Solutions

RED

Fall 2012
Physics 123 section 2
Final Exam
Colton 2-3669

Please write your CID here __________

No time limit. No notes, no books. Student calculators OK.

Constants and conversión factors which you may or may not need:

$g = 9.8$ m/s^2
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
$k_B = 1.381 \times 10^{-23}$ J/K
$N_A = 6.022 \times 10^{23}$
$R = k_B \cdot N_A = 8.314$ J/mol·K
$\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
$c = 3 \times 10^8$ m/s
$m_{electron} = 9.11 \times 10^{-31}$ kg
Density of water: 1000 kg/m^3
1 inch = 2.54 cm
1 m^3 = 1000 L
1 atm = 1.013×10^5 Pa = 14.7 psi
1 eV = 1.602×10^{-19} J
$T_F = 9/5\, T_C + 32$
$T_K = T_C + 273.15$

Other equations which you may or may not need to know:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A_{sphere} = 4\pi r^2$

$V_{sphere} = 4/3\, \pi r^3$

$(1+x)^n \approx 1 + nx$

$\Delta L = \alpha L_0 \Delta T$; $\Delta V = \beta V_0 \Delta T$; $\beta = 3\alpha$

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_B T}$$

$v_{most\ probable} = \sqrt{\dfrac{2k_B T}{m}}$

$v_{avg} = \int_0^\infty v f(v)\, dv = \sqrt{\dfrac{8k_B T}{\pi m}}$

$v_{rms} = \sqrt{\int_0^\infty v^2 f(v)\, dv} = \sqrt{\dfrac{3k_B T}{m}}$

Mean free path: $l = \dfrac{1}{\sqrt{2}\pi d^2 n}$

Ave time between collisions: $\tau = l/v_{avg}$

$P = \dfrac{\Delta Q}{\Delta t} = \dfrac{kA\Delta T}{L} = \dfrac{A\Delta T}{R}$; $R = L/k$

$P = \dfrac{\Delta Q}{\Delta t} = e\sigma A T^4$

$e_{Otto} = 1 - \dfrac{1}{r^{\gamma-1}}$; $r = V_{max}/V_{min}$

$S = k_B \ln W$

$\#\, microstates\,(k = \#\, heads) = \dbinom{N}{k} = \dfrac{N!}{k!(N-k)!}$

$\#\ microstates\ (total) = 2^N$

$P = \frac{1}{2}\mu\omega^2 A^2 v$

$r = \dfrac{v_2 - v_1}{v_2 + v_1} = \dfrac{n_1 - n_2}{n_1 + n_2}$; $t = \dfrac{2v_2}{v_2 + v_1} = \dfrac{2n_1}{n_1 + n_2}$

$R = |r|^2$; $T = 1 - R$

$v_{string} = \sqrt{T/\mu}$; $\mu = m/L$

$v_{rod} = \sqrt{Y/\rho}$; $Y = \dfrac{stress}{strain} = \dfrac{F/A}{\Delta L/L}$

$v_{sound} = \sqrt{B/\rho}$

$v_{sound} = 343\,\frac{\text{m}}{\text{s}} \sqrt{\dfrac{T}{293\text{K}}}$

$\beta = 10\log\left(\dfrac{I}{I_0}\right)$; $I = I_0 10^{\beta/10}$; $I_0 = 10^{-12}$ W/m^2

$f' = f\dfrac{v \pm v_o}{v \pm v_s}$

$\sin\theta = 1/\text{Mach}\#$

$\Delta x \Delta k \geq \frac{1}{2}$; $\Delta x \Delta p \geq \hbar/2$

$\Delta t \Delta \omega \geq \frac{1}{2}$; $\Delta t \Delta E \geq \hbar/2$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n x}{L}\right)$$

$a_0 = \dfrac{1}{L}\int_0^L f(x)\,dx$

$a_n = \dfrac{2}{L}\int_0^L f(x)\cos\left(\dfrac{2\pi n x}{L}\right)dx$

$b_n = \dfrac{2}{L}\int_0^L f(x)\sin\left(\dfrac{2\pi n x}{L}\right)dx$

musical half step: $f_2/f_1 = 2^{1/12}$

$\tan\theta_{Brewster} = n_2/n_1$

$f = R/2$

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

(R_1 = pos, R_2 = neg if convex-convex)

$\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$

(p = pos if object in front of surface, q = pos if image in back of surface, R = pos if center of curvature in back of surface)

