys 471, that turns out to be exactly the right thing to do. That does mean, however, that the constant in front of the integral no longer has units of electric field.

(c) Show that your answer can be written as proportional to $a\,\text{sinc}(x)$, where $x = \frac{\pi a \sin\theta}{\lambda}$. The "sinc" function is probably not one you are familiar with yet. It is defined as: $\text{sinc}\, x = \frac{\sin x}{x}$. Hint: Recall that $\sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$.

(d) The intensity pattern, being the square of the electric field pattern, is thus

$$I(\theta) = I_0 \left( \text{sinc}\left( \frac{\pi a \sin\theta}{\lambda} \right) \right)^2.$$

Using a program such as Mathematica, plot the intensity pattern for angles between $-30°$ and $30°$, with $I_0 = 1$ and $a/\lambda = 10$. Set your vertical scale to be from 0 to 1. Note: Sinc[x] is a built-in function in Mathematica, just like Sin[x] or Cos[x].

(e) Use the intensity pattern formula you just derived to prove the formula given in the book for the angle of the $m$th minimum in a single slit pattern:

$$\sin \theta_{\text{dark}} = \frac{m\lambda}{a}$$

34-4. (Paper only.) Below is a photograph of the interference pattern (intensity) on a screen for a particular two-slit experiment, and a plot of what the detector measured as it scrolled across the pattern. The $x$ axis is in centimeters, the $y$ axis has arbitrary units. A 633 nm laser was used. The intensity pattern is simply the two-slit (infinitely narrow) pattern, times a single finite-width slit pattern.
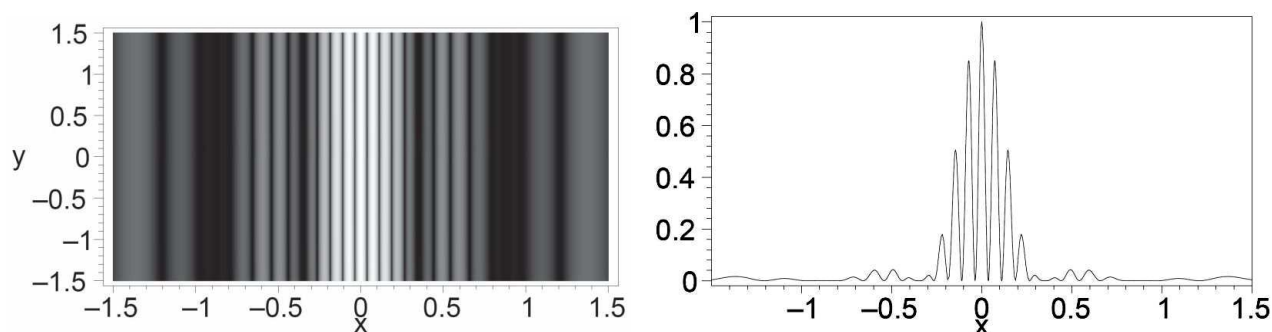


If the screen/detector was positioned 0.75 m away from the slits, (a) what was the separation between slits, and (b) what was the width of each slit? Hint: You will have to read some quantities off of the graph. The grader will allow some margin of error when judging whether your answers are correct or not, but be as accurate as you can.

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35-1. On the night of April 18, 1775, a signal was sent from the Old North Church steeple to Paul Revere who was 1.8 miles away: "One if by land, two if by sea." If in the dark, Paul's pupils had [01] _____-mm diameters, what is the minimum possible separation between the two lanterns that would allow him to correctly interpret the signal? Assume that the predominant wavelength of the lanterns was 580 nm.

NOTE: Although the wavelength is modified by the refractive index within the eye, the angles between incident rays are also modified by a similar amount. The two effects cancel each other, so you need not worry about it.

35-2. (a) Two stars have an angular separation of [02] ————— $\mu$rad as viewed from Earth. Assuming that most of the light from the star is yellow with a wavelength around 580 nm, how big must the diameter of a telescope be in order to see that there are two stars, not one? (b) If my eye has a pupil which is 4 mm in diameter, what must the angular magnification (magnitude) of the telescope be, for me to be able to see that there are two stars rather than one?

35-3. (a) Grote Reber, a pioneer in radio astronomy, constructed a radio telescope with a 10 m diameter receiving dish. What was the telescope's angular resolution when observing [03] ————— m radio waves? (b) The Hubble telescope has a primary objective with approximately 1 m diameter. What is the angular resolution when observing visible light with wavelength [04] ————— nm?

35-4. A diffraction grating contains 15,000 lines/inch. We pass a laser beam through the grating. The wavelength of the laser is 633 nm. On a screen [05] ————— m away, we observe spots of light. (a) How far from the central maximum ($m = 0$) is the first-order maximum ($m = 1$) observed? (b) How far from the central maximum ($m = 0$) is the second-order maximum ($m = 2$) observed? DO NOT use the "small-angle approximation," $y_{\text{bright}} = (\lambda L/d)m$. The angles are too large for $\sin \theta \approx \tan \theta$ to be a very good approximation.

35-5. In this problem, we will find the ultimate resolving power of a microscope. First of all, in order to obtain a large magnification, we want an objective lens with a very short focal length. Second, in order to obtain maximum resolution, we also want that lens to have as large a diameter as possible. These two requirements are conflicting, since a lens with a short focal length must have a small diameter. It is not practical for a lens to have a diameter much larger than the radius of curvature of its surfaces. Otherwise, the lens starts looking like a sphere. So, let us assume that the objective lens has a diameter $D$ equal to the radius of curvature of the two surfaces, like the lens in the figure to the right. (a) If the lens is made of glass with index of refraction [06] ―――――, find the focal length $f$ in terms of the diameter $D$ of the lens. (b) The distance between the sample to be observed and the objective lens is approximately equal to the focal length $f$. Find the distance between two points on the sample which can be barely resolved by the lens. Use the result from part (a) to eliminate $f$ from the expression. You should find that $D$ is also eliminated from the expression and that the answer is given entirely in terms of the wavelength $\lambda$ of the light. You may use the small angle approximation, $\sin\theta \approx \tan\theta \approx \theta$.

35-6. The diagram depicts a standard spectrometer setup. A lens (or concave mirror) following a slit creates collimated light that strikes the grating (usually a reflective grating rather than a transmission grating as shown). Suppose that you observe light from a sodium lamp which has two strong emission lines at $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm. Your grating has [07] ―――――― lines/mm.
(a) In the first diffraction order, what is the difference between the diffraction angles of the two wavelengths (ignore the second lens)? Don't use the small angle approximation.
(b) To spatially separate the two wavelengths, it is necessary to let the light travel to a faraway screen. Because this can require very large distances, it is convenient to image what would have appeared on the faraway screen to a closer screen using a lens (or concave mirror). As proved in one of the additional problems below, the image appears at the focus of the lens and the angular separation is preserved, referenced from the position of the lens. If the final lens has a 30 cm focal length, how far apart on the detector screen are the two sodium wavelengths?

(c) As mentioned in the 6th edition of Serway and Jewett (but not in subsequent editions), gratings are characterized by their resolving power $R$, defined as their ability to distinguish between two nearly equal wavelengths: $R = \lambda_{ave}/\Delta\lambda$ ($\Delta\lambda$ is how close the wavelengths can be to each other without "blurring" together). As more and more grating lines contribute towards the overall interference pattern, the diffraction spots sharpen and the resolving power increases. By calculating the interference pattern for a very large number of contributing slits, like we did in a previous homework problem for three slits, a simple relationship can be derived: $R = Nm$, where $N$ is the number of slits being illuminated, and $m$ is the order of the diffraction spot being used. In order for the two sodium peaks to be cleanly separated from each other in the first diffraction order, what is the minimum number of slits that must be illuminated?
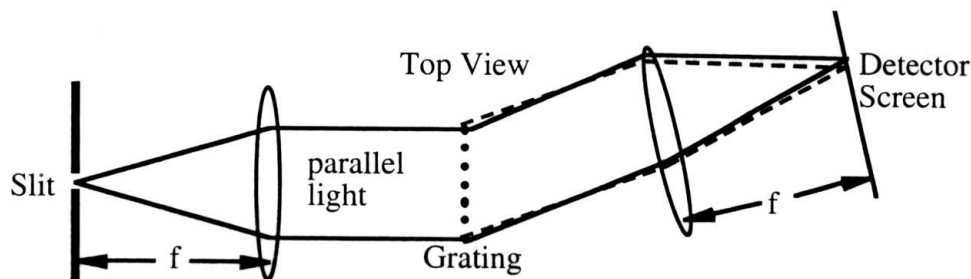


35-7. Monochromatic x-rays ($\lambda = $ [08] _____ nm) are incident on a potassium chloride (KCl) crystal surface. The spacing between planes of atoms in KCl for this orientation is 0.314 nm. At what angle (relative to the surface) should the beam be directed for a second-order maximum to be observed?

**Extra problems I recommend you work (not to be turned in):**

- A hobby telescope uses a concave mirror with a diameter of 12.4 cm. Find the distance between two points on the moon that can be resolved by this telescope. Use 550 nm for the wavelength of visible light. (Answer: 2078 m.)

- A television screen generates images which are composed of little red, green, and blue dots. Far from the screen, the dots blend together. (a) If there are 28 dots per cm, how close would you have to put your eye to the screen in order to see the individual dots? (b) If you can't focus on anything closer than 25 cm from your eye, how close must the dots be such that you cannot possibly distinguish them from each other? Use a wavelength of 520 nm, and assume that your pupil has a diameter of 4 mm. (Answers: 2.252 m; 39.65 $\mu$m, which is 252 dots/cm.)

