Exam 3 Solns Fall 2011

Instructions:

- Record your answers to the multiple choice questions ("Problem 1") on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly. Draw a picture if possible. Be clear about what equations you are using, and why. Prove that you understand what is going on in the problem. It's generally a good idea to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Double-check your calculator work. Think about whether your answer makes sense; if not, go over your work again or try working the problem a different way to double-check things.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(36 pts) **Problem 1**: Multiple choice questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper for your own reference.

1.1. A power plant takes in steam at 250°C (523K) to power turbines and then exhausts the steam at 130°C (403K). Each second the turbines transform 100 megajoules of heat energy from the steam into usable work. If the power plant operates at the theoretical maximum possible efficiency, what will its power output be?

a. 0 - 5 megawatts
b. 5 - 10
c. 10 - 15
d. 15 - 20
e. 20 - 25
f. 25 - 30
g. 30 - 35
h. 35 - 40
i. 40 - 45
j. 45 - 50 megawatts

$e_{max} = 1 - T_c/T_h = 1 - 403/523 = 22.9\%$

$W_{net} = e Q_h = 22.9$ MJ (per sec)

1.2. A sound wave passes from medium A to medium B, at normal incidence (the wave travels perpendicular to the boundary). In which case will you get the most transmitted sound energy?

a. $v_A < v_B$
b. $v_A = v_B$ → then $r = 0$ and $t = 1$
c. $v_A > v_B$

1.3. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.

a. A sound wave is a transverse wave. X it's longitudinal
b. Intensity of a sound at a given location usually varies inversely with the distance from that location to the source of the sound: I ~ 1/r X $I \sim 1/r^2$
c. Sound waves travel faster on a warm day than a cool day. True!
d. The fundamental frequency of a guitar string corresponds to the standing wave pattern in which there is a complete wavelength within the length of the string. X = ½ wavelength
e. An "open-closed" air column that can play a fundamental frequency of 250 Hz will not have 500 Hz as a harmonic. True! Open-closed just gets the odd harmonics

1.4. In a vacuum, radio waves have a greater __________ compared to visible light

a. speed
b. wavelength
c. frequency
d. a and b
e. a and c
f. b and c
g. a, b, and c

Speed for both is c
radio waves have longer λ
Since $v = \lambda f$, means radio has smaller f.

1.5. If carbon tetrachloride has an index of refraction of 1.461, what is the speed of light through this liquid?

a. Less than 1.7×10^8 m/s
b. 1.7 - 1.9
c. 1.9 - 2.1
d. 2.1 - 2.3
e. 2.3 - 2.5
f. 2.5 - 2.7
g. More than 2.7×10^8 m/s

$v = c/n = \frac{3 \cdot 10^8 \text{ m/s}}{1.461} = 2.05 \cdot 10^8$ m/s

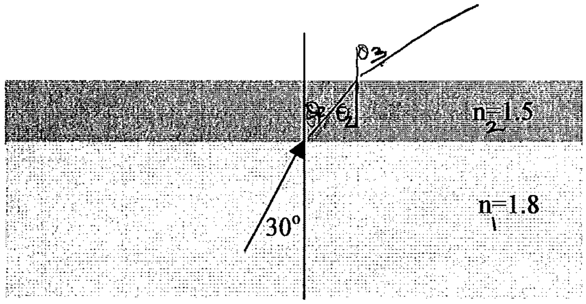
Snell for 1→2 : $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Snell for 2 -> 3 : $n_2 \sin\theta_2 = n_3 \sin\theta_3$

plug in: $n_1 \sin\theta_1 = n_3 \sin\theta_3$

$(1.8)(\sin 30°) = (1)(\sin\theta_3)$

$\theta_3 = \underline{64.16°}$



1.6. A ray from a microscope sample travels as shown in oil (n=1.8) to glass (n = 1.5), and then from glass to air. At what angle does it enter the air? _______ degrees with respect to the vertical

a. 0 to 10
b. 10 to 20
c. 20 to 30
d. 30 to 40
e. 40 to 50
f. 50 to 60
g. 60 to 70
h. 70 to 80
i. 80 to 90

1.7. Which of the following describes what will happen to a light ray traveling in air, when it hits an air-to-glass boundary ?

a. total reflection
b. total transmission
c. partial reflection, partial transmission Self explanatory ?
d. partial reflection, total transmission

1.8. If the critical angle for internal reflection inside a certain transparent material is found to be 48.0 degrees (measured from the perpendicular), what is the index of refraction of the material? (Air is outside the material).

a. Less than 0.9
b. 0.9 - 1.0
c. 1.0 - 1.1
d. 1.1 - 1.2
e. 1.2 - 1.3
f. 1.3 - 1.4
g. More than 1.4

