Phys123 Fall 2011 Final Exam Solns

Instructions:

- Record your answers to the multiple choice questions ("Problem 1" on the next page) on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(42 pts) **Problem 1:** Multiple choice conceptual questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. A boat is on a lake. If an anvil (that sinks) is pushed from the boat into the water, will the overall water level of the lake rise, fall or stay the same? (compared to when the anvil was in the boat)
   a. rise
   b. fall
   c. stay the same

In water it displaces its own volume. In boat it displaces its weight (buoyant force gets created), which is a larger volume of water than its own volume.

1.2. Three cubes of the same size and shape are put in water. They all sink. One is lead, one is steel and one is a dense wood (ironwood). $\rho_{lead} > \rho_{steel} > \rho_{ironwood}$. On which cube is the buoyant force the greatest?
   a. lead
   b. steel
   c. wood
   d. same buoyant force

B = weight of displaced water, same for all three

1.3. When a gas expands in a piston-like container:
   a. The gas does positive work on its surroundings.
   b. The surroundings do positive work on the gas.
   c. Work is not necessarily done.

Gas pushes against surroundings and does work on them

1.4. The "garden hose equation" ($A_1v_1 = A_2v_2$) is a statement of:
   a. conservation of energy
   b. conservation of linear momentum
   c. conservation of angular momentum
   d. conservation of mass/volume
   e. probability

$\frac{\text{volume}}{\text{sec}} = \frac{\text{volume}}{\text{sec}}$

1.5. Bernoulli's Law is a statement of:
   a. conservation of energy
   b. conservation of linear momentum
   c. conservation of angular momentum
   d. conservation of mass/volume
   e. probability

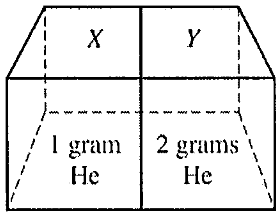
1.6. The first law of thermodynamics is a statement of:
   a. conservation of energy
   b. conservation of linear momentum
   c. conservation of angular momentum
   d. conservation of mass/volume
   e. probability

1.7. The second law of thermodynamics is a statement of:
   a. conservation of energy
   b. conservation of linear momentum
   c. conservation of angular momentum
   d. conservation of mass/volume
   e. probability

1.8. If a gas undergoes a thermodynamic change whereby it somehow ends up in the same state it started in:

a. The internal energy of the gas will be less than when it started.
b. The internal energy of the gas will be greater than when it started. b/c it is a "state variable"
c. The internal energy of the gas will be the same as when it started.
d. The change in internal energy will depend on the direction of the change (clockwise vs. counter-clockwise).

1.9. Cubical tanks X and Y have the same volumes and share a common wall as in the figure. There is 1 gram of helium gas in tank X and 2 grams of helium gas in tank Y. Everything is at the same temperature. Which of the following is the same for the gas in X compared to the gas in Y?



a. The average speed of the molecules → just depends on temperature
b. The density of the helium
c. The mean free path of the molecules
d. The number of collisions per second on the common wall
e. The pressure exerted by the helium

1.10. A thin rectangular piece of copper is 3 cm × 4 cm, with an area of 12 cm^2. (Take those numbers as exact.) It is heated from 0°C to 50°C. By how much does its area increase? The linear coefficient of expansion for copper is 17×10^{-6} /°C.

$\Delta A = (2\alpha) A \Delta T$
$= 2(17 \times 10^{-6})(12\ cm^2)(50)$
$= \boxed{.0204\ cm^2}$

a. Less than 0.009 cm^2
b. 0.009 - 0.012
c. 0.012 - 0.015
d. 0.015 - 0.018
e. 0.018 - 0.021
f. 0.021 - 0.024
g. 0.024 - 0.027
h. 0.027 - 0.030
i. More than 0.030 cm^2

1.11. A power plant takes in steam at 325°C (598 K) to power turbines and then exhausts the steam at 110°C (383 K). Each second the turbines transform 100 megajoules of heat energy from the steam into usable work. If the power plant operates at the theoretical maximum possible efficiency, what will its power output be?

$e_{max} = 1 - T_c/T_h = 1 - 383/598$
$= .3595$
$e = \frac{W}{Q_h} \to W = eQ_h = \boxed{35.95\ MJ}$

a. 0 - 5 megawatts
b. 5 - 10
c. 10 - 15
d. 15 - 20
e. 20 - 25
f. 25 - 30
g. 30 - 35
h. 35 - 40
i. 40 - 45
j. 45 - 50 megawatts