- Coherent light with wavelength 500 nm passes through a round hole and propagates to a faraway screen. (a) Draw a picture of the setup and resulting pattern. (b) If a converging lens is placed close to the hole (after the hole), where will the diffraction pattern appear? Hint: Use the image from the hole (at infinity, or close to it) as the object for the lens, then find the image of the lens. (c) Show that the new diffraction pattern is just the same as the old pattern, but with distances scaled down by $f/L$, where $L$ was the original distance to the screen where the pattern was viewed. Hint: find the magnification. (d) Show that the angular separation between features on the new diffraction pattern (angles measured relative to the center of the lens) is the same as the angular separation was on the previous diffraction pattern (angles measured relative to the center of the hole). Hint: think similar triangles.

- A HeNe laser as a wavelength of 633 nm. If the collimated beam has a diameter of 0.50 cm, estimate the radius of the beam after it travels 10 km. (Answer: 1.545 m.)

- In a sodium chloride crystal, NaCl (the kind of salt we put on food), the sodium ions form a cubic lattice. The lattice constant $a$ (the distance by which each sodium ion is separated from its nearest neighbors) is 0.565 nm at room temperature. You send a beam of x-rays at a crystal of NaCl and find that the $m = 1$ diffraction order off of the Bragg planes which are spaced by $a$ is traveling in a direction which is 20° from the direction of the incident beam of x-rays. You then shoot the same beam of x-rays at a crystal of potassium chloride, KCl, which has a similar structure to NaCl. You find that this particular diffraction order now makes an angle of 17.9° with respect to the incident beam (as shown below). (a) What is the wavelength of the x-rays? (b) What is the lattice constant for the KCl crystal? (Answers: 0.386 nm, 0.629 nm.)

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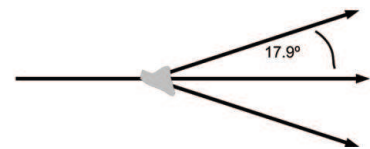
36-1. (Paper only.) A particular transverse traveling wave has the form:

$$\vec{y}(x,t) = A\hat{x}\cos\left(k\left(\frac{y+2z}{\sqrt{5}}\right) - \omega t\right)$$

In terms of the given quantities $A$, $k$, and $\omega$:

(a) What is the amplitude of the wave?

(b) What is the wavelength?

(c) What is the period?

(d) What is the direction and magnitude of the velocity?

(e) Between what two directions does the wave oscillate?

36-2. (Paper only.) Write down the proper mathematical expression for a plane wave traveling in the $\frac{\hat{x}+5\hat{y}}{\sqrt{26}}$ direction, oscillating at a frequency of 10000 Hz, having a wavelength of 5 meters, and an amplitude of 3. It is a transverse wave, with its amplitude oscillating back and forth from the $\frac{-5\hat{x}+\hat{y}}{\sqrt{26}}$ direction to the $\frac{5\hat{x}-\hat{y}}{\sqrt{26}}$ direction. Don't worry about phase shifts.

**Extra problems I recommend you work (not to be turned in):**

• A drop of water falls into a perfectly calm pond generating ripples that travel out in circular rings. Prove that as each ring expands, the amplitude of the ring drops off as $1/\sqrt{r}$, where $r$ is the radius of the ring. Assume that no energy is lost, and that the waves propagate non-dispersively (i.e., the radial thickness of a ring doesn't change as the ring expands).

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37-1. A ball is thrown at [01] —————— m/s inside a boxcar moving along the tracks at 45.8 m/s. What is the speed of the ball relative to the ground if the ball is thrown (a) forward? (b) backward? (c) out the side door?

37-2. (Paper only.) A car of mass $m_1 = 1000$ kg is sliding without friction on ice at a velocity $u_1 = 20$ m/s when it strikes another car of mass $m_2 = 1200$ kg which was standing still. The two cars lock together and slide together without friction after the collision. (a) Use the principle of momentum conservation to find the velocity of the two cars after the collision. (b) An observer riding on a bicycle in the same direction as the cars watches the collision while traveling at a velocity $v = 10$ m/s. Find the initial and final velocities of the two cars as measured in the reference frame of the bicyclist. (c) Show that the velocities that you found in part (b) satisfy momentum conservation in the bicycle rider's reference frame.

37-3. (Paper only.) A 1 kg object ($m_1$) collides with a 2 kg object ($m_2$) on a frictionless surface. Before the collision, $m_1$ is traveling at 9 m/s to the right and $m_2$ is at rest with respect to the ground. The collision is *elastic* and $m_1$ bounces straight back to the left. (a) Figure out the final velocities of both masses after the collision. Hint: You have two unknowns and thus need two equations. One of the two equations comes from conservation of momentum. The other could come from conservation of energy, but in practice it's easier to use what I call the "velocity reversal equation" in its place; that's Serway equation 9.20 (8th edition) and was derived from conservation of energy. (b) A bicycle rider moving at 5 m/s to the right (relative to the ground) observes the collision. Show that both kinetic energy and momentum are also conserved in her frame of reference.

37-4. (Paper only.) An observer at rest with respect to the Earth finds that objects falling under gravity accelerate at a constant rate of 9.8 m/s². According to Galilean relativity: (a) Will an observer on a train moving at 10 m/s see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate? (b) Will an observer on a rocket moving vertically away from the surface of the earth at 10 m/s see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate? (c) Will an observer on a rocket accelerating vertically away from the surface of the earth at 10 m/s² see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate?

38-1. A jet plane is [01] —————— m long. How much shorter is the plane when it travels at the speed of sound, 343 m/s? You will find the following approximation useful: $\sqrt{1-x} \approx 1 - \frac{1}{2}x$ when $x \ll 1$.

38-2. How fast must a meter stick move for it to appear to be only [02] —————— cm long?

38-3. Muons are unstable subatomic particles, somewhat similar to electrons (but heavier). When produced in laboratories, they have a measured average lifetime of 2.2 $\mu$s before they decay. Muons also get produced by cosmic rays in the atmosphere above the Earth. Based on the 2.2 $\mu$s lifetime, they would be expected to decay long before they reach the surface of the Earth. However, they don't! This is one of the classic proofs of Einstein's theory of relativity: there is simply no other explanation for why muons created in the upper atmosphere reach the Earth's surface.
Suppose that a muon is created [03] —————— km above the Earth. What fraction of the speed of light must the muon be traveling if it is to reach the ground in its expected lifetime of 2.2 $\mu$s? Express your answer as a number times $c$. Give your answer to 6 significant figures. Hint: you can think about this either in the muon's frame of reference or in the Earth's frame. Both views yield the exact same answer.

38-4. Astronauts travel at $0.950c$ from Earth to a star which is [04] —————— ly (light years) away.
(a) How long does it take them to reach the star as observed by people on Earth? (Earthlings know how much time it takes for a signal to reach them from the star, and they *do not* include it as part of the travel time for the astronauts.)
(b) How long does the trip take from the perspective of the astronauts?
(c) How far apart are Earth and the star from the perspective of the astronauts as they travel?

38-5. (Paper only.) Just because I can't travel faster than the speed of light, it doesn't mean that I can't travel any further than about 100 light years from Earth before I die (where 100 years is about the longest someone might live). (a) Why not? (b) If I travel at speed $\beta$ $(= v/c)$, how far will I get in 100 years? You can leave your answer in terms of $\beta$ and/or $\gamma$ $(= 1/\sqrt{1 - \beta^2})$.

**Extra problems I recommend you work (not to be turned in):**

- A moving rod is observed to have a length of 2.00 m, and to be oriented at an angle of 30° with respect to the direction of motion (see figure). The rod has a speed of 0.995c. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame? (Answers: 17.4 m, 3.3°.)

  2.00 m

  30.0°

  Direction of motion
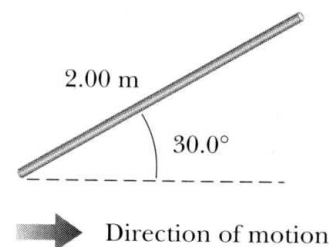
- At Stanford Linear Accelerator, Dr. Peatross's former Ph.D. advisor was involved in experiments where a high-intensity laser is aimed at electrons which approach almost straight-on with a velocity of $v = (1 - 3 \times 10^{-10})c$. In the lab frame, the time interval necessary for an electron to cross sequential wavecrests of the light wave is $T_{lab} = \lambda_{lab}/(c + v)$, as you would expect. (a) Show that *in the rest frame of the electron,* that time interval is

$$T_e = \frac{\lambda_{lab}}{c} \sqrt{\frac{1 - v/c}{1 + v/c}},$$

and hence the wavelength experienced by the electron is

$$\lambda_e = \lambda_{lab} \sqrt{\frac{1 - v/c}{1 + v/c}}.$$

This formula describes the Doppler shift of light, where $v$ is the relative motion between the source and observer. Hint: $1 - v^2/c^2 = (1 + v/c)(1 - v/c)$.

(b) If the laser's wavelength in the lab frame is 800 nm, what is the wavelength in the rest frame of the electrons?

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39-1. You run a red light. You are pulled over. You explain to the traffic officer that you didn't know that the light was red—because you were moving, the red light ([01] —————— nm) was Doppler-shifted to appear green ([02] —————— nm). If you were telling the truth, how fast were you going?