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

θ_c when $\theta_2 = 90°$

$n_{glass} \sin 48° = 1 \sin 90°$

$n_{glass} = \underline{1.346}$

1.9. At what angle is the sun above the horizon if its light is found to be completely polarized when it is reflected from the top surface of a horizontal slab of glass (n = 1.65)?

a. 0 to 10 degrees
b. 10 to 20
c. 20 to 30
d. 30 to 40
e. 40 to 50
f. 50 to 60
g. 60 to 70
h. 70 to 80
i. 80 to 90 degrees

Brewster: $\tan\theta_B = n_2/n_1$

$\tan\theta_B = 1.65/1$

$\theta_B = 58.78$

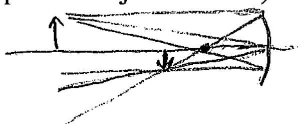
Angle above horizon $= 90° - \theta_B = \underline{31.22°}$

1.10. Which of the following best describes the image formed by a plane mirror? (Assume a positive object distance.)

a. real, inverted, and reduced
b. real, inverted, and the same size as object
c. real, upright, and reduced
d. real, upright, and the same size as object
e. virtual, inverted, and reduced
f. virtual, inverted, and the same size as object
g. virtual, upright, and reduced
h. virtual, upright, and the same size as object

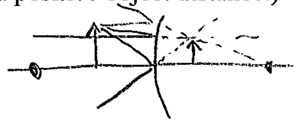
1.11. Which of the following best describes the image formed by a *converging* mirror when the object is at a distance of <u>3× the focal length</u> of the mirror? (Assume a positive object distance.)

a. real, inverted, and enlarged
b. real, inverted, and reduced
c. real, upright, and enlarged
d. real, upright, and reduced
e. virtual, inverted, and enlarged
f. virtual, inverted, and reduced
g. virtual, upright, and enlarged
h. virtual, upright, and reduced



1.12. Which of the following best describes the image formed by a *diverging* mirror when the object is at a distance of <u>half the focal length</u> of the mirror? (Assume a positive object distance.)

a. real, inverted, and enlarged
b. real, inverted, and reduced
c. real, upright, and enlarged
d. real, upright, and reduced
e. virtual, inverted, and enlarged
f. virtual, inverted, and reduced
g. virtual, upright, and enlarged
h. virtual, upright, and reduced



1.13. A converging mirror with a focal length of 10 cm creates a real image 30 cm away, on its principal axis. How far from the mirror is the corresponding object?

a. Less than 10 cm
b. 10 - 13
c. 13 - 16
d. 16 - 19
e. 19 - 22
f. 22 - 25
g. More than 25 cm

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \longrightarrow p = \left(\frac{1}{f} - \frac{1}{q}\right)^{-1}$

$p = \left(\frac{1}{10} - \frac{1}{30}\right)^{-1}$

$\underline{p = 15 \text{ cm}}$

1.14. If a man's face is 30 cm in front of a converging shaving mirror which creates a virtual upright image 1.5 times as large as the object, what is the mirror's focal length?

$M = -q/p$

so $q = -45$ cm

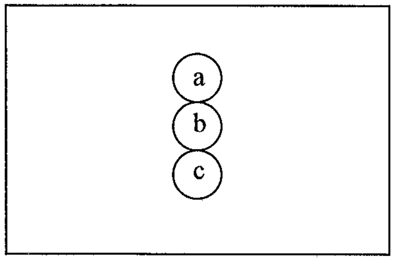
a. Less than 67 cm
b. 67 - 77
c. 77 - 87
d. 87 - 97
e. 97 - 107
f. 107 - 117
g. More than 117 cm

$f = \left(\frac{1}{p} + \frac{1}{q}\right)^{-1} = \left(\frac{1}{30} + \frac{1}{-45}\right)^{-1}$

$= \underline{90} \text{ cm}$

1.15. If a marble is embedded inside a thick sheet of glass at point b, about where will the observer see the image? (Note the position of the observer.)

a. point a
b. point b
c. point c



observer

As per class discussion, objects in water (or glass) appear closer than they really are, because the light rays bend away from the perpendicular when they leave the water (or glass)

1.16. The index of refraction of a lens is somehow doubled, from 1.5 to 3.0. How will the focal length of the lens change?

a. $f_{new} = 1/8\ f_{old}$
b. $f_{new} = 1/6\ f_{old}$
c. $f_{new} = 1/4\ f_{old}$
d. $f_{new} = 1/2\ f_{old}$
e. $f_{new} = f_{old}$
f. $f_{new} = 2\ f_{old}$
g. $f_{new} = 4\ f_{old}$
h. $f_{new} = 6\ f_{old}$
i. $f_{new} = 8\ f_{old}$

lensmaker's eqn: $\frac{1}{f} \sim n-1$

$\frac{f_{new}}{f_{old}} = \frac{n_{old}-1}{n_{new}-1} = \frac{.5}{2} = \frac{1}{4}$