1.12. If you double the number of microstates available to a thermodynamic system, but how much does the entropy change?

$S = k_B \ln W$
$S_{new} = k_B \ln(2W_{old})$
$= k_B(\ln 2 + \ln W_{old})$
$= k_B \ln 2 + S_{old}$

a. $S_{new} = 2S_{old}$
b. $S_{new} = ½\, S_{old}$
c. $S_{new} = S_{old} \ln 2$
d. $S_{new} = S_{old} / \ln 2$
e. $S_{new} = S_{old} + 2k_B$
f. $S_{new} = S_{old} - 2k_B$
g. $S_{new} = S_{old} + k_B \ln 2$
h. $S_{new} = S_{old} - k_B \ln 2$

1.13. A wave packet is constructed by adding together an infinite number of cosine waves, all having amplitude 1:

$$f(x,t) = \cos\left(k_1(x - v_1 t)\right) + \cos\left(k_2(x - v_2 t)\right) + \ldots$$

$\bar{k} = 2$

The spatial frequencies present in the summation range from k = 1.5 to 2.5 and they are spaced infinitely close together. (All numbers in this problem are given in standard SI units.) Due to dispersion in the medium, waves with different k values travel at different velocities. For a given cosine term, the relationship between the velocity and the k-value is given by: $v = 3 + 7k^{-2}$. What is the phase velocity of the wave packet?

This is the eqn for phase velocity
$v = 3 + \frac{7}{2^2} = \boxed{4.75\ \frac{m}{s}}$

a. Less than 0.6 m/s
b. 0.6 - 1.2
c. 1.2 - 1.8
d. 1.8 - 2.4
e. 2.4 - 3.0
f. 3.0 - 3.6
g. 3.6 - 4.2
h. 4.2 - 4.8
i. 4.8 - 5.6
j. More than 5.6 m/s

1.14. Same situation. What is the group velocity of the wave packet?

From previous problem
$\frac{\omega}{k} = 3 + \frac{7}{k^2}$
$\omega = 3k + \frac{7}{k}$
$\frac{d\omega}{dk} = 3 - \frac{7}{k^2}$
$v_g = 3 - \frac{7}{2^2} = \boxed{1.25\ \frac{m}{s}}$

a. 0 - 0.6 m/s
b. 0.6 - 1.2
c. 1.2 - 1.8
d. 1.8 - 2.4
e. 2.4 - 3.0
f. 3.0 - 3.6
g. 3.6 - 4.2
h. 4.2 - 4.8
i. 4.8 - 5.6
j. More than 5.6 m/s

1.15. A certain sound undergoes a 15 dB increase in volume. By how much has the intensity of the sound increased?
I_{new}/I_{old} = ________ $I \sim 10^{\beta/10}$

+15 dB = $10^{15/10}$ = $\boxed{31.62}$

a. Less than 7
b. 7 - 12
c. 12 - 17
d. 17 - 22
e. 22 - 27
f. 27 - 32
g. 32 - 37
h. 37 - 42
i. 42 - 47
j. More than 47

1.16. A ray of white light, incident upon a glass prism, is dispersed into its various color components. Which one of the following colors experiences the greatest angle of refraction?

→ largest n, which is shortest λ Blue!

a. Blue
b. Green
c. Orange
d. Red
e. Yellow

1.17. A single mirror can produce a virtual image that is much smaller than the object. What type of mirror is it?

~~could be either~~ → but this can't be converging lens

a. converging
b. diverging
c. could be either

1.18. A single lens can take a large object and reduce it to a much smaller real image. What type of lens is it?

→ must be converging lens

a. converging
b. diverging
c. could be either

1.19. Which of the following best describes the image formed by a thin converging lens when the object is at a distance of ¼ the focal length? (Assume a positive object distance.)

$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{f} - \frac{1}{\frac{1}{4}f}\right)^{-1} = \left(\frac{-3}{f}\right)^{-1} = \frac{-f}{3}$ (virtual)

→ $M = -q/p = -\frac{-f/3}{f/4} = (4/3)$ enlarged, upright

a. real, inverted, and enlarged
b. real, inverted, and reduced
c. real, upright, and enlarged
d. real, upright, and reduced
e. virtual, inverted, and enlarged
f. virtual, inverted, and reduced
g. virtual, upright, and enlarged
h. virtual, upright, and reduced

1.20. The index of refraction of a lens is somehow doubled, from $n = 2$ to $n = 4$. How will the focal length of the lens change?