39-2. Ever since the Big Bang, different parts of the universe have been flying away from each other. Astronomers can figure out how fast a star is moving away from us by looking at atomic emission lines and measuring how much the lines have been Doppler-shifted from the wavelength of the lines which we measure in experiments on Earth. They often characterize the amount of Doppler shift using a parameter $z$, frequently just called the "redshift". It is defined as

$$z = \frac{\lambda_m - \lambda_0}{\lambda_0}$$

where $\lambda_m$ is the wavelength they measure for the light coming from the star, and $\lambda_0$ is the wavelength measured in an experiment in which the atoms emitting the light are at rest. (a) Use the equation for the Doppler shift for light to show that the parameter $z$ and the speed of the astronomical object $v$ are related by:

$$z = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1$$

Hint: recall that the astronomical object is moving *away* from the Earth.

(b) The largest red shifts measured are those of quasars. Quasars are the most luminous objects in the universe, so they are the farthest-away objects that can be seen. Hence they are the fastest-moving objects that can be seen. Imagine that you look at a particular quasar and measure its redshift to be [03] _____. How fast is that quasar moving relative to the Earth? (According to Wikipedia, the record for the largest redshift as of Dec 2007 was a quasar with $z = 6.43$. Its speed is $0.907c$.)

(c) Edwin Hubble discovered that (as one would expect in an explosion), the velocity at which various galaxies are receding from the Earth—as measured by their redshift—is very consistently proportional to their distance from us. Specifically, Hubble's Law says $v = H_0 d$. $H_0$ is the "Hubble constant", which according to the best current measurements is about 70.8 km/s per Mpc. (1 Mpc, a "megaparsec", is $3.2616 \times 10^6$ light years.) According to Hubble's Law, how far away from us is the quasar? (Side note: as I understand it, the distance $d$ in Hubble's law corresponds to the distance the light actually traveled. That is larger than the distance the quasar was from us when the light was emitted, because the space between the Earth and the quasar has been expanding the whole time the light has been traveling. However, it is smaller than the distance the

quasar is from us now, because—due to this same expansion—distances covered by the light ray at the start of its path are now larger than they were then.)

(d) How long did it take the light from your quasar to reach us? For the record-holding quasar mentioned above, the answer is about 13.3 billion years. That sets a lower limit on how long it has been since the Big Bang. (The best current estimates for the Big Bang are that it occurred about 13.7 billion years ago.)

39-3. A red light flashes at position $x_R = 3.31$ m and time $t_R = $ [04] —————— s, and a blue light flashes at $x_B = 5.15$ m and $t_B = 9.03 \times 10^{-9}$ s (all values are measured in the S reference frame). Reference frame S$'$ has its origin at the same point at S at $t = t' = 0$; frame S$'$ moves constantly to the right. Both flashes are observed to occur at the same place in S$'$. (a) Find the relative velocity between S and S$'$. (b) Find the location of the two flashes in frame S$'$. (c) At what time does the red flash occur in the S$'$ frame?

39-4. (Paper only.) Barn paradox: Suppose that Lee is carrying a 20 m long ladder (rest length) and running extremely fast towards a 10 m long barn (rest length). Cathy is watching the process and is stationary with respect to the barn. Because of length contraction, Cathy sees the ladder fit completely inside the barn. She flips a switch and "instantaneously" closes the front and back barn doors. Is the ladder completely inside the barn now? If the answer is yes, it seems paradoxical, because from Lee's point of view it is the barn that has shrunk because of length contraction. This is discussed in the textbook before the Lorentz transformation equations are given, where it is called the "pole in the barn paradox".

Like all good physics "paradoxes", there is no paradox when you look at things the right way. In this case, the solution to the paradox is that from Lee's point of view, the front and back doors of the barn do not close simultaneously and so the 20 m (to Lee) ladder is not trapped inside the really narrow (to Lee) barn. You will prove that in this problem.

(a) How fast must Lee be going so that his ladder is 10 m long to Cathy?

(b) Suppose Lee is going that speed. In Cathy's frame of reference, Lee's ladder can "fit" inside the barn at a particular instant in time, because both the ladder and the barn are 10 m long. Let's call that instant "t=0"; let's call the middle of the barn "x=0". Thus we can talk about two events in Cathy's frame of reference: event 1 = "front end of the ladder gets to the end of the barn," which is at $(ct, x) = (0, 5)$; event 2 = "back end of the ladder gets to the start of the barn," which is at $(0, -5)$. Determine the times and

places these two events occur, in Lee's frame of reference. You should find that event (2) occurs after event (1), so that while Cathy knows that Lee's ladder is completely inside the barn at a particular moment, to Lee himself, the ladder is not! Thus the paradox is resolved. How much later does event 2 occur than event 1 to Lee?

(c) Sketch Cathy's world line, Lee's world line, and the two events on a space-time diagram for (1) Cathy's frame of reference, and (2) for Lee's frame of reference. Also sketch the world lines of the right and left edges of the ladder.

**Extra problems I recommend you work (not to be turned in):**

- Read the short book *Mr. Tompkins in Wonderland*, by George Gamow (also sold under the name *Mr. Tompkins in Paperback*). You will enjoy it, and you will better understand relativity and quantum mechanics. Plus it's on the list of extra credit books in the syllabus! The HBLL library has several copies under the following call numbers: QC 71.S775 1999, QC 71.G25 1965, QC 173.5.G36x, and QC 6.G23 1940.

- Suppose we are on a space ship generating a beam of electrons. In our reference frame, the electrons are traveling in the $+x$ direction with a speed of $0.87c$. Our space ship passes by the earth. In the earth's reference frame, the space ship is traveling in the $+x$ direction with a speed of $0.70c$. Find the speed of the electrons in the earth's reference frame. (Answer: $0.9758c$.)

- Jimmy Neutron is returning from a trip to the center of the galaxy, traveling at a speed of $0.871c$ relative to the Earth. Carl, one of Jimmy's friends standing still on Earth, is monitoring Jimmy's trip. Right as Jimmy flies past the Earth Carl verifies that his watch is synchronized with Jimmy's. In both Carl's and Jimmy's frames the Earth is at a location $x = 0$, $y = 0$, and $z = 0$ at time $t' = t = 0$. Carl sets up his coordinate system such that Jimmy is moving in the $+x$ direction, and Jimmy sets his up such that his $y$ and $z$ directions are the same as Carl's and such that Carl is moving in the $-x$ direction. After Jimmy has passed Earth, Carl observes a supernova looking through a telescope. Taking into account the time that it took for light to reach him, he determines that the supernova occurred at a time $t = -1.45 \times 10^9$ seconds at a location of $x = 3.29 \times 10^{17}$ m, $y = 1.53 \times 10^{17}$ m, and $z = 1.69 \times 10^{17}$ m. (a) As measured in Carl's reference frame, how far is Jimmy from the supernova when it occurs? (b) As measured in Jimmy's reference frame, how far is Jimmy from the supernova when it occurs? (Answers: $7.434 \times 10^{17}$ m, $1.458 \times 10^{18}$ m.)

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40-1. If two objects are both traveling at [01] _____$c$ but in opposite directions, find the speed of one object in the reference frame of the other object.

40-2. (Paper only.) (Modified from Griffiths, *Introduction to Electrodynamics.*) A policeman ($v = \frac{1}{2}c$, relative to the ground) is chasing an outlaw ($v = \frac{3}{4}c$, relative to the ground). The policeman fires a bullet whose speed is $\frac{1}{3}c$ (relative to the policeman). Find the speed of each object: ground, policeman, outlaw, and bullet, in each of the four reference frames. Be careful with positives and negatives. Show that in each frame of reference the bullet does not catch up to the outlaw because its speed is less than the outlaw's. (That's good! If there were a reference frame where the bullet hit and killed the outlaw, relativity would be much more confusing than is already the case.) Present your results in this sort of summary table:

| Speed of → Relative to ↓ | Ground | Policeman | Outlaw | Bullet | Escape? |
|---|---|---|---|---|---|
| Ground | 0 | | | | |
| Policeman | | 0 | | | |
| Outlaw | | | 0 | | |
| Bullet | | | | 0 | |

40-3. (Paper only.) John, Lee, and Henry are in a train which is speeding past Cathy at $0.4c$. Lee and Henry are standing still, 10 m apart, but John is running forward in the train at $0.3c$ (relative to the train). Right as John passes Lee on the train, the two also pass Cathy outside the train. Call this instant $x = 0$ and $t = 0$. John passes Henry a short instant later. Make three accurate space-time diagrams: one from Cathy's point of view, one from Lee's, and one from John's. On each diagram draw the world lines of John, Lee, Henry, and Cathy. On each diagram accurately label event 1 (John passes Lee) and event 2 (John passes Henry).

40-4. A physics professor on the Earth gives an exam to her students, who are on a rocket ship traveling at speed [02] _____$c$ relative to the Earth. The moment the ship passes the professor, she signals the start of the exam. She wishes her students to have 50.0 min (rocket time) to complete the exam. At the appropriate time, she sends a light signal telling the students to stop. (a) Draw a space-time diagram from the perspective of the Earth, containing the world-lines of the professor, the students, and the light signal. Mark the exam start and stop events, along with the "professor sends signal" event. (b) Draw a similar space-time diagram from the perspective of the rocket. (c) How long did the professor wait (Earth time) before sending the light signal?

40-5. Suppose we are on our way to Proxima Centauri, which is 4.2 ly away (in the reference frame of the earth). We are in a space ship which is traveling with a speed of [03] _____$c$. When we are half-way there, we send a signal to both the earth and Proxima Centauri. The signals travel at the speed of light $c$. In the reference frame of the earth and Proxima Centauri, both signals travel 2.1 ly and thus arrive at their destinations at the same time.
(a) Draw two space-time diagrams of the situation, one from the Earth's point of view and one from our point of view. On each diagram include worldines for the Earth, Proxima Centauri's worldline, and the space ship. Also mark the "signals sent", "signal received by Earth", and "signal received by Proxima Centauri" events.
(b) Although the two events (the signal reaching earth and the signal reaching Proxima Centauri) are simultaneous in the Earth's reference frame, as should be obvious from your space-time diagrams they are not simultaneous in our reference frame. How much time elapses between these two events in our reference frame?