$\phi = 2\pi \Delta PL/\lambda$

$\Delta PL = d\sin\theta$

$E = E_0\,(e^{i\phi_1} + e^{i\phi_2} + \ldots)$

$I \sim |E|^2$

2 narrow slit: $I = I_0 \cos^2\left(\dfrac{2\pi}{\lambda}\dfrac{d}{2}\sin\theta\right)$

1 wide slit: $I = I_0 \text{sinc}^2\left(\dfrac{\pi a \sin\theta}{\lambda}\right)$

circular: $\theta_{min.resolve} = 1.22\lambda/D$

grating: $R = \lambda_{ave}/\Delta\lambda = \#\text{slits} \times m$

Bragg: $2d\sin\theta_{bright} = m\lambda$ (θ from horizontal)

$f' = f\sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}}$

$x_{frame2} = \gamma x_{frame1} \pm \gamma\beta(ct)_{frame1}$

$(ct)_{frame2} = \pm\gamma\beta x_{frame1} + \gamma(ct)_{frame1}$

$\begin{pmatrix} x \\ ct \end{pmatrix}_{frame2} = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}_{frame1}$

$E^2 = (pc)^2 + (mc^2)^2$; $E = pc$

$1/p'_{photon} - 1/p_{photon} = 2/(m_{electron}c)$

Scores: (for grader to fill in). **100 total points.**

Problem 1 __________

Problem 2 __________

Problem 3 __________

Problem 4 __________

Problem 5 __________

Problem 6 __________

Problem 7 __________

Problem 8 __________

Extra Credit __________

Total __________

Instructions:

- Record your answers to the multiple choice questions ("Problem 1" on the next page) on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(27 pts) **Problem 1**: Multiple choice conceptual questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

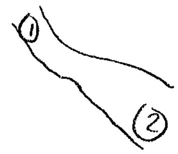
1.1. As an airplane flies horizontally at a constant elevation, the pressure above a wing is __________ the pressure below the wing.

a. larger than
b. smaller than
c. the same as

1.2. Water (no viscosity, incompressible) flows from a little pipe into a big pipe while also decreasing in height. That is, the water is flowing downhill. The volume flow rate (m^3/s) in the little pipe will be ______ in the big pipe.

a. greater than
b. the same as
c. less than
d. cannot be determined from the information given

VFR is constant as long as the density isn't changing



1.3. Same situation. The flow speed (m/s) in the little pipe will be ______ in the big pipe.

a. greater than
b. the same as
c. less than
d. cannot be determined from the information given

Think "garden hose eqn"

1.4. Same situation. The pressure in the little pipe will be ______ in the big pipe.

a. greater than
b. the same as
c. less than
d. cannot be determined from the information given

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_1 = P_2 + \underbrace{\rho g(h_2 - h_1)}_{\text{negative}} + \underbrace{\tfrac{1}{2}\rho(v_2^2 - v_1^2)}_{\text{also negative}}$$

$$P_1 < P_2$$

1.5. You have two balloons that are the same size, one filled with air and one filled with helium. If you put both balloons into a tub of liquid nitrogen, which one will end up with the largest volume?

a. the air balloon
b. the helium balloon
c. they will end up with the same volume

Demo done in class - the air inside the air balloon condenses into a liquid

1.6. Gas A is composed of molecules which are twice as massive as the molecules in Gas B. Gas A is also at twice the temperature (kelvin) as gas B. Which is true about the rms speed of molecules in gas A compared to gas B?

a. $v_{rms,A} = \frac{1}{4} v_{rms,B}$
b. $v_{rms,A} = \frac{1}{2} v_{rms,B}$
c. $v_{rms,A} = \frac{1}{\sqrt{2}} v_{rms,B}$
d. $v_{rms,A} = v_{rms,B}$
e. $v_{rms,A} = \sqrt{2}\, v_{rms,B}$
f. $v_{rms,A} = 2 v_{rms,B}$
g. $v_{rms,A} = 4 v_{rms,B}$

$$\frac{v_A}{v_B} = \frac{\sqrt{3kT_a/m_a}}{\sqrt{3kT_b/m_b}} = \underbrace{\sqrt{\frac{T_a}{T_b}}}_{\sqrt{2}} \times \underbrace{\sqrt{\frac{m_b}{m_a}}}_{\sqrt{\frac{1}{2}}}$$

1.7. How many degrees of freedom does each atom in a solid have, according to the Dulong-Petit law?

a. 0
b. 1
c. 2
d. 3
e. 4
f. 5
g. 6
h. 7
i. 8
j. 9

3 vibrational KE
+3 vibrational PE

1.8. The probability density function for the Maxwell-Boltzmann velocity distribution is: $f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$.