Lensmaker's Eqn: $\frac{1}{f} \sim n-1$

$\frac{f_{new}}{f_{old}} = \frac{n_{old}-1}{n_{new}-1} = \frac{1}{3}$

a. $f_{new} = 1/8\ f_{old}$
b. $f_{new} = 1/7\ f_{old}$
c. $f_{new} = 1/6\ f_{old}$
d. $f_{new} = 1/5\ f_{old}$
e. $f_{new} = 1/4\ f_{old}$
f. $f_{new} = 1/3\ f_{old}$
g. $f_{new} = 1/2\ f_{old}$
h. $f_{new} = f_{old}$

1.21. What type of image is produced by the eyepiece of a telescope?

a. real
b. virtual

It's on the same side of lens as object, so must be virtual

1.22. A puddle of water (n = 1.33) is covered with a very thin layer of oil (n = 1.20) with a slightly varying thickness across the puddle. Light with a wavelength of 550 nm is shined at the oil perpendicular to the surface, and in one place the light reflects back especially brightly (due to interference). What is the thinnest the oil could be in that region?

n = 1
n = 1.2
n = 1.33
180° 180° } same phase shift

ΔOPL = mλ → thinnest when m = 1

→ (1.2)(2t) = 550 nm

$t = \frac{550}{2.4}$ nm

$\boxed{t = 229.2\text{nm}}$

a. 80 - 120 nm
b. 120 - 160
c. 160 - 200
d. 200 - 240
e. 240 - 280
f. 280 - 320
g. 320 - 360
h. 360 - 400
i. 400 – 440 nm

1.23. A spaceship is heading toward an enemy space station at 0.5c and launches a missile at it that the spaceship crew sees leaving at 0.8 c. If the length of the missile is measured by various observers, who would measure the length to be the shortest?

space contraction's most for someone moving fastest relative to the missile

a. the ship's crew
b. the station's crew
c. an ant living on the missile

1.24. Sally is on earth watching Harry move toward earth on a rocket at 0.9c. Harry sees the earth moving toward him with a speed:

a. less than 0.9c

b. more than 0.9c

c. 0.9c

a velocity of .9c in the opposite direction, to be precise

1.25. Gonzor makes several bombs on Planet Zyzyx and attaches timers to them. He sets them to go off in 1 year. He leaves Planet Zyzyx to return his home planet a few light years away, quickly gets up to speed, and travels at ~~0.6c~~ 0.59c. However, he accidentally takes one of the bombs with him on his ship. According to the residents of Planet Zyzyx which goes off first?

a. bombs on Planet Zyzyx

b. bomb on Gonzor's ship

c. both go off together

→ They see Gonzor's time as dilated, so his bomb goes off later

1.26. Same situation. If Gonzor travels in a straight line, how far will he be from Planet Zyzyx when the bomb he carries goes off, as measured by the people on the planet?

a. 0 to 0.2 light years

b. 0.2 to 0.4

c. 0.4 to 0.6

d. 0.6 to 0.8

e. 0.8 to 1.0

f. 1.0 to 1.2

g. 1.2 to 1.4

h. 1.4 to 1.6

i. more than 1.6 light years

He's traveling at 0.59c, so you'd perhaps think $x = vt = (.59c)(1yr)$. However, his time is dilated, so

$x = (\gamma)(.59c)(1yr) = \frac{.59}{\sqrt{1-.59^2}}\,ly = .731\ ly$

1.27. A piece of sticky clay of mass M moves to the right at speed 0.8 c, and collides with another piece of clay of mass 2M, moving to the left at a speed of 0.4c. The clay sticks together (very strong clay), and this new mass will ________. (Hint: how do the momenta of the two pieces compare?)

a. move to the right

b. move to the left

c. be at rest

$p_1 = \gamma_1 m v_1 = \frac{1}{\sqrt{1-.8^2}} M \cdot .8c = +1.33Mc$

$p_2 = \gamma_2 (2m) v_2 = \frac{1}{\sqrt{1-.4^2}} (2M)(-.4c) = -.87$

$|p_1| > |p_2|$, so m_1 "wins"

1.28. When a one-megaton nuclear bomb is exploded, approximately 4.5×10^{15} J of energy is released. How much of the bomb's mass was converted into energy?

a. Less than 1 kg

b. 1 - 10

c. 10 - 100

d. 100 - 1,000

e. More than 1,000 kg

$E = mc^2$

$m = \frac{E}{c^2} = \frac{4.5 \cdot 10^{15}\ J}{(3 \cdot 10^8 \frac{m}{s})^2} = .05\ kg$

(9 pts) **Problem 2**. A particular electric stove burner has a surface area of 320 cm^2 (0.032 m^2) and an emissivity of 1. It is turned on and reaches a temperature of 500°F (533.15 K). The surroundings are at 300 K. How much electrical energy (in joules) is required to maintain the burner at that temperature for an hour? Disregard energy loss through convection and conduction. Hint: conservation of energy says that the electrical energy supplied + energy absorbed from environmental radiation = energy lost through radiation.

$$P_{supplied} + e\sigma A T^4_{surroundings} = e\sigma A T^4_{object}$$

$$P_{supplied} = e\sigma A\left(T^4_{obj} - T^4_{surround}\right)$$

$$= (1)\left(5.67\cdot 10^{-8}\ \frac{W}{m^2K^4}\right)(.032m^2)\left[(533.15K)^4 - (300K)^4\right]$$

$$P = 131.902\ W$$

$$E = P\cdot t$$

$$= (131.902\ W)(3600\ \text{seconds})$$

$$= \boxed{474{,}848\ J}$$

(10 pts) **Problem 3**. A nuclear power plant has an electrical power output of 420 MW and operates with an efficiency of 29%. The waste heat is carried away from the plant by a river. If the waste heat is sufficient to make the river water's temperature rise by 1.5°C, what is the flow rate (kg/s) of the river? The specific heat of water is 4186 J/kg·°C.