$f_{new} = \frac{1}{4} f_{old}$

1.17. To correct nearsightedness, you need to choose a lens that will create a ______ image at ______, of an object at ______

a. real; the near point; the far point
b. real; the far point; infinity
c. real; infinity; the far point
d. real; infinity; 25 cm
e. virtual; the near point; the far point
f. virtual; the far point; infinity
g. virtual; infinity; the far point
h. virtual; infinity; 25 cm

Nearsighted: can't see farther than far point

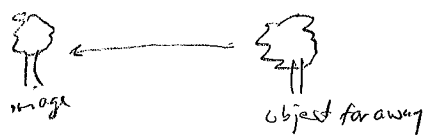
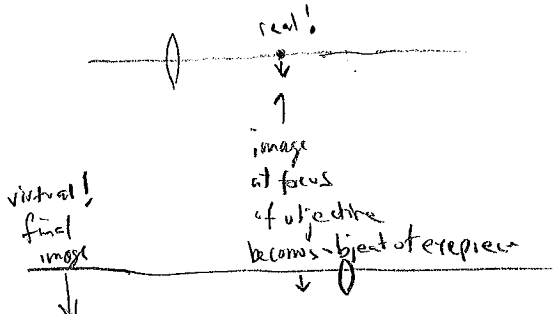


image must be brought to "far point"

1.18. In a telescope, the objective lens creates a _______ image, and the eyepiece creates a _______ final image which is viewed by the eye.

a. real, real
b. real, virtual
c. virtual, real
d. virtual, virtual



1.19. A telescope has an objective with f = +50 cm and an eyepiece with f = +3 cm. If you use the telescope to look at Mars at a time of year where Mars has an (unassisted) angular width of 1.4×10^{-5} rad, what will the angular width be as you look through the telescope?

ang. mag $m = \frac{f_{obj}}{f_{eye}} = \frac{50}{3}$

a. Less than 1.0×10^{-4} rad
b. 1.0 - 1.2
c. 1.2 - 1.4
d. 1.4 - 1.6
e. 1.6 - 1.8
f. 1.8 - 2.0
g. 2.0 - 2.2
h. 2.2 - 2.4
i. 2.4 - 2.6
j. More than 2.6×10^{-4} rad

$\theta_{final} = \theta_{init} \times m$
$= (1.4 \times 10^{-5})(\frac{50}{3}) = 2.33 \cdot 10^{-4}$ rad

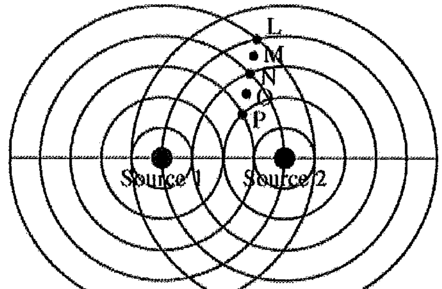
1.20. A film of oil (n = 1.3) lies on top of thick glass (n = 1.5). Light shines from the air onto the film at normal incidence, and some light is reflected at each interface. There is a λ/2 phase shift in the reflected light at which boundaries?

a. air to oil
b. oil to glass
c. both
d. neither

n=1
n=1.3
n=1.5

normal incidence : phase shift of 180°
when going from small n to large n.

1.21. If the circles represent wavefronts (the "crests"; the separation of circles is therefore one wavelength), then at point O we expect:



a. constructive interference
b. destructive interference
c. interference that is neither completely constructive nor destructive
d. no interference

trough + trough = larger trough.

Constructive!
(destructive is when peak + trough cancel out)

1.22. An observer looks through a telescope of width 5 cm at two small but very bright LEDs a distance of 1 km away. The LEDs emit light with λ = 500 nm. The minimum separation of the LEDs at which the observer can resolve the two light sources will be closest to:

Rayleigh : $\theta = 1.22\,\lambda/D \approx \frac{500\,nm}{5\,cm} = 1 \cdot 10^{-5}$ rad

a. 0.01 m
b. 0.1
c. 1
d. 10
e. 100 m

small angle : $\theta = \frac{separation}{distance}$ → separation $= (10^{-5}\,rad)(1000\,m)$
$= .01\,m$

1.23. A 500 nm laser shines directly on a diffraction grating, and several diffraction spots are formed on a wall that is 5 m away. If the grating has a spacing of 1200 lines/mm, how far apart will the zeroeth order and the first order spots be?