**Extra problems I recommend you work (not to be turned in):**

- Suppose our Sun is about to explode and we escape in a spaceship toward the star Tau Ceti, which is 12 light years away (not including Lorentz contraction). We travel at $v = 0.82c$. When we reach the midpoint of our journey, we see our Sun explode and, unfortunately, we see Tau Ceti explode as well (we observe the light arriving from each explosion). (a) Draw two space-time diagrams: one from the Sun's frame of refernece, and one from our frame of reference. In each diagram, include world lines for the Sun, Tau Ceti, our spaceship, and the light rays arriving from each explosion. Mark these three events on each diagram: Sun explodes, Tau Ceti explodes, and the instant we observe both explosions. (b) In the rest frame of the Sun (and Tau Ceti), did the explosions occur simultaneously? (c) In the spaceship frame of reference, did the explosions occur simultaneously? (d) In the spaceship frame of reference, how long before we saw the Sun explode did it actually explode? (e) In the spaceship frame of reference, how long before we saw Tau Ceta explode did it actually explode? (Answers to parts (d) and (e): 1.89 years, 19.08 years.)

- A deep space probe is launched from the Earth and passes a deep space station on its way into the unknown. The probe travels at a constant velocity of $0.811c$ relative to the station. It has an on-board atomic clock connected to a computer which is programmed to send a microwave signal back to the station exactly one year later (as measured in the probe's frame of reference). (a) Draw two space-time diagrams: one from the station's frame of refernece, and one from the probe's frame of reference. In each diagram, include world lines for the station and the probe, and the microwave signal sent by the probe. Mark these two events on each diagram: probe sends signal, and station receives signal. (b) From the reference frame of someone on the space station, how much time elapses from the time the probe passes by, to when the microwave signal arrives back at the station? (Answer to part (b): 3.10 years.)

- The Lorentz transformation equations transform a line in Cathy's space-time diagram to a line in Lee's space-time diagram. Prove this: given a line in Cathy's frame, $ct = mx + b$, determine its equation in Lee's frame. Lee is moving to Cathy's right, with speed $v$. Specifically, in terms of $\beta$ $(= v/c)$, $\gamma$ $(= 1/\sqrt{1 - \beta^2})$, $m$, and $b$, prove that the slope and the y-intercept of the line in Lee's frame of reference are:

$$m_{\text{Lee}} = \frac{m - \beta}{1 - m\beta}$$

$$b_{\text{Lee}} = \frac{b}{\gamma(1 - m\beta)}$$

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41-1. An electron has a kinetic energy [01] _____ times greater than its rest energy. Find (a) its total energy and (b) its speed.

41-2. Find the work required to increase the velocity of an electron by $0.010c$ (a) if its initial velocity is [02] _____ $c$ and (b) if its initial velocity is [03] _____ $c$. Remember that the work done is equal to the change of kinetic energy.

41-3. Suppose that a particle accelerator accelerates two electrons in opposite directions, such that they each have a speed of [04] _____ $c$ (in the laboratory frame of reference). (a) In the laboratory frame, what is the total kinetic energy before the collision? (It's just the sum of the two particle's kinetic energies.) (b) In the frame of reference of either particle, what is the total kinetic energy before the collision? (In Galilean relativity, the answer to part (b) would be only twice as large as the answer to part (a).)

41-4. A $^{57}$Fe nucleus at rest emits a [05] _____-keV photon. Use conservation of relativistic momentum to determine the kinetic energy of the recoiling nucleus. (Use $Mc^2 = 8.60 \times 10^{-9}$ J for the final state of the $^{57}$Fe nucleus.)

41-5. (Paper only.) A particle with a rest mass of $m$, traveling at a speed of $0.9600c$ to the right has a collision with a particle of mass $3m$ which is initially at rest. The larger mass moves away after the collision with a velocity of $0.8202c$ to the right. (a) Will the smaller mass be traveling to the left or to the right after the collision? (b) What is the final speed of the smaller mass (in terms of $c$)?

41-6. (Paper only.) A photon collides with an electron that is at rest. The photon imparts momentum to the electron, and a new photon recoils in the opposite direction (exactly backward relative to the incoming photon). Show that the momentum of the recoiling photon $p'_{\text{photon}}$ is related to the momentum of the incident photon $p_{\text{photon}}$ through

$$\frac{1}{p'_{\text{photon}}} - \frac{1}{p_{\text{photon}}} = \frac{2}{m_e c}.$$

This shift in photon momentum (and hence wavelength, as you will learn) is called the *Compton shift*.

Hint: Write down two equations: conservation of energy and conservation of momentum. One should contain a $\gamma$, the other a $v\gamma$, which equals $\beta\gamma c$. You have to figure out how to combine the two equations and eliminate $\beta$ and $\gamma$ at the same time. One way is to put $\beta$ in terms of $\gamma$, then solve one equation for $\gamma$ and substitute it into the other. It takes a bunch of algebra, but eventually gets you the right answer.

When I myself did the problem, I did it a slightly different way. My way also took a lot of algebra, so I'm not sure it was any easier. What I did was to use the conservation of energy equation to give me an expression for $p - p'$ and the conservation of momentum equation to give me an expression for $p + p'$. Then by adding the two equations I was able to solve for $p$ (in terms of $\gamma$ and $\beta$), and by subtracting the two equations I was able to solve for $p'$. I then took the inverse of $p$, the inverse of $p'$, and added them together to get the desired equation (that was the step that took all the algebra).

**Extra problems I recommend you work (not to be turned in):**

- (a) According to Newtonian physics (kinetic energy $= \frac{1}{2}mv^2$), how much work is required to accelerate an electron from rest to $0.99c$? (b) If we do that much work on an electron, what will its final speed actually be? (Answers: $4.012 \times 10^{-14}$ J, $0.7414c$.)

- In an accelerator, an electron experiences a constant electric field of $E = 1.00$ MV/m. What is its speed (a number times $c$) after 11.1 ns, assuming it starts from rest? Hint: The force on the electron is $F = qE$, where $q = 1.602 \times 10^{-19}$ C is the electron charge. Note that a megavolt per meter is the same as $10^6$ N/C. Eq. (39.20) is trivially integrated since $F$ is constant. (Answer: $0.9884c$.)

- Consider an electron which has been accelerated to a total energy of 30 GeV. (a) What fraction of the electron's energy is due to its rest mass (i.e., the ratio of rest energy to total energy)? (b) What is the momentum in kg·m/s and (c) in GeV/c? (d) The speed of the electron can be written in the form $v = (1 - \delta)c$. Find the value of $\delta$.
  Hint: $\sqrt{1 + x} \approx 1 + \frac{1}{2}x$ when $x$ is small. (Answers: $1.703 \times 10^{-5}$, $1.60 \times 10^{-17}$ kg·m/s, 30 GeV/c, $1.451 \times 10^{-10}$.)

- A 1 kg block of copper is heated from a temperature of 25°C to a temperature of 150°C. How much does the mass of the copper change? (Answer: $5.38 \times 10^{-13}$ kg.)

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Answers to Homework Problems, Physics 123, Fall Semester, 2011
Section 2, John Colton

1-1a. $1.00 \times 10^5$, $3.00 \times 10^5$ Pa
1-1b. $2.00 \times 10^5$, $4.00 \times 10^5$ Pa
1-2. 90, 140 cm$^2$
1-3. 150, 400 tons
1-4. 1.0, 20.0 lb
1-5. 10.0, 30.0 cm
1-7. 25.0, 50.0 kN
2-1. 40, 100 kg
2-2. 1.40, 2.30 cm
2-3b. 0.170, 0.320 m
2-4a. 500.0, 1100.0 N
2-4b. 500.0, 1100.0 N
2-4c. 5.0, 12.0 N
2-4d. 80.0, 100.0 N
2-5. 3.50, 6.00 g/cm$^3$
2-6. $-1.00$, $-2.00$ g
3-1. 10.0, 25.0 min
3-2a. 1000, $1900 \pm 10$ Pa
3-2b. 100, 300 lb
3-3. $1.00 \times 10^5$, $2.20 \times 10^5$ Pa
3-4a. 1.20, 2.50 cm
3-4b. 10.0, 20.0 m/s
4-1. $-200.0$, $-400.0$ °B
4-2b. 1.60010, 1.6050 cm
4-3. 500, 1100 gal
4-4a. $-0.20$, $-1.50$ mm
4-4c. 5.0, 50.0 s
4-5. 9.9920, 9.9970 cm
4-6. 100, 900 balloons
4-70. $1.5 \times 10^{12}$, $3.5 \times 10^{12}$
4-9. 100, 600 atm
5-1a. 0.60, 0.90 N
5-1b. 1.00, 1.40 Pa
5-2a. 60.00, 65.00 mi/h
5-2b. 60.00, 65.00 mi/h
5-3a. $2.50 \times 10^{23}$, $4.00 \times 10^{23}$
5-3b. $5.50 \times 10^{-21}$, $7.50 \times 10^{-21}$ J
5-3c. 1200, $1500 \pm 10$ m/s
5-3d. 1200, $1500 \pm 10$ m/s
5-4a. $4.00 \times 10^6$, $7.00 \times 10^6$ m
5-4b. 2.00, 4.00 h
6-1. 40, 70°C