$$e = \frac{W}{Q_h} \longrightarrow Q_h = \frac{W}{e} \qquad Q_c = Q_h - W = \frac{W}{e} - W$$

$$= W\left(\frac{1}{e} - 1\right)$$

so $$\frac{Q_c}{time} = \frac{W}{time}\left(\frac{1}{e} - 1\right)$$

Also $Q_c = mc\Delta T$ since exhaust heat warms the water

So $$\frac{mc\Delta T}{time} = \frac{W}{time}\left(\frac{1}{e} - 1\right)$$

$$\frac{m}{time} = \frac{Power}{c(\Delta T)}\left(\frac{1}{e} - 1\right)$$

$$= \frac{420 \cdot 10^6\ W}{\left(4186\ \frac{J}{kg\,°C}\right)(1.5°C)}\left(\frac{1}{.29} - 1\right)$$

$$= \boxed{163764\ \frac{kg}{s}}$$

(10 pts) **Problem 4.**

(a) Superman flies towards you at half the speed of sound. He sings a pure sinusoidal tone of the A above middle C, 440 Hz (in his frame). (Apparently Superman is a high tenor.) What frequency do you hear? (Use 343 m/s as the speed of sound.)

Sound Doppler: $f' = f \dfrac{v \mp v_o}{v \pm v_s}$

$= (440\text{ Hz}) \underbrace{\dfrac{343}{343 - \frac{1}{2}(343)}}_{\frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2}$

$f' = 2 \times 440\text{ Hz}$

$= \boxed{880\text{ Hz}}$

(b) Superman flies towards you at 0.5 c. He turns on a flashlight with λ = 500 nm (in his frame). What wavelength do you see?

Light Doppler: $f' = f \sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}}$

$f = \dfrac{c}{\lambda}$, so

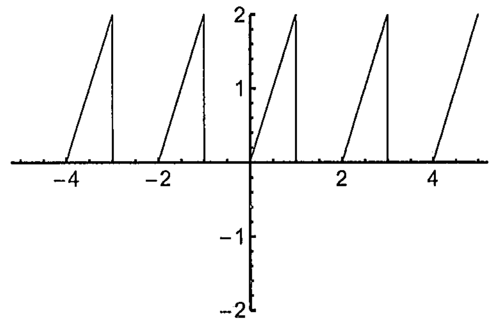
$\dfrac{\cancel{c}}{\lambda'} = \dfrac{\cancel{c}}{\lambda} \underbrace{\sqrt{\dfrac{1+.5}{1-.5}}}_{\sqrt{\frac{1.5}{.5}} = \sqrt{3}}$

$\lambda' = \dfrac{\lambda}{\sqrt{3}} = \dfrac{500\text{ nm}}{\sqrt{3}} = \boxed{288.7\text{ nm}}$

(10 pts) **Problem 5**. The function f(x), graphed on the right, is defined as follows:

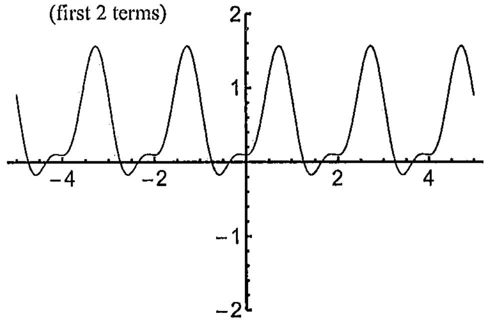
$$f(x) = \begin{cases} 2x, & \text{for } x \text{ between } 0 \text{ and } +1 \\ 0, & \text{for } x \text{ between } -1 \text{ and } 0 \end{cases}$$

(repeated with a period of $L = 2$)

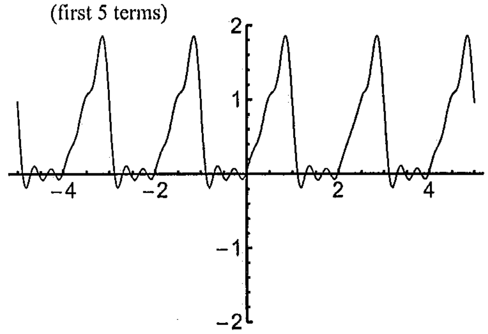


As with any periodic function, this can be turned into a Fourier series. Plots of f(x) for increasing numbers of terms in the Fourier summation are also shown.