What would be the appropriate integral to calculate how many molecules have speeds between 200 and 250 m/s?

a. $N_{tot} \times \int_{200}^{250} f(v)dv$ = Probability any given one is between 200 → 250 $\frac{m}{s}$

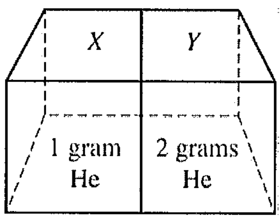
b. $N_{tot} \times \int_{200}^{250} (f(v))^2 dv$

c. $N_{tot} \times \int_{200}^{250} v \cdot f(v)dv$

d. $N_{tot} \times \int_{200}^{250} v^2 \cdot f(v)dv$

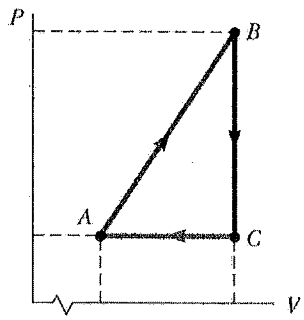
e. $N_{tot} \times \sqrt{\int_{200}^{250} v^2 \cdot f(v)dv}$

1.9. Cubical tanks X and Y have the same volumes and share a common wall as in the figure. There is 1 gram of helium gas in tank X and 2 grams of helium gas in tank Y. Everything is at the same temperature. Which of the following is the same for the gas in X compared to the gas in Y?



a. The average speed of the molecules ✓ Just depends on temperature
b. The density of the helium X volume is same but mass is different
c. The mean free path of the molecules X depends on # per volume
d. The number of collisions per second on the common wall X depends on P, and T
e. The pressure exerted by the helium X $PV = nRT \rightarrow V$ and T are the same but n is different.
f. More than one of the above

1.10. For the next three problems, consider the cyclic process described by the figure. For A to B: is $W_{\text{on gas}}$ positive, negative, or zero?



a. Positive
b. Negative volume is expanding so gas is doing work and W_{on} = neg.
c. Zero
d. Can't tell without more details

1.11. For B to C: does the internal energy increase, decrease, or stay the same?

a. Increase
b. Decrease Temp is decreasing
c. Stays the same ($\Delta E_{int} = 0$)
d. Can't tell without more details

1.12. For C to A: is heat added or taken away from the gas? ΔE = neg

a. Added
b. Taken away Temp is decreasing and work is +
c. Neither ($Q_{added} = 0$) $\Delta E_{int} = Q + W_{on}$
d. Can't tell without more details neg = Q + pos → Q = neg.

1.13. Two ideal gases undergo an adiabatic compression where the volume decreases to 50% of the original amount. They each have one mole of molecules, but gas A is monatomic whereas gas B is diatomic. Which gas will end up at the higher temperature?

a. Gas A
b. Gas B
c. Same temperature
d. Can't tell without more details

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$\left(\frac{nRT_1}{V_1}\right) V_1^{\gamma} = \left(\frac{nRT_2}{V_2}\right) V_2^{\gamma}$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Now can compare: monot $T_2 = T_1 (2)^{5/3-1} = 1.587 T_1$ higher!

diat $T_2 = T_1 (2)^{7/5-1} = 1.320 T_1$

773K 413K

1.14. A large power plant takes in steam at 500°C to power turbines and then exhausts the steam at 140°C. Each second the fuel powering the turbines produces 100 megajoules of heat energy, which is then used to produce work. If the power plant operates at the theoretical maximum possible efficiency (according to the Carnot theorem), what will its power output be?

a. 0 - 5 megawatts
b. 5 - 10
c. 10 - 15
d. 15 - 20
e. 20 - 25
f. 25 - 30
g. 30 - 35
h. 35 - 40
i. 40 - 45
j. 45 - 50 megawatts

$e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{413}{773} = 46.6\%$

$e = \frac{W}{Q_h} \rightarrow W = eQ_h = \boxed{46.6\ MJ}$

1.15. Which of the following statements about the Carnot cycle is false?

a. A Carnot cycle contains two isotherms and two adiabats. True!
b. An engine using a Carnot cycle can be operated in reverse as a refrigerator without changing the environment. True!
c. Because of their high efficiency, Carnot cycles are commonly used in commercial power stations. False! Not used at all
d. Carnot cycles are often useful for theoretical comparisons with other cycles. True!