6-2. 4.00, 9.00 km
6-3. 2.00, 5.00 h
6-5. 10.0, 20.0°C
6-6. 10.0, 20.0 g
7-1. 0.0600, 0.1100 J/s·m·°C
7-2. 10.0, 25.0 J/s
7-3. 30, 100°C
8-1a. $1.00 \times 10^5$, $1.30 \times 10^5$ Pa
8-1b. $-17.0$, $-21.0$ J
8-2a. $-600$, $-850$ J
8-2b. $-400$, $-650$ J
8-2c. $-200$, $-450$ J
8-4a. $-100$, $-140$ J
8-4b. 0 J
8-4c. 100, 140 J
9-1a. 3000, $7000 \pm 20$ J
9-1b. 3000, $7000 \pm 20$ J
9-2a. $3.00 \times 10^4$, $6.00 \times 10^4$ Pa
9-2b. 1000, $2000 \pm 10$ J
9-2c. 1000, $2000 \pm 10$ J
9-3a. 100, 500°C
9-3b. 10, 90 J
9-3c. 10, 90 J
9-4a. 5.0, 12.0 kJ
9-4b. $3.0 \times 10^6$, $9.5 \times 10^6$ Pa
9-4c. 700, $1200 \pm 10$ K
9-4d. 5.0, 12.0 kJ
10-1a. 600, 950 J
10-1b. 400, 650 J
10-2. 0.20, 0.50°C
10-3a. 10.0, 30.0 kJ
10-3b. $-10.0$, $-20.0$ kJ
10-3c. 4.0, 8.0 kJ
10-3d. 20.0, 30.0%
10-4a. $1.200 \times 10^5$, $1.500 \times 10^5$ J/s
10-4b. 25.0, 30.0 mi/gal
11-1a. 15.0, 30.0 J
11-1b. 100, 200 J
11-2. 1900, $2900 \pm 10$ J
11-3. 2000, $3000 \pm 10$ W
11-4a. 7.00, 9.99
11-4b. 30.0, 50.0 W

11-4c. 20.0, 40.0 dollars
12-1a. 3.00, 7.00 J/K
12-1b. 3.00, 7.00 J/K
12-2a. 6.0, 11.0 J/K
12-2b. 6.0, 11.0 J/K
12-2c. 6.0, 11.0 J/K
12-3. 1.5, 3.5 J/K
12-4. 5.00, 8.00 J/K
13-1a. $1.5 \times 10^{27}$, $3.0 \times 10^{27}$
13-1b. $4.5 \times 10^4$, $9.5 \times 10^4$ J/K
14-1a. 210, 340 m
14-1b. 2.80, 3.60 m
14-2a. 10, 30 cm
14-2b. 1.0, 3.0 $s^{-1}$
14-2c. 0.010, 0.025 $cm^{-1}$
14-2d. 3.0, 4.0 rad
14-3a. 0.80, 1.30 Hz
14-3b. 5.00, 8.00 rad/s
14-3c. 1.00, 2.00 m
14-3d. 3.00, 7.00 rad/m
14-3e. 0.50, 1.50 cm
14-3f. 0.50, 3.00 m/s
15-1. 100, 300 N
15-2. 25.0, 55.0 J
16-1a. 0.00, 9.99
16-1b. 0.00, 1.60 rad
16-1c. 0.00, 9.99
16-1d. 0.00, 9.99
17-1. 1.00, 9.99 %
17-2a. 0.60, 0.90
17-2b. 1.00, 1.50
17-2c. 0.950, 0.999
17-3. 0.300, 0.600
18-1a. $1.50 \times 10^{-4}$, $1.90 \times 10^{-4}$ W/m$^2$
18-1b. 82.0, 84.0 dB
18-2a. 5.0, 70.0 mW/m$^2$
18-2b. 95.0, 110.0 dB
18-2c. 0.50, 3.00 m
18-3. 0.0100, 0.0600 J
19-1. 42.20, 42.70 kHz
19-2a. 320, 380 Hz
19-2b. 460, 520 Hz
19-3. 20.0, 35.0 m/s
19-4b. $1.00 \times 10^8$, $2.00 \times 10^8$ m/s
19-5. 1.3, 3.0

19-6. 1.20, 2.00 s
19-7a. 0.10, 0.60 m
19-7b. 0.50, 1.50 m
20-1. 2.0, 7.0 g/m
20-2a. 100, 200 lb
20-2b. 10.0, 20.0 tons
20-3a. 0.400, 0.600 m
20-3b. 300, 450 Hz
20-4a. 23.00, 25.00 cm
20-4b. 0.30, 0.70 cm
20-4c. 365.0, 371.0 Hz
21-1. 9.0, 17.0 Hz
21-2a. 525.0, 529.0 Hz
21-2b. $-1.0$, $-2.0$%
21-3. 2.0, 6.0 Hz
21-4. 1.0, 4.0 cm
21-5a. 10, 25
21-5b. 300, 350 Hz
21-5c. 1, 10
21-5d. 1450, 1490 Hz
21-5e. 15, 25
25-1a. 34.0, 38.0°
25-1b. 60.0, 70.0°
25-2a. 49.00, 50.00°
25-2b. 49.00, 51.00°
25-2c. 48.00, 50.00°
26-1b. 1.00, 2.00 m
26-2. 25.0, 85.0 degrees
27-1. 30, 50°
27-2a. 0.00, 6.00 W/m$^2$
27-2b. 0.00, 6.00 W/m$^2$
27-2c. 0.00, 6.00 W/m$^2$
27-2d. 0.00, 6.00 W/m$^2$
27-3b. 36.00, 38.00 degrees
28-1a. 3.70, 4.30 cm
28-1b. $-0.20$, $-0.50$
28-1c. $-0.50$, $-0.90$ cm
28-2a. $-2.0$, $-9.9$ cm
28-2b. 1.0, 5.0
28-2c. 1.0, 5.0 cm
28-3a. $-1.50$, $-1.80$ cm
28-3b. 0.140, 0.200
28-3c. 0.50, 0.99 cm
28-4a. 25.0, 35.0 cm
28-4c. $-25.0$, $-35.0$ cm

29-1. 50, 80 cm
29-2a. 5.00, 6.00 cm
29-3a. 4.00, 9.50 cm
29-3b. $-1.00$, $-4.00$
29-3c. $-1.00$, $-4.00$ cm
29-4a. $-3.0$, $-9.0$ cm
29-4b. 1.5, 2.5
29-4c. 1.5, 2.5 cm
29-5a. $-2.50$, $-3.50$ cm
29-5b. 0.30, 0.50
29-5c. 1.50, 2.50 cm
29-6a. 10.0, 20.0 cm
29-7a. $-20.0$, $-30.0$ cm
29-7b. 2.0, 4.0
29-8a. 110, 220 cm
29-8c. 0.800, 0.900
30-1. $-40$, $-60$ cm
30-2. 25.0, 35.0 cm
30-3a. 40.0, 70.0 mm
30-3b. 0.10, 0.70 mm
30-3c. 0.040, 0.099 mm
30-3d. 0.010, 0.040 mm
30-4. 10.0, 50.0 cm
30-5. 7.00, 9.00 cm
31-1a. 0.100, 0.150°
31-1b. 0.150, 0.200°
31-1c. 1.30, 1.60
31-2a. 2.00, 4.00
31-2b. 2.00, 4.00
31-3a. 0.150, 0.250 mm
31-3b. 1.00, 1.60°
31-3c. 30.0, 60.0 cm
31-4b. 0.2, 5.0 mm
32-1. 160, 300 Hz
32-2. 400, $800 \pm 10$ nm
32-3. 0.000, 0.100 W/cm$^2$
33-1. 480, 660 nm
33-2. 200, 400 nm
33-3b. 6.0, 9.0 $\mu$m
33-4. 400, 999 nm
34-1. 400, $800 \pm 10$ nm
34-2. 50.0, 99.9 $\mu$m
35-1. 0.1, 1.0 m
35-2a. 5.0, 25.0 cm
35-2b. 10.0, 60.0

35-3a. 5, 30°
35-3b. $3.0 \times 10^{-5}$, $5.0 \times 10^{-5}$ degrees
35-4a. 1.00, 1.50 m
35-4b. 2.00, 4.00 m
35-5a. 0.80, 1.30 $D$
35-5b. 1.00, 1.60 $\lambda$
35-6a. 0.040, 0.120°
35-6b. 200, $600 \pm 10$ $\mu$m
35-6c. 900, 1100 slits
35-70. 15.0, 45.0°
37-1a. 50.0, 80.0 m/s
37-1b. 10.0, 40.0 m/s
37-1c. 40.0, 60.0 m/s
38-1. $6.0 \times 10^{-12}$, $9.9 \times 10^{-12}$ m
38-2. 0.999880, 0.999950$c$
38-3. 0.999850$c$, 0.999950$c$
38-4a. 6.00, 9.50 y
38-4b. 1.50, 3.50 y
38-4c. 1.50, 3.50 ly
39-1. 0.150, 0.300 c
39-2b. $2.50 \times 10^8$, $3.00 \times 10^8$ m/s
39-2c. $1.20 \times 10^{10}$, $1.35 \times 10^{10}$ light years
39-2d. $1.20 \times 10^{10}$, $1.35 \times 10^{10}$ y
39-3a. $2.20 \times 10^8$, $2.70 \times 10^8$ m/s
39-3b. 4.70, 5.70 m
39-3c. $-1.10 \times 10^{-8}$, $-1.60 \times 10^{-8}$ s
40-1. 0.65, 0.95$c$
40-4c. 14.0, 19.0 min
40-5b. 40.0, 70.0 years
41-1a. 2.00, 3.50 MeV
41-1b. 0.960, 0.990$c$
41-2a. 0.50, 0.99 keV
41-2b. 50, 99 keV
41-3a. $1.00 \times 10^{-12}$, $4.00 \times 10^{-12}$ J
41-3b. $1.00 \times 10^{-11}$, $9.00 \times 10^{-11}$ J
41-4. $1.50 \times 10^{-3}$, $4.00 \times 10^{-3}$ eV

Physics 123                                    Identification Number _____
Lab #1
Pressure in a Fluid

In this lab you will measure the density of an unknown liquid. You do this by forcing the
liquid up a tube using a known amount of pressure (see figure).