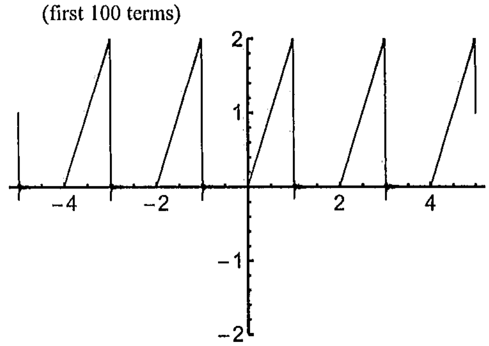
(first 2 terms)



(first 5 terms)



(first 100 terms)



(a) What will the constant term of the series be equal to?

$$a_0 = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\,dx = \frac{1}{2}\underbrace{\int_0^1 2x\,dx}_{x^2\big|_0^1 = 1-0 = 1}$$

$$= \boxed{\frac{1}{2}}$$

(b) Will there be sine terms in the series, cosine terms, or both? Justify your answer. The function is not odd and not even, nor can it be made odd/even by adding/subtracting a constant. Therefore both sine and cosine terms will be present.

(c) What k-value will all the spatial frequencies in the series be multiples of?

$$k_0 = \frac{2\pi}{L} = \frac{2\pi}{2} = \boxed{\pi}$$

(d) Write down an equation that you could use to determine the coefficient of the 3rd cosine term of the series, assuming it is non-zero. You don't have to solve the equation, but it should be simplified enough that you could just type it into Mathematica to obtain its numerical value.

$$a_3 = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\cos(3k_0 x)\,dx$$

$$= \frac{2}{2}\int_0^1 2x\cos(3\pi x)\,dx$$

$$= \boxed{2\int_0^1 x\cos(3\pi x)\,dx}$$

(e) Suppose instead of representing a function of x, the function represents a voltage changing in time. That is, the x-coordinate is time and the y-coordinate is a voltage. As discussed in one of the homework problems, an AC voltage can be filtered (with a capacitor) to create a DC voltage. If you wanted to make a 5 V DC power supply out of this AC voltage, it would clearly not work since the peak AC voltage is much lower than 5 V (as shown, it is only 2 V). How large would you need to make the peak AC voltage amplitude to create a 5 V DC power supply?

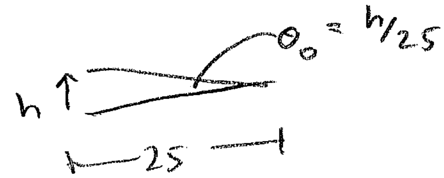
↳ a_0 would need to be increased by ×10

So the peak amplitude would need to be increased from 2 to $\boxed{20V}$

(12 pts) **Problem 6.** Suppose you want to use a lens with a focal length of 5 cm as a magnifying glass to look at an ant which is sitting on your finger. You first look at the ant from 25 cm away. (You cannot focus on the ant when it is closer than 25 cm to your eye.) You then put your eye up to the lens and adjust the position of the ant until the image of the ant is 35 cm from your eye. What is the angular magnification m, that you have obtained?

$\rightarrow q = -35$

initial

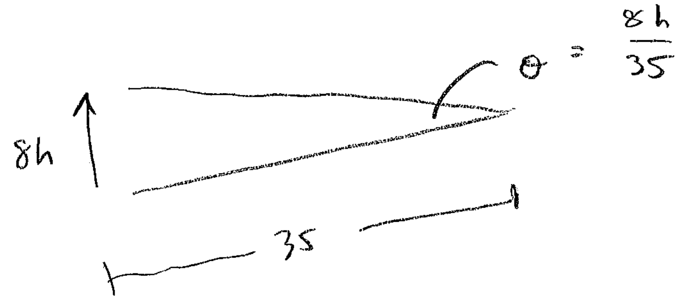


(using small angle approx.)

with lens

$$p = \left(\frac{1}{f} - \frac{1}{q}\right)^{-1} = \left(\frac{1}{5} - \frac{1}{-35}\right)^{-1} = 4.375$$