$d = 10^{-3}m/1200$

a. Less than 2.2 m
b. 2.2 - 2.6
c. 2.6 - 3.0
d. 3.0 - 3.4
e. 3.4 - 3.8
f. 3.8 - 4.2
g. More than 4.2 m

grating max : $d\sin\theta = m\lambda$
zeroeth order when $\theta = 0°$
1st order when $\theta = \sin^{-1}\left(\frac{1 \cdot 500 \cdot 10^{-9}}{10^{-3}/1200}\right)$
$\theta = 36.9°$

$\tan\theta = y/5$ → $y = 5\tan(36.9°)$
5m
$y = 3.75\,m$

1.24. If a diffraction grating is used underwater with a laser beam, what changes (compared to using it in air)?

a. the diffraction angles decrease
b. the diffraction angles increase
c. nothing changes

$c = \lambda f$ → if c decreases under water
then λ decreases (because f doesn't change)

grating : $d\sin\theta = n\lambda$
→ if λ decreases
then θ also decreases

(10 pts) **Problem 2**. Give short answers/explanations to the following questions:

(a) What do we mean when we said that a particular ray of light is "linearly polarized"?

That means the electric field in the light wave oscillates in a plane as it travels.

Or, at a fixed spot, the electric field oscillates up and down in a straight line ↕

(light wave coming towards the viewer)

(b) Explain where the equation $\theta_{\text{min.resolve}} = 1.22\lambda/D$ came from, and what its significance is.

When light diffracts through a circle it forms an intensity pattern like:

the angle $\Theta = \frac{1.22\lambda}{D}$ is the position of the first minimum

It (roughly) marks the point where you would be able to resolve a second source of light.

Called "Rayleigh's criterion"

(c) A certain transverse wave is traveling in the (1, -1, 3) direction, and oscillates in the (2, 2, 0) direction. Note that neither of those vectors has been normalized. The wave's amplitude is 5, its speed is 7, and its wavelength is 13 (in some set of units). Write down the proper "wave function" which describes this wave. (Don't worry about units.)

$\vec{k} = \underset{2\pi/\lambda}{k} \cdot \underbrace{\frac{(1,-1,3)}{\sqrt{11}}}_{\text{unit vector}}$

polarization unit vector $\hat{p} = \frac{(2,2,0)}{\sqrt{8}}$

$$\boxed{f = 5\left(\frac{2\hat{x} + 2\hat{y}}{\sqrt{8}}\right)\cos\left(\frac{2\pi}{13}\left[\left(\frac{x - y + 3z}{\sqrt{11}}\right) - 7t\right]\right)}$$

(11 pts) **Problem 3.**

(a) How far would an object need to be placed from a converging lens of focal length 10 cm if it is to produce a *virtual* image which is 20 cm from the mirror?

$f = 10$

$q = -20$

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

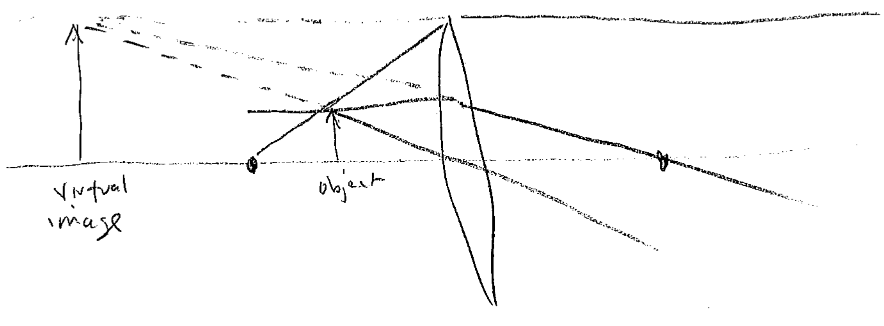
$p = \left(\frac{1}{f} - \frac{1}{q}\right)^{-1} = \left(\frac{1}{10} - \frac{1}{-20}\right)^{-1} = \boxed{6.67\text{ cm}}$

(b) What would be the magnification of the image? Would the image be upright or inverted?

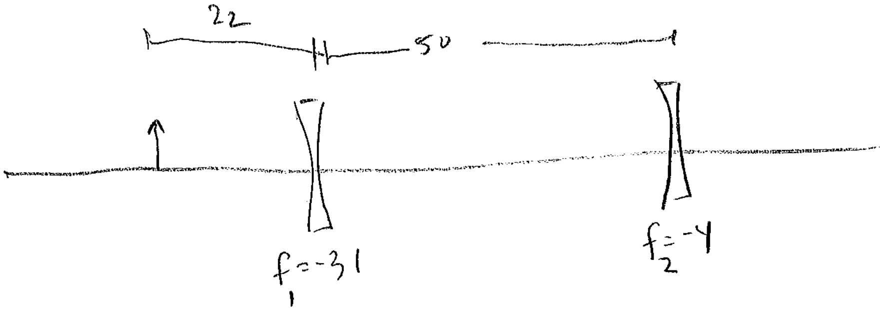
$M = -\frac{q}{p} = -\frac{-20}{6.67} = \boxed{+3}$

upright

(c) Draw a ray diagram for the situation.