Pressurize the bottle of liquid by squeezing the hand pump repeatedly. The liquid should be
forced up the tube. Be sure that the silver air release value is closed (twist it clockwise). Increase
the pressure until the level of the liquid in the tube is almost 2 m above the floor. If you
overshoot 2 m, you may lower the level of the liquid by opening the air release valve (twist it
counter-clockwise).

Using the 2-meter stick, measure $h_1$ and $h_2$ (relative to the bottom of the bottle) and calculate
$\Delta h = h_2 - h_1$. Record the results below. Record the pressure measured by the gauge. (Note that
this is the pressure $P - P_0$ relative to the atmospheric pressure $P_0$. Also note that the units of
pressure measured by the gauge is oz/in$^2$. 16 oz = 1 lb.) Using $P = P_0 + \rho g h$, calculate the density
$\rho$ of the liquid and record the result below. Your result should be accurate to the nearest
0.01 g/cm$^3$. Please release the air pressure when you are finished.

$h_1 = $ _____

$h_2 = $ _____

$h = $ _____

$P - P_0 = $ _____

$\rho = $ _____
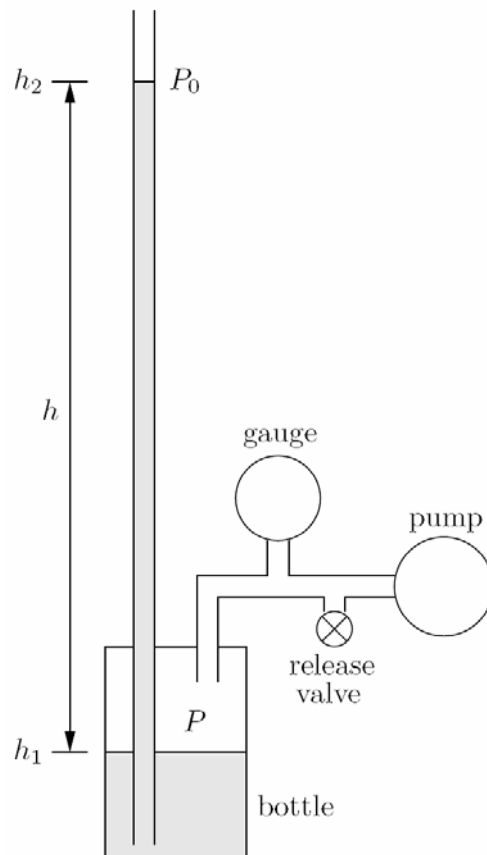
Physics 123                                    Identification Number _____
Lab #2
Heat Capacity of a Solid

In this lab, you will measure the specific heat of aluminum. A
strap is wound around an aluminum cylinder of mass $m = 216$ g
and radius $r = 1.00$ inch. One end of the strap is attached to a
weight of mass $M = 1.00$ kg, and the other end is secured to a
fixed support. As you turn the cylinder, the weight is lifted up
slightly. The strap slips around the cylinder, and the weight is
lifted due to a frictional force $Mg$ between the strap and cylinder.
When you turn the cylinder one revolution, the work done by the
friction is equal to $W = (Mg)(2\pi r)$. This work becomes heat
which causes the temperature of the cylinder to rise.
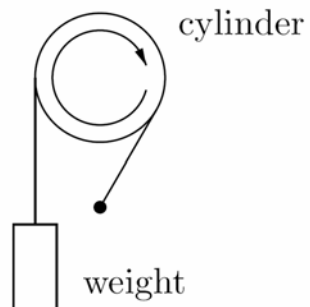
The temperature of the cylinder is measured using a thermocouple wire which is connected to a
digital meter. Insert the wire into the shallow hole at the center of the red circle drawn on the end
of the aluminum cylinder. Hold it there for 30 seconds. If you hold it with your fingers, then be
sure to keep your fingers at least two inches away from the end of the wire so that the heat from
your fingers does not influence the reading of the temperature.

In order to minimize the effect of the heat flow between the cylinder and the surrounding air, we
first cool down the cylinder to a few degrees below room temperature. This is done by pressing a
piece of cold aluminum supplied with the apparatus against the rotating cylinder for about two
seconds. If the temperature is still not below room temperature (perhaps because someone else
had just finished the lab and left the cylinder hot), press the piece of cold aluminum against the
cylinder for another two seconds or so. Do not lower the temperature below about 18°C.

Record the initial temperature $T_i$ below. Turn the crank on the cylinder 100 times. Note that
every revolution of the crank produces 12 revolutions of the cylinder, so the cylinder has actually
gone through 1200 revolutions. Record the final temperature $T_f$. Calculate the change in
temperature $\Delta T$. Calculate the work $W$ per revolution done. Calculate the total work $W$ done.
Calculate the specific heat $c$ of the cylinder.

$T_i =$ _____

$T_f =$ _____

$\Delta T =$ _____

$W$/revolution = _____

total $W =$ _____

$c =$ _____

In this lab you will use a computer simulation to study how wave packets propagate in linear media. You will study both non-dispersive media in which sine-waves of all wavelengths travel at the same speed (like, for example, light traveling in a vacuum) as well as dispersive media (like light traveling through a piece of glass, electron quantum waves traveling through space, and just about every other real system).

The first step is to go to the class website and click the "Lab 3 - Dispersion" link. You can run the applet and get additional help there. Once the applet is running, you should see a screen with two graphs and some text. The next step is to click on the red "get help" button in the upper left-hand corner and read the instructions for the software. Before proceeding, you may want to play with the program for a bit to make sure that you understand how it works.

**Uncertainty** First let's explore the uncertainty which is inherent in waves. To do this, first click on "Reset All." In the upper graph you should see a depiction of a Gaussian wave packet (a little "burst" of a sine-wave with a Gaussian-shaped "envelope"). In the lower graph you can see the spectrum of the pulse (the amplitude of each of the sine waves which the computer added together to make the wave packet in the upper graph). On the far right-hand side of the program the computer displays $\Delta x$; (the standard deviation of the pulse in space), $\Delta k$ (the standard deviation of the pulse's spectrum), and the product of the two.

We learned in class that in order to make pulses which were very narrow in space, we have to add a wide band of frequencies or wavenumbers together, making it difficult to state with certainty what the frequency of the pulse is. To make a wave packet with a very well defined frequency or wavenumber we have to let the packet extend over a large range in space such that it is difficult to assign a location to the packet with precision. Furthermore, we learned that if we defined uncertainty to be the RMS standard deviation, the uncertainties in $x$ and $k$ follow the uncertainty relation $\Delta x \Delta k \geq \frac{1}{2}$.

Notice that our wave satisfies the above uncertainty relation. Now type in a different value for the pulse width ($w$). Notice that as the pulse shrinks, its spectrum widens. The uncertainty relation should still hold. Now change the central wavenumber ($k$) and see what happens.

Now click "Reset All," enter 150 for $k$, and enter `squarepulse(x/w)` for the "Envelope." Now try different values for the pulse width and fill in the table below. Then answer the question below the table.

| $w$ | $\Delta x$ | $\Delta k$ | $\Delta x \Delta k$ |
|------|-----------|-----------|--------------------|
| 0.02 | | | |
| 0.05 | | | |
| 0.08 | | | |
| 0.1  | | | |

- Do the values in this table satisfy the uncertainty relation above?

Note that the physical size of the pulse on the screen is about 4 times larger than $\Delta x$. This is just due to the fact that we have chosen to define uncertainty as the RMS standard deviation. This is the most commonly used but not always the most useful definition. So, you see, there is uncertainty in our definition of uncertainty! As a result, the uncertainty relation is often written in the less precise form: $\Delta x \Delta k \gtrsim 1$.

**Non-dispersive media**. In this part of the lab we will examine what happens when wave pulses travel in non-dispersive media. In non-dispersive media the angular frequency of a sine wave is simply proportional to the wavenumber of the wave: $\omega(k) = vk$, where $v$ *is* the velocity that waves travel through the medium. Wait a minute... is that the phase or group velocity? Think about this for one minute, and then answer the following two questions in the space provided.

- The dispersion relation of light traveling through a vacuum is just $\omega(k) = ck$, where $c$ is equal to $2.9979 \times 10^8$ m/s. What is the phase velocity for a pulse of light whose central wavelength is 657 nm?

- What is the group velocity for such a light pulse?

Now let's use the computer simulation to see what happens to a Gaussian-shaped pulse as it propagates through a non-dispersive medium. First click on the "Reset All" button. There should now be a pretty pulse displayed in the upper graph, with a nice spectrum centered around a wavenumber of 75 m$^{-1}$ in the lower graph. Now click on the "Go!" button to let time run and see what happens. The dispersion relation, shown just below the "Reset All" button, is $\omega(k) = 0.1$ m/s $\cdot$ $k$. Use this dispersion relation to answer the following question.

- What is the group velocity for a pulse in this medium centered at 75 m$^{-1}$?