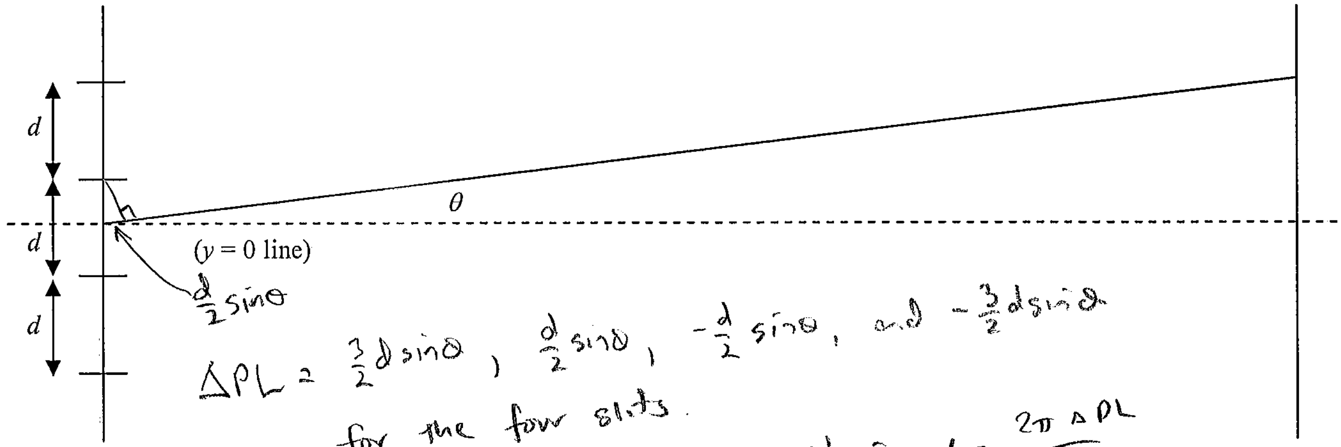
$$M = -q/p = -\frac{-35}{4.375} = +8$$



(using small angle approx.)

$$m = \frac{\theta}{\theta_0} = \frac{8\cancel{h}/35}{\cancel{h}/25} = \frac{8 \cdot 25}{35} = \boxed{5.714}$$

(12 pts) **Problem 7.** Derive the diffraction pattern for a 4-slit situation: successive slits are separated by a distance d as in the figure. Obtain the intensity as a function of θ, d, and λ that would show up on a screen a distance away from the slits as shown. Put your answer in terms of an arbitrary overall maximum intensity, I_0. Simplify your answer as much as you can. Hint: measure your phase relative to the center of the four slits. Remember these two identities: $\cos x = (e^{ix} + e^{-ix})/2$; $\sin x = (e^{ix} - e^{-ix})/(2i)$.



$\Delta PL = \frac{3}{2} d\sin\theta$, $\frac{d}{2}\sin\theta$, $-\frac{d}{2}\sin\theta$, and $-\frac{3}{2}d\sin\theta$

for the four slits.

For a given ΔPL, the phase shift is $\phi = \frac{2\pi\,\Delta PL}{\lambda}$

The electric field at the position on the screen is:

$$E_{tot} = E_0\left[\underbrace{e^{i\frac{2\pi}{\lambda}\left(\frac{3}{2}d\sin\theta\right)} + e^{i\frac{2\pi}{\lambda}\left(-\frac{3}{2}d\sin\theta\right)}}_{=2\cos\left(\frac{2\pi}{\lambda}\frac{3}{2}d\sin\theta\right)} + \underbrace{e^{i\frac{2\pi}{\lambda}\left(\frac{d}{2}\sin\theta\right)} + e^{i\frac{2\pi}{\lambda}\left(-\frac{d}{2}\sin\theta\right)}}_{=2\cos\left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)}\right]$$

$$E_{tot} = 2E_0\left[\cos\left(\frac{3\pi d}{\lambda}\sin\theta\right) + \cos\left(\frac{\pi d}{\lambda}\sin\theta\right)\right]$$

Since $I \sim |E|^2$

$$\boxed{I = I_0\left[\cos\left(\frac{3\pi d}{\lambda}\sin\theta\right) + \cos\left(\frac{\pi d}{\lambda}\sin\theta\right)\right]^2}$$

or really to make $I = I_0$ at the maximum $(\theta = 0)$, it should be

$$\boxed{I = \frac{I_0}{4}\left[\cos\left(\frac{3\pi d}{\lambda}\sin\theta\right) + \cos\left(\frac{\pi d}{\lambda}\sin\theta\right)\right]^2}$$

(11 pts) **Problem 8.** Muons are known to have an average lifetime of 2.197×10^{-6} s before decaying into an electron plus two neutrinos. That measurement is for muons at rest relative to the observer. However, in a classic experiment in 1943, researchers measured the lifetime of muons moving through the atmosphere to be much longer than that, due to relativistic time dilation. The momentum of the muons in their experiment was measured to be approximately 5×10^8 eV/c. The rest mass of a muon is now known to be 105.658×10^6 eV/c^2.