(13 pts) **Problem 4.** An object is placed 22 cm to the left of lens 1 (diverging, f = –31 cm). Lens 1 is placed 50 cm to the left of lens 2 (diverging, f = –4 cm). Where will the final image be formed? Will it be real or virtual? What will be the total magnification? You do not have to provide ray diagrams for this problem, although you are certainly welcome to draw them if that will help you visualize the situation.



$$q_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{-31} - \frac{1}{22}\right)^{-1} = -12.87$$

12.87 to left of lens 1

$$M_1 = -\frac{q_1}{p_1} = -\frac{-12.87}{22} = +.5849$$

$$p_2 = 12.87 + 50 = 62.87$$

$$q_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-4} - \frac{1}{62.87}\right)^{-1} = \boxed{-3.76}$$

3.76 to left of lens 2

$$M_2 = -\frac{q_2}{p_2} = -\frac{-3.76}{62.87} = +.05982$$

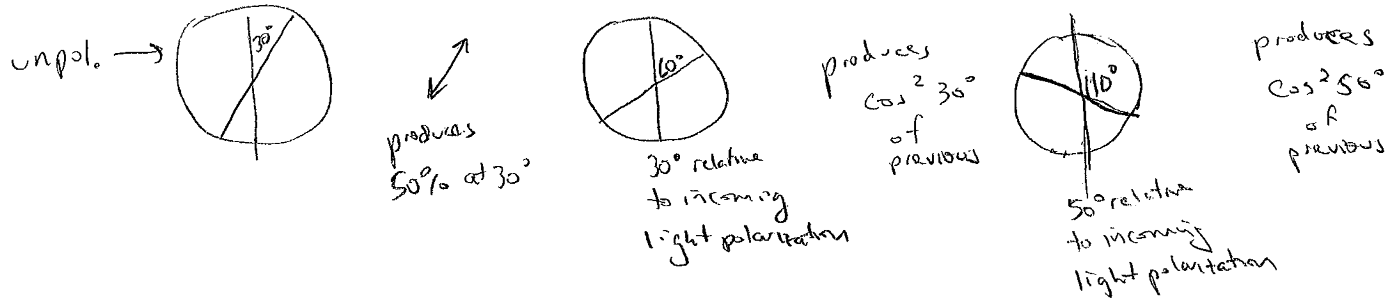
$$M_{tot} = M_1 \times M_2$$
$$= (.5849)(.05982)$$
$$= \boxed{.03499}$$

q_{final} = – 3.76 cm relative to lens 2 (use a negative sign if to the left)

real vs. virtual: virtual

M_{tot} = .03499

(9 pts) **Problem 5.** An unpolarized light beam is shined onto a linear polarizer oriented with the polarization axis at 30° from the vertical. A second polarizer is placed after the first one, with its transmission axis rotated 30° from the transmission axis of the first polarizer; that is, at 60° from the vertical. Finally, a third polarizer is placed after the second one, with its transmission axis rotated 50° from the transmission axis of the second polarizer; that is, at 80° from the first polarizer and 110° from the vertical ~~(which is the same as 20° from the vertical~~). What fraction of the original light's intensity will pass through the third polarizer? Assume all polarizers are perfect.



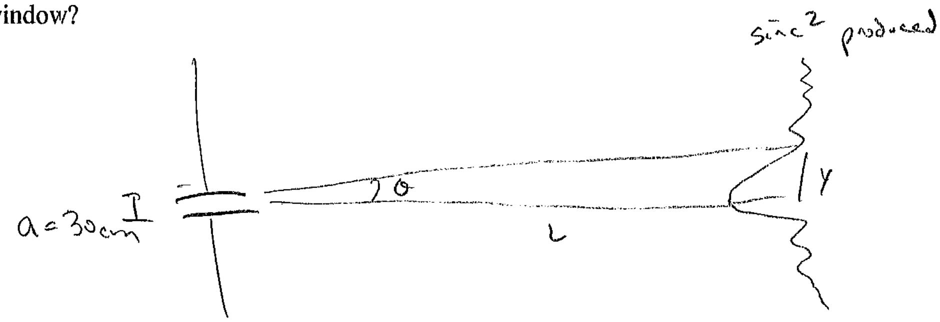
$$\text{Total} = (.5)(\cos^2 30°)(\cos^2 50°)$$

$$= .1549$$

$$\boxed{15.49\% \text{ of original light}}$$

(14 pts) **Problem 6**

(a) Microwaves of wavelength 8 cm enter a long, narrow window in a building that is otherwise opaque to the microwaves. If the window is 30 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 7 m from the window?