Now click on the "Stop" button to stop the simulation if it hasn't already stopped, and click on the "Reset t=0" button to set time back to zero. Now plug the group velocity you calculated above into the "x-Axis Velocity" box to make our "view window" move with the pulse. Click on "Go!". If you did your calculation correctly, the pulse should stand still in the window.

Based on what you have seen, answer the following question.

- What happens to the spatial size of a pulse and the spread of frequencies or wavenumbers in a pulse as it travels in a non-dispersive medium?

**Dispersive Media.** Now let's pick a dispersion relation which is a little more interesting. Click on "Reset All," and then enter the dispersion relation `0.001*k^2`. Before you do anything else, use this dispersion relation to calculate the group and phase velocities for a pulse centered around $k = 75\text{m}^{-1}$.

- Group Velocity

- Phase Velocity

Now click on "Go!" and see what happens. Now stop the simulation, set time to $t = -10$, and set the "x-Axis Velocity" equal to the group velocity you calculated above. Press "Go!" again and watch what happens. Now stop the simulation, set time to $t = -2.5$, and set the "x-Axis Velocity" equal to the phase velocity calculated above. Press "Go!" and see what happens (hint: this is the part of the lab where the vertical blue line in the center of the graph is useful). Finally, based on what you saw and in you own words explain what phase and group velocity represent:

- Group velocity is…

- Phase velocity is…

Now, based on what you have seen, answer the following question.
- What happens to the spatial "size" of a pulse when it travels through a dispersive medium?

- What happens to the spectrum of a pulse when it travels through a dispersive medium?

That's the end of the lab, but I recommend that you take some additional time to play around with this simulation. If you can develop a solid understanding of dispersion, uncertainty, and group and phase velocities, you will be able to better understand many more concepts that you will learn in future courses in physics, chemistry, engineering, etc. After all, quantum mechanics tells us that everything is a wave, and that even a vacuum is dispersive for waves that represent matter!

Physics 123                                    Identification Number _____
Lab #4
Standing Waves in a Wire

In this lab, you will produce standing waves in a wire. This is done by placing the wire through
the poles of a magnet and passing an alternating current (60.00 Hz) through the wire. The
resulting force of the magnetic field on the current drives the wire into a vertical oscillation at
60.00 Hz. The tension in the wire is equal to the weight hanging at the end. At certain tensions,
the wire will resonate and produce visible standing waves.
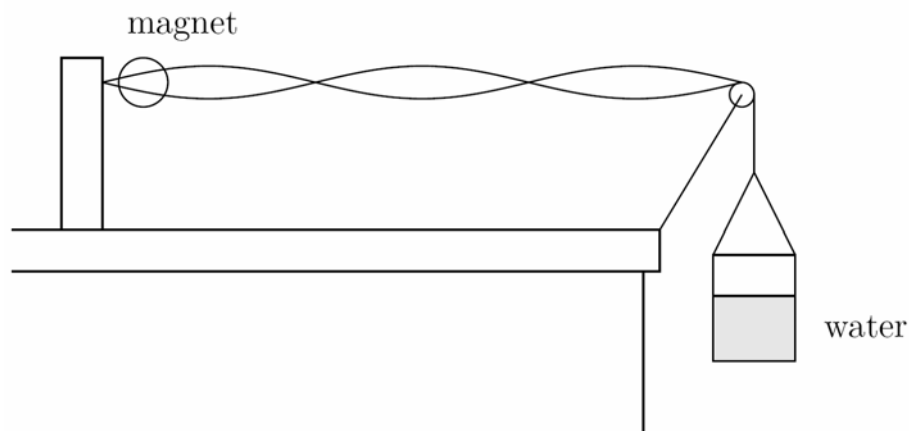


Produce a standing wave by adjusting the amount of water in the container and thus changing the
tension in the wire. (Don't add any additional weight beside water. You may break the wire.)
Adjust the tension until the amplitude of the antinodes is as large as possible (even though the
nodes may not be as well defined). Using a meter stick, measure the wavelength $\lambda$ of the
standing wave. Calculate the velocity $v$ of the waves in the wire. Weigh the container of water to
obtain its mass $m$. Calculate the tension $F$ in the wire. From $F$ and $v$, calculate the linear mass
density $\mu$ of the wire. Repeat this for a different standing wave.

<table>
<tr><td></td><td>1st Standing Wave</td><td>2nd Standing Wave</td></tr>
<tr><td>$\lambda =$</td><td>_____</td><td>_____</td></tr>
<tr><td>$v =$</td><td>_____</td><td>_____</td></tr>
<tr><td>$m =$</td><td>_____</td><td>_____</td></tr>
<tr><td>$F =$</td><td>_____</td><td>_____</td></tr>
<tr><td>$\mu =$</td><td>_____</td><td>_____</td></tr>
</table>

Physics 123                                    Identification Number _____
Lab #5
Standing Waves in a Pipe

In this lab, you will produce standing waves in a pipe. This is done by placing a speaker at an open end of the pipe and driving the speaker with an oscillator as shown below:
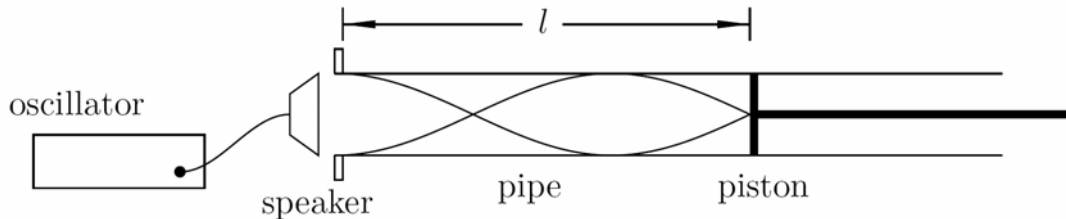


A piston is inserted into the other end of the pipe. At certain positions of the piston, the speaker will cause the pipe to resonate, thus producing standing waves.

Set the frequency $f$ of the oscillator at approximately 700 Hz. Read the frequency shown on the counter and record it below. Starting with the piston at the end of the pipe, push it in slowly. You will notice that at certain positions, the sound of the speaker is enhanced. This is caused by standing waves in the pipe. Use the sound meter to accurately determine the position of the piston where the enhanced sound is loudest. Measure the distance $l$ between the piston and the open end of the pipe at all positions of the piston for which this occurs and record it below. You ought to find 5 of them.

For each standing wave, the piston is at a position of a displacement node. From the data, you can thus obtain the distance between nodes and consequently the wavelength $\lambda$. Using the wavelength and frequency, calculate the velocity of sound to the nearest m/s (three significant figures). Record these results below.

$f =$ _____

$l =$ _____ _____ _____ _____ _____

$\lambda =$ _____

$v = f \lambda =$ _____

Physics 123                                    Identification Number _____
Lab #6
Fourier Transforms

In this lab you will study the relationship between time dependent signals and their frequency spectrum (i.e., their Fourier transform). You will do this using a computer program which can generate or record waveforms or read-in pre-recorded waveforms. This program will display the waveform along with its Fourier transform.

The first step is to go to the class website and click the "Lab 6 - Fourier transforms" link. You can run the applet and get additional help there. The next thing to do is to play with the program and make sure that you understand how to use it. In particular, make sure you understand how to zoom in and out on the graphs, and how to find the exact value of a point by right-clicking on it.

**Musical Octaves**. Click on "RESET ALL". This will set up the program to work with a "user defined" waveform and set the waveform equal to `sin(2*pi*440*t)`. This will generate a sine wave at 440 Hz (the A above middle C). Now, adjust the frequency (the 440) until you hear a tone which is one octave higher. Note the frequency below. Adjust the frequency again until the tone is another octave higher. Note the frequency below. Now think to yourself— does this agree with what we studied in class?

$f_{\text{One Octave Up}} =$

$f_{\text{Two Octaves Up}} =$

**Generating a Square Wave**. Now enter `squarewave(2*pi*440*t)` as the user defined waveform to generate a 440 Hz square wave. Zoom in on the wave until you can see that it is, indeed, a square wave. Play the wave and hear what it sounds like. Now, using the "Its spectrum" graph, find the frequency and amplitude of the four lowest-frequency Fourier components and record them in the table below. Also record the frequency divided by the fundamental frequency (440 Hz). (Hint, $f$/440Hz should be an integer for all of the components, and should equal 1 for the lowest frequency component.)

|      | $f$ | $A$ | $f$/440 Hz |
|------|-----|-----|------------|
| 1.   |     |     |            |
| 2.   |     |     |            |
| 3.   |     |     |            |
| 4.   |     |     |            |

Now let's see what happens when we add together four sine waves with the above frequencies and amplitudes. Type

```
A*sin(2*pi*fa*t) + B*sin(2*pi*fb*t) + C*sin(2*pi*fc*t) +
                 D*sin(2*pi*fd*t)
```

in as the user defined waveform, where A, B, C and D are the amplitudes you measured above, and $f_a$, $f_b$, $f_c$, and $f_d$ are the frequencies which go with each amplitude. Click on "Recalc/Record" and then zoom in on the graph of the wave to see if it looks like a square wave. Sketch what you see below:

For kicks, you might want to see what the wave looks like as you add more and more sine terms together. You can get a pretty decent looking square wave!

**Uncertainty Relations**. Now let's make a short pulse of sound and explore the topic of "wave uncertainty". Enter `sin(2*pi*440*t)*exp(-10000*(t-0.5)^2)` as the user defined waveform and click on "Recalc/Record." Click on "Zoom to fit," and take a look at the wave and its spectrum. Then play the wave. Now zoom in on the wave and on its spectrum and estimate $\Delta t$ and $\Delta f$. Now calculate $\Delta \omega$ from $\Delta f$, and calculate the uncertainty product $\Delta \omega \, \Delta t$ and record everything below.