(a) How many kg·m/s is 5×10^8 eV/c ?

$$p = 5\cdot 10^8 \frac{\cancel{eV}}{c} \times \frac{1.602\cdot 10^{-19}\,J}{1\,\cancel{eV}} \times \frac{1c}{3\cdot 10^8 \frac{m}{s}}$$

$$p = \boxed{2.67\cdot 10^{-19}\ kg\frac{m}{s}}$$

(b) How many kg is 105.658×10^6 eV/c^2 ?

$$m = 105.658\cdot 10^6 \frac{\cancel{eV}}{c^2} \times \frac{1.602\cdot 10^{-19}\,J}{\cancel{eV}} \times \left(\frac{1c}{3\cdot 10^8 \frac{m}{s}}\right)^2$$

$$= \boxed{1.881\cdot 10^{-28}\ kg}$$

(c) How fast were the muons in the experiment traveling?

$$p = \gamma m v = \gamma m(\beta c)$$

$$5\cdot 10^8 \frac{\cancel{eV}}{\cancel{c}} = \gamma\left(105.658\cdot 10^6 \frac{\cancel{eV}}{\cancel{c^2}}\right)\beta \cancel{c}$$

$$\gamma\beta = 4.7322 \rightarrow \text{call this "x"}$$

$$\frac{\beta}{\sqrt{1-\beta^2}} = x$$

$$\beta = x\sqrt{1-\beta^2}$$

$$\beta^2 = x^2(1-\beta^2)$$

$$\beta^2 = x^2 - x^2\beta^2$$

$$\beta^2(1+x^2) = x^2$$

$$\beta^2 = \frac{x^2}{1+x^2}$$

$$\beta = \sqrt{\frac{x^2}{1+x^2}}$$

$$= \sqrt{\frac{4.7322^2}{1+4.7322^2}}$$

$$\beta = .9784$$

$$\boxed{v = .9784c}$$

(d) What was the average lifetime of the muons in their experiment?

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-.9784^2}} = 4.8368$$

Lifetime of $2.197\cdot 10^{-6}$ s is dilated by factor

$$\text{measured life-time} = (2.197\cdot 10^{-6}\,s)(4.8368) = \boxed{1.0626\cdot 10^{-5}\,s}$$

10.63 μs

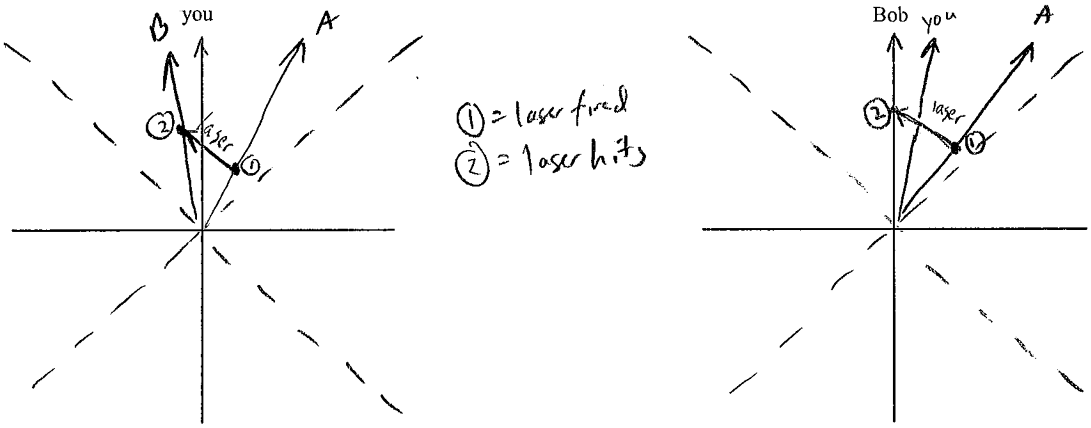
(14 pts) **Problem 9.** You are watching Alice and Bob go past the earth in two rocket ships: Alice at 0.5c and Bob at –0.3c. (I.e., Alice is traveling to your right and Bob is traveling to your left.) They both go past you at the same instant in time, call that $t = 0$. Exactly 1 second later (in your reference frame), Alice fires a laser beam at Bob.

(a) How fast is Alice going in Bob's reference frame?

$$\beta_{AB} = \frac{\beta_{A\text{-}you} + \beta_{you\text{-}B}}{1 + \beta_{A\text{-}you} \times \beta_{you\text{-}B}} = \frac{.5 + .3}{1 + (.5)(.3)} = \boxed{.6957}$$

$V = .6957c$

(b) Draw two fairly accurate space-time diagrams of the situation, one for your frame of reference and one for Bob's frame. Include all three worldlines (you, Alice, Bob), the laser beam, and the "laser gets fired" and "laser hits" events.