Single slit: $I = I_0 \,\text{sinc}^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$ → central max at $\theta = 0°$

→ 1st min when $\frac{\cancel{\pi} a \sin\theta}{\lambda} = \cancel{\pi}\ 1$

$a \sin\theta = \lambda$

$\theta = \sin^{-1}(\lambda/a) = \sin^{-1}\left(\frac{8}{30}\right)$

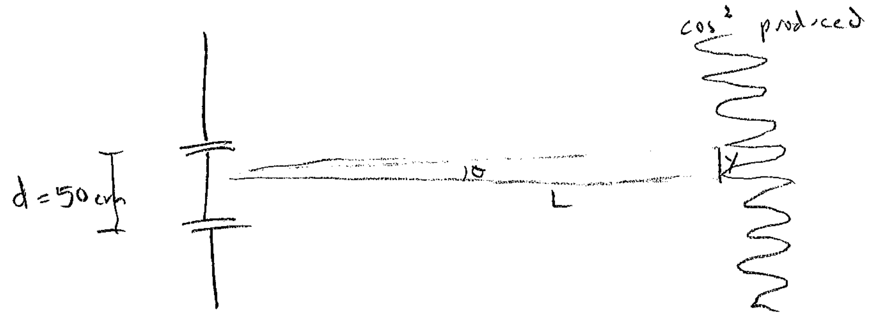
$\theta = 15.47°$

$\tan\theta = Y/L$

$Y = L\tan\theta = (7)(\tan 15.47°) = \boxed{1.937\,m}$

(b) Same situation, but now the window is only 0.5 cm wide and there is a second window placed in a "two-slit" configuration 50 cm away from the first window. What is the distance from the central maximum to the first-order minimum along a wall 7 m from the window?

Call this "infinitely narrow"
So we just have a pure two-slit pattern



double slit: $I = I_0 \cos^2\left(\frac{\cancel{2}\pi}{\lambda}\frac{d}{\cancel{2}}\sin\theta\right)$ → central max at $\theta = 0$

→ first min when $\frac{\pi d}{\lambda}\sin\theta = \frac{\pi}{2}$

$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{8}{2 \cdot 50}\right)$

$\theta = 4.589°$

$Y = L\tan\theta = (7)(\tan 4.589°) = \boxed{.562\,m}$

Alternatively, it's a small angle so $\sin\theta \simeq \tan\theta \simeq \theta$ (in radians)

$Y = L \times \frac{\lambda}{2d} = .560\,m$

(7 pts) **Problem 7.**

(a) (2 pts; no partial credit) A wave at a particular location in space is described by the sum of 10 cosine waves:

$$f(t) = A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\omega_0 t + \phi_2) + \ldots + A_{10} \cos(\omega_0 t + \phi_{10})$$

Describe what $f(t)$ will look like. Specifically, insofar as it is possible from the given information, describe its shape, period/angular frequency, amplitude, and phase.

It will be a sinusoidal wave

Frequency ω_0, period $2\pi/\omega_0$

Amp + phase would need to be determined by e.g. adding $A_1\angle\phi_1 + A_2\angle\phi_2 + \ldots$ as vectors (complex numbers)

(b) (2 pts; no partial credit) A wave at a particular location in space is described by the sum of an infinite number of cosine waves:

$$f(t) = A_1 \cos(\omega_0 t) + A_2 \cos(2\omega_0 t) + A_3 \cos(3\omega_0 t) + \ldots$$

Describe what $f(t)$ will look like. Specifically, insofar as it is possible from the given information, describe its shape, period/angular frequency, amplitude, and phase.

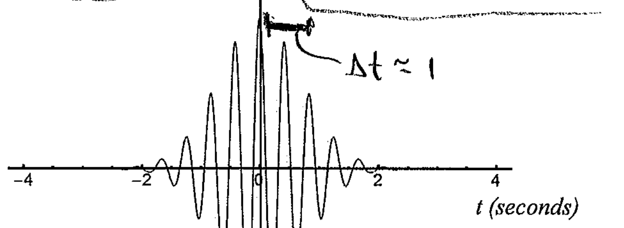
This is a Fourier series!

$f(t)$ will be an unknown non sinusoidal shape (but periodic) (and even)

with fundamental frequency ω_0, period $= 2\pi/\omega_0$

There's not a way to determine the amplitudes, but the phase would be zero (in some sense)

(c) (3 pts; partial credit possible) A wave at a particular location in space is described by the graph on the right; the wave is zero for $t > 4$ and $t < -4$. It is a sum of cosine terms. Explain what cosine functions you would have to add together in order to create this type of resulting wave; specifically, insofar as it is possible from the given information, describe the number of terms as well as the period/angular frequencies and amplitudes of the terms being added together.



This is a wave packet made up of an infinite number of infinitely closely spaced frequencies.