$\Delta t$:

$\Delta f$:

$\Delta \omega$:

$\Delta \omega \, \Delta t$:

Now make the pulse shorter and longer in time by changing the 10000 in the waveform to other numbers. Change it by at least a factor of 20 in both directions (smaller and larger). Describe below what happens to the width of the spectrum when you change the duration of the pulse in time. Why does this happen?

Describe below what happens to the tone of the note as you change the duration of the pulse in time. Why does this happen**?**

**Playing Around**. You have now finished the lab. But for your own learning experience I recommend that you play around with the program. In particular, you should do the following things.

(1) Record the sound of your hands clapping (or use the pre-recorded sound of my hands clapping, available under the "Waveform" drop-down box) and see if the uncertainty product $\Delta\omega\Delta t$ makes sense.

(2) Listen to the various pre-recorded waveforms and note their spectral properties. Notice that most of the instruments have a spectrum which looks like a harmonic series and ask yourself why that is the case. Also notice that the percussive instruments do not have a spectrum which looks like a harmonic series. Not even the timpani which seems to generate a specific tone! Ask yourself why a timpani's waveform does not consist of a harmonic series of frequencies.

(3) Try to generate different waveforms by adding sine waves together. You might want to actually calculate the Fourier transform of some waveform, and then plug the results in and see what you get.

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Lab #7
Brewster's Angle

In this lab, you will measure the Brewster angle for two different materials. From these measurements, you will then calculate the index of refraction for each material.

As shown in the figure below, a laser beam is directed towards the surface of a sample. The sample is mounted on a platform which can be rotated. The pointer attached to the platform points in a direction perpendicular to the surface of the sample. The incident angle $\theta$ of the beam can be read from a scale on the apparatus.

The reflected beam passes through a sheet of Polaroid and hits a white screen. The transmission axis of the Polaroid is horizontal. When the angle of the incident beam is equal to the Brewster angle, the reflected beam is polarized vertically and thus will not pass through the Polaroid. At this angle, the illuminated spot on the screen will disappear. (Actually, since the sample and the Polaroid are not ideal, the spot will not disappear completely, but will have a minimum intensity.)
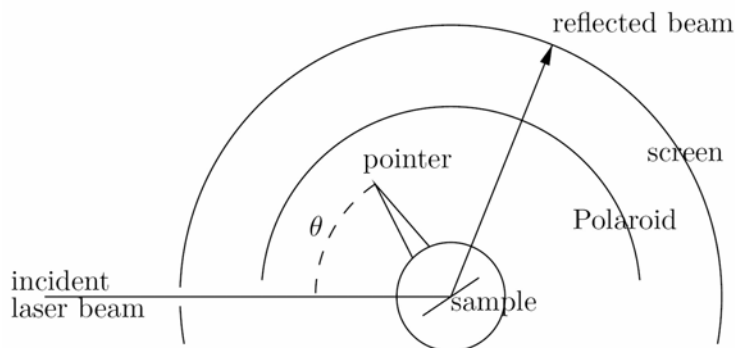


There are two samples. One is ordinary glass, and the other is zirconium oxide ($ZrO_2$). First insert the glass into the sample holder. Rotate the sample platform and find the orientation where the reflected beam has a minimum intensity. Be sure that the Polaroid sheet is in place so that the reflected beam passes through it. Read the incident angle from the scale and record it below. This is the Brewster's angle $\theta_p$. Determine the index of refraction from $n = \tan\theta_p$ and record it below. Repeat this for the $ZrO_2$ sample.

**Warning**: Do not touch the sample surfaces. Fingerprints on the samples will affect your measurements. Wipe off any fingerprints with the tissues provided.

Glass sample:    $\theta_p =$ _____        $n =$ _____

ZrO2 sample:    $\theta_p =$ _____        $n =$ _____

When you are finished, remove the Polaroid sheet and notice how intense the reflected beam is. Then place a small circular Polaroid sheet in the path of the reflected beam and observe how its intensity changes as you rotate the sheet.

Physics 123                    Identification Number _____
Lab #8
Telescope

In this lab, you will construct a simple telescope using two lenses. Mount the source (illuminated arrow) and the screen on the optical bench, and mount one of the lenses between them. Adjust their positions until a real image of the arrow is focused on the screen. For best results, adjust the positions so that the lens is about half-way between the object and the image. Measure $p$ and $q$. Calculate $f$ from the thin lens equation,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \; .$$

Repeat for the other lens. Record your results below.

Construct a telescope by mounting the two lenses a distance $f_1 + f_2$ apart. Use the lens with the smaller focal length for the eyepiece. View the large scale mounted on the wall across the room. The distance between the two lenses may be adjusted to bring the image into better focus.

Measure the angular magnification $m$ of the telescope by viewing the scale through the telescope with one eye and looking directly at the scale with the other eye. In this way, you ought to be able to see both the magnified and unmagnified scale superimposed on each other.

Finally, calculate $m$ from the measured focal lengths.

                    lens 1                         lens 2

$p =$    _____        _____

$q =$    _____        _____

$f =$    _____        _____

$m =$    _____        measured

$m =$    _____        calculated

Physics 123                                    Identification Number _____
Lab #9
Michelson Interferometer

In this lab, you will use a Michelson interferometer to measure the index of refraction of a gas. A chamber which can be evacuated is placed in one arm of the interferometer. All of the air is first evacuated from the chamber. As the gas to be studied is slowly allowed to enter the chamber, the number of fringes passing by the center of the screen is counted.
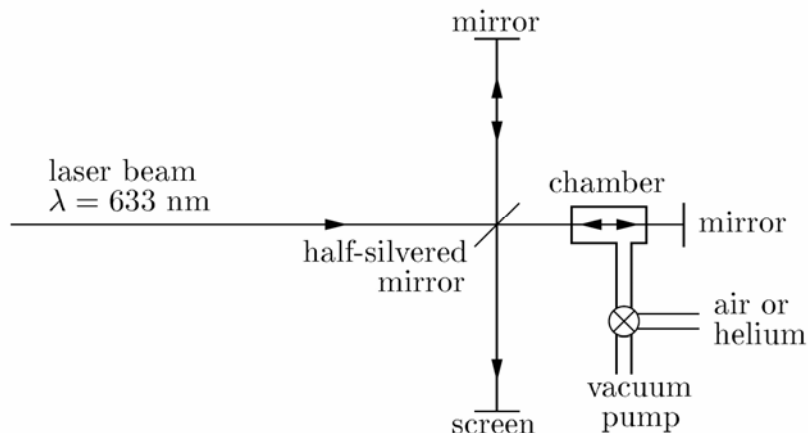


The index of refraction $n$ of the gas is given by

$$n = 1 + \frac{N\lambda}{2L},$$

where $N$ is the number of fringes counted, $\lambda$ is the wavelength of the laser in vacuum, and $L$ is the length of the chamber. See the Supplement on the next page for the derivation.

Turn on the vacuum pump and evacuate the chamber. Pump for at least a couple of minutes to obtain a good vacuum. Valve off the vacuum pump and *slowly* open the chamber to air and count the fringes. (You will probably open the valve too fast the first time you try, and the fringes will go by too quickly to count. If this happens, evacuate the chamber again and start over.) Repeat using helium gas instead of aid. Measure $L$ and calculate $n$ for each gas.

              Air                       Helium

$L =$   _____    _____

$N =$   _____    _____

$n =$   _____    _____

Supplement to Michelson Interferometer

When the chamber is evacuated, the number of wavelengths along its length $L$ is given by

$$N_{vac} = \frac{L}{\lambda_{vac}} \, ,$$

where $\lambda_{vac}$ is the wavelength of the laser light in vacuum. When the chamber is filled with some gas, the number of wavelengths along its length is now given by

$$N_{gas} = \frac{L}{\lambda_{gas}} \, ,$$

where $\lambda_{gas}$ is the wavelength of the laser light in the gas.

Each time one arm of the interferometer gets behind (or ahead) by one wavelength, one fringe passes by the screens. As we fill the chamber with gas, that arm of the interferometer will get behind by $N = 2(N_{gas} - N_{vac})$ wavelengths. (The factor 2 is included since the light passes through the chamber twice, once going and once coming back.) From the two above equations, we thus obtain

$$N = 2\left( \frac{L}{\lambda_{gas}} - \frac{L}{\lambda_{vac}} \right).$$

We also know that

$$\lambda_{gas} = \frac{\lambda_{vac}}{n} \, ,$$

where $n$ is the index of refraction of the gas. Using this to solve for $n$, we obtain

$$n = 1 + \frac{N\lambda_{vac}}{2L} \, .$$

Physics 123                                   Identification Number _____
Lab #10
Diffraction Grating

In this lab, you will observe the interference pattern produced by shining a laser beam through a diffraction grating. From the distance between peaks in the pattern, you will determine the distance between the slits in the grating.

The He-Ne laser used in this lab produces red light of wavelength 633 nm. Turn on the laser. Its beam should pass through the diffraction grating. You should observe the interference pattern on the wall.

Use a meter stick to measure the distance $\Delta x$ between peaks in the interference pattern. Average this distance over several adjacent peaks so that your measurement will be as accurate as possible. Record your result below.

Use the tape measure to determine the distance $L$ between the diffraction grating and the interference pattern on the wall and record your result below.

Calculate the angle $\theta$ between adjacent bright spots in the interference pattern and record your result below.

Using $d \sin\theta = \lambda = 633$ nm, calculate the distance $d$ between the slits in the grating and record your result below.

$\Delta x =$ _____

$L =$ _____

$\theta =$ _____

$d =$ _____