① = laser fired
② = laser hits

(c) When/where does Alice fire the laser in your frame? (use units of seconds and light-seconds)

$t = 1$ sec

$x = vt = (.5c)(1 \text{ sec}) = .5$ light seconds

$(x, t) = \boxed{(.5, 1)}$

(d) When/where does Alice fire the laser in Bob's frame? (use units of seconds and light-seconds) → then $c = 1$

$$\begin{pmatrix} x \\ t \end{pmatrix}_{Bob} = \begin{pmatrix} \gamma & +\gamma\beta \\ +\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}_{you}$$

From you to Bob, $\beta = .3$

$\gamma = \frac{1}{\sqrt{1 - .3^2}} = 1.0483$

$\gamma\beta = .31449$

$$= \begin{pmatrix} 1.0483 & .31449 \\ .31449 & 1.0483 \end{pmatrix} \begin{pmatrix} .5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} .83864 \\ 1.2055 \end{pmatrix}$$

$\boxed{\begin{array}{l} x = .83864 \text{ light seconds} \\ t = 1.2055 \text{ seconds} \end{array}}$

(10 pts) **Problem 10.**

$\gamma = 1.40028$ $\quad \gamma = 7.08881$

(a) How much work (in joules) is required to accelerate an electron from 0.70*c* to 0.990*c*?

$$W = \Delta KE = (\gamma_{new}-1)mc^2 - (\gamma_{old}-1)mc^2$$

$$= (\gamma_{new} - \gamma_{old})mc^2$$

$$= (7.08881 - 1.40028)(9.11\cdot10^{-31}\text{ kg})(3\cdot10^8 \tfrac{m}{s})^2$$

$$= \boxed{4.664\cdot10^{-13}\text{ J}}$$

(b) The momentum of a photon is related to its wavelength through the equation: $p = \frac{h}{\lambda}$, where p is the momentum, λ is the wavelength, and h is "Plank's constant" equal to 6.626×10^{-34} J·s (joules × seconds). If a 0.0100 nm photon (x-ray) collides with an electron and then it recoils straight backwards, what will its new wavelength be? (Hint: this relates to the "Compton shift" homework problem.)

Compton shift eqn for a straight backwards recoil:

$$\frac{1}{p'_{photon}} - \frac{1}{p_{photon}} = \frac{2}{m_e c}$$

$$\frac{\lambda'}{h} - \frac{\lambda}{h} = \frac{2}{m_e c}$$

$$\lambda' = \lambda + \frac{2h}{m_e c}$$

$$= .01\text{ nm} + \frac{2(6.626\cdot10^{-34})}{(9.11\cdot10^{-31})(3\cdot10^8)}\text{ m}$$

$$= .01\text{ nm} + .00485\text{ nm}$$

$$= \boxed{.01485\text{ nm}}$$

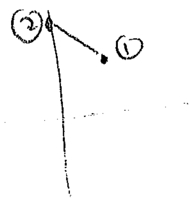
$1.485\cdot10^{-11}$ m

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)

(a) For the situation in problem 9, when/where does the laser reach Bob in your frame? In Bob's frame? (You must answer all four of these correctly for credit.)

Bob's frame

① $(x,t) = (.83864, 1.2055)$



45° line ⟹

② $= (0, .83864 + 1.2055)$

$= (0, 2.0441)$

$\boxed{x = 0,\ t = 2.0441 \text{ secs}}$

Your frame

$$\begin{pmatrix} x \\ t \end{pmatrix}_{you} = \begin{pmatrix} 1.0483 & -.31449 \\ -.31449 & 1.0483 \end{pmatrix} \begin{pmatrix} 0 \\ 2.0441 \end{pmatrix} = \begin{pmatrix} -.64285 \\ 2.14283 \end{pmatrix}$$

$\boxed{\begin{array}{l} x = -.64285 \text{ light seconds} \\ t = 2.14283 \text{ seconds} \end{array}}$

(b) A piece of rubbery clay of mass M moves to the right at speed 0.8c, and collides with another piece of rubbery clay of mass 2M, moving to the left at a speed of 0.4c. The clay pieces bounce off of each other elastically but remain intact (very strong clay). Set up two equations that you could use to solve for the two final velocities of M and 2M. Call the two velocities v_1 and v_2, respectively. The two equations must have no other unknowns in them except for v_1 and v_2, so that you could (for example) type them into Mathematica to solve. (For simplicity, please use β's in your equations instead of v's.) If you are curious, I believe the two velocities turn out to be $v_1 = -0.965c$, $v_2 = 0.581c$.

cons momentum

$$\gamma \cancel{m} v + \gamma(2\cancel{m})v = \gamma \cancel{m} v + \gamma(2\cancel{m})v$$

$$\boxed{\frac{.8}{\sqrt{1-.8^2}} - \frac{2(.4)}{\sqrt{1-.4^2}} = \frac{\beta_1}{\sqrt{1-\beta_1^2}} + \frac{2\beta_2}{\sqrt{1-\beta_2^2}}}$$

cons. energy

$$\gamma \cancel{m} \cancel{c^2} + \gamma(2\cancel{m})\cancel{c^2} = \gamma \cancel{m} \cancel{c^2} + \gamma(2\cancel{m})\cancel{c^2}$$

$$\boxed{\frac{1}{\sqrt{1-.8^2}} + \frac{2}{\sqrt{1-.4^2}} = \frac{1}{\sqrt{1-\beta_1^2}} + \frac{2}{\sqrt{1-\beta_2^2}}}$$