The main frequency component (and probably the largest amplitude) is at $\omega_0 = \frac{2\pi}{.4} \approx 15$ rad/s

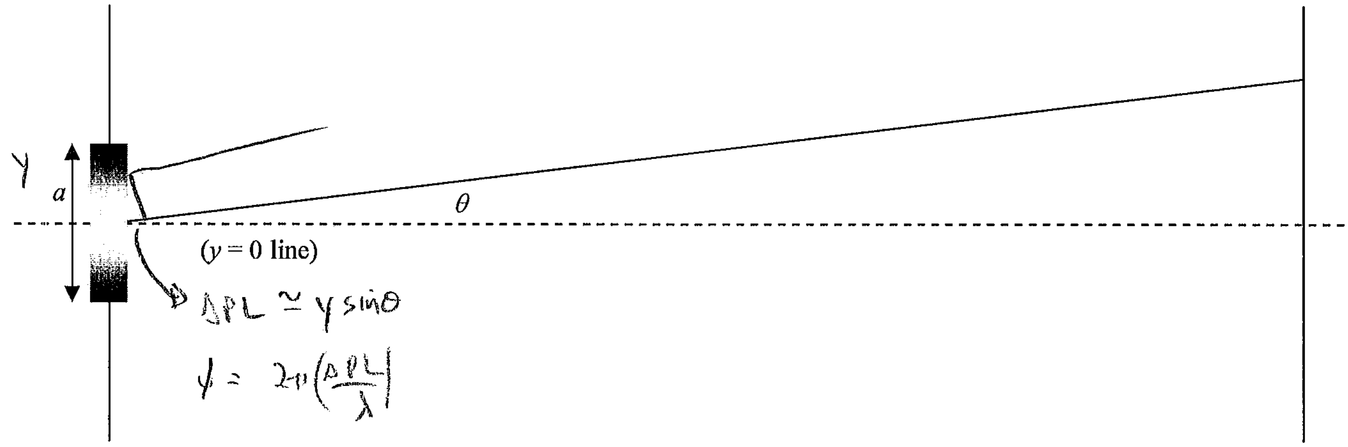
The spread in frequencies can be estimated by $\Delta t \Delta\omega \approx \frac{1}{2}$

$(1)(\Delta\omega) \approx \frac{1}{2} \rightarrow \Delta\omega \approx .5$ rad/s

So frequency components will extend from about $14.5 \frac{\text{rad}}{s}$ up to $15.5 \frac{\text{rad}}{s}$

(5 pts, no partial credit) **Extra Credit**. The technique of "apodization" is sometimes used to affect diffraction patterns. This technique involves placing some sort of filter mask over the aperture. In the picture below, for example, the mask serves to preferentially allow light through from the center of the aperture. When integrating over the aperture it causes some y-values to be weighted more heavily than others. Mathematically, the weighting function that I have plotted is $f(y) = \cos(\pi y/a)$.

Among other things, this trick can potentially change the regular sinc^2 single-slit function into something that peaks more (or less) narrowly. Derive the diffraction pattern, intensity vs. angle θ, for the slit (width a) and the plotted weighting function. Simplify your answer by identifying quantities that are either 0 or 1.



Hint: Here's an integral you will need: $\int_{-a/2}^{a/2} \cos(k_1 y) e^{ik_2 y}\, dy = \frac{2}{k_1^2 - k_2^2}\left(k_1 \cos(k_2 a/2)\sin(k_1 a/2) - k_2 \cos(k_1 a/2)\sin(k_2 a/2)\right)$

$E = E_0\left(e^{i\phi_1} + e^{i\phi_2} + \ldots\right)$ for discrete slits

For continuous slit, must integrate

$E = E_0 \int_{-a/2}^{a/2} e^{i\phi(y)}\, dy$ but need to weight each y value's component according to the apodization function

$$\boxed{E = E_0 \int_{-a/2}^{a/2} \cos\frac{\pi y}{a}\, e^{i\frac{2\pi}{\lambda} y \sin\theta}\, dy}$$

→ fits given integral with $k_1 = \frac{\pi}{a}$, $k_2 = \frac{2\pi}{\lambda}\sin\theta$

integral $= \frac{E_0\, 2}{\frac{\pi^2}{a^2} - \frac{4\pi^2}{\lambda^2}\sin^2\theta}\left[\frac{\pi}{a}\cos\left(\frac{2\pi}{\lambda}\sin\theta\,\frac{a}{2}\right)\underbrace{\sin\left(\frac{\pi}{a}\cdot\frac{a}{2}\right)}_{\sin\frac{\pi}{2} = 1} - \frac{2\pi}{\lambda}\sin\theta\,\underbrace{\cos\left(\frac{\pi}{a}\cdot\frac{a}{2}\right)}_{\cos\frac{\pi}{2} = 0}\sin\left(\frac{2\pi}{\lambda}\sin\theta\,\frac{a}{2}\right)\right]$

$$E = \frac{2E_0}{\frac{\pi^2}{a^2} - \frac{4\pi^2}{\lambda^2}\sin^2\theta}\left[\frac{\pi}{a}\cos\left(\frac{\pi a}{\lambda}\sin\theta\right)\right]$$

Square to get intensity

$$\boxed{I = I_0 \frac{\cos^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi^2}{a^2} - \frac{4\pi^2\sin^2\theta}{\lambda^2}\right)^2}}$$

or better, so that I_0 represents $I(\theta = 0)$

$$\boxed{I = \frac{I_0\cos^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(1 - \frac{4a^2}{\lambda^2}\sin^2\theta\right)^2}}$$

(it's a different I_0)

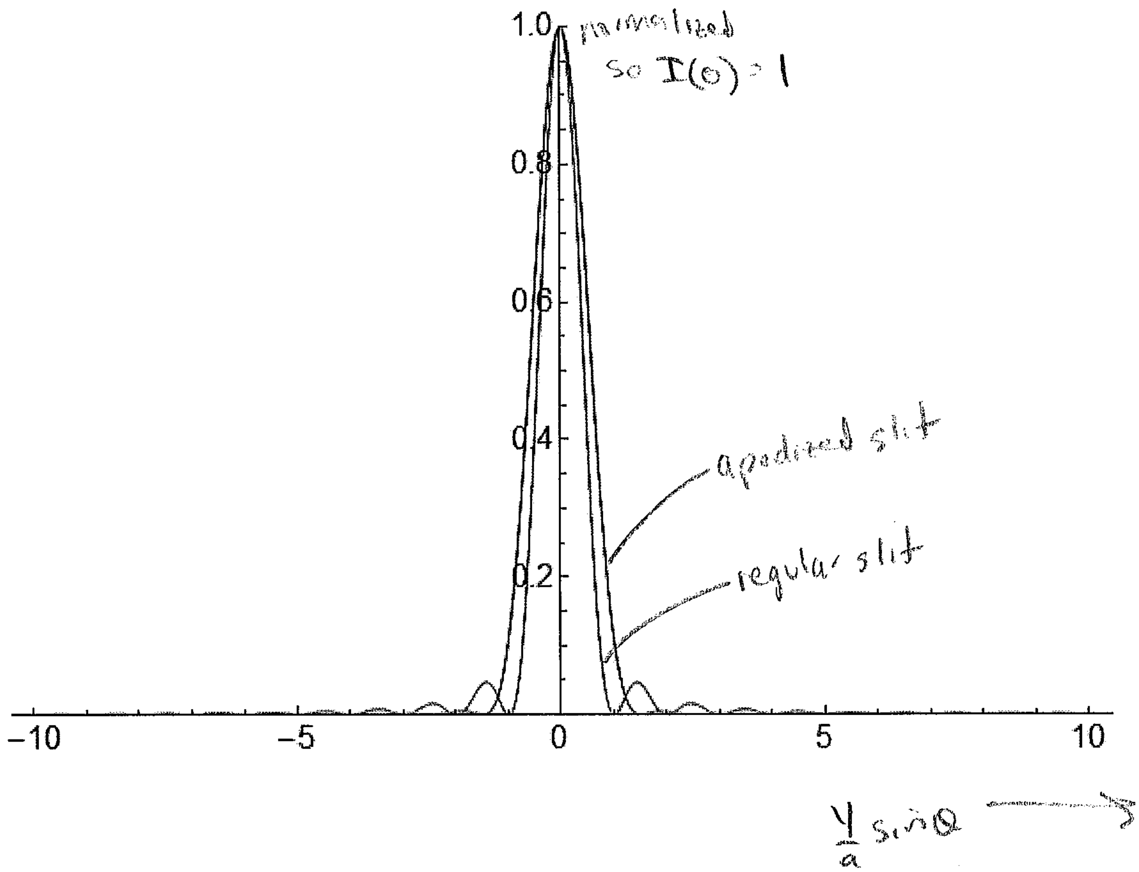
See graph on next page.

In[10]:= `apodizedintensity[b_] = Cos[Pi b]^2 / (Pi^2 - 4 Pi^2 b^2)^2`

Out[10]= $\frac{\mathrm{Cos}[b\,\pi]^2}{\left(\pi^2 - 4\,b^2\,\pi^2\right)^2}$

In[11]:= `Plot[{apodizedintensity[b] / apodizedintensity[0], Sinc[Pi b]^2}, {b, -10, 10}, PlotRange → {0, 1}]`

Out[11]=



Notice that although the apodized slit pattern is slightly wider than the regular slit pattern, the side lobes have been drastically cut down. I can imagine that that could be very beneficial