1.16. As I'm taking data in my lab, I typically average data for 1 second per point. Suppose I decide to average the data for 10 seconds per point instead, so the scan takes ten times as long. How much better is my signal-to-noise ratio likely to be?

a. $\sqrt{10}$ times better
b. 5 times better
c. 10 times better
d. 20 times better
e. 100 times better

As per the HW problem, Sig to Noise $\sim \sqrt{time}$

1.17. Consider the following two changes to an ideal gas: (1) the temperature is doubled in a constant pressure process, vs. (2) the temperature is doubled in a constant volume process. In which of the two changes is there the largest change in entropy?

a. Constant pressure
b. Constant volume
c. Same change in entropy

$\Delta S = \int \frac{dQ}{T} = \int \frac{nC\,dT}{T} = nC \ln T_f/T_i$

could be either C_p or C_v

C_p always $> C_v$, so ΔS biggest for C_p

1.18. A gas undergoes an adiabatic expansion. The gas and the thermal reservoir which surrounds it are isolated from the rest of the universe. Which of the following is true?

a. The entropy of the gas will increase. The entropy of the reservoir will increase.
b. The entropy of the gas will increase. The entropy of the reservoir will decrease.
c. The entropy of the gas will increase. The entropy of the reservoir will stay the same.
d. The entropy of the gas will decrease. The entropy of the reservoir will increase.
e. The entropy of the gas will decrease. The entropy of the reservoir will decrease.
f. The entropy of the gas will decrease. The entropy of the reservoir will stay the same.
g. The entropy of the gas will stay the same. The entropy of the reservoir will increase.
h. The entropy of the gas will stay the same. The entropy of the reservoir will decrease.
i. The entropy of the gas will stay the same. The entropy of the reservoir will stay the same.

The gas is surrounded by a reservoir, but is not exchanging heat with it (adiabatic $\rightarrow Q=0$),

Since $\Delta S = \int \frac{dQ}{T}$, ($\Delta S$ must be zero) (for both)

(9 pts) **Problem 2.** Give short answers/explanations to the following questions:

(a) Why is there a maximum length to how long a straw can be (and still function)?

There is no "sucking force", just a "pushing force" from the atmosphere. That's a non-infinite number, so there's a non-infinite distance the atmosphere can push the liquid.

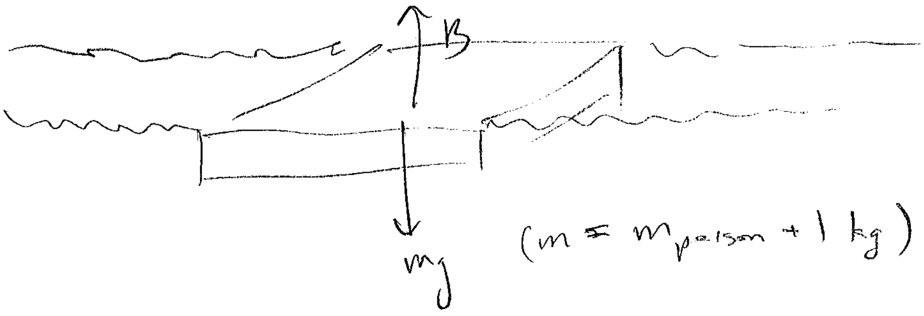
(b) The thermal conductivities of air and fiberglass are listed in your book as 0.0234 and 0.045 J/(s·m·°C), respectively. It looks like air is a better thermal insulator than fiberglass is… so why is it beneficial to use fiberglass insulation in your attic?

Fiberglass helps reduce convection, not conduction.

(c) Why is $\int_0^\infty v \cdot f(v)dv$ the appropriate formula to use to calculate the average speed of all molecules (as given on pg 1 of the exam)? What does this equation "mean"?

It's adding up all the velocities, weighted by how many molecules are at a given velocity. That's just like when calculating the average height of the class you need to multiply each height by how many students are that tall.

(10 pts) **Problem 3**. A rectangular floatable air mattress is designed for use in a swimming pool. It is 1.5 m long, 0.7 m wide, and 0.1 m deep, and has a mass of 1 kg. What is the maximum weight (or mass, really) of a person that it can support?



$(m = m_{person} + 1\text{ kg})$

Equilibrium $\rightarrow mg = B$

max B when completely submerged.

$m\cancel{g} = \rho_{fl} V_{obj} \cdot \cancel{g}$

$(m_{person} + 1\text{ kg}) = (1000)(1.5 \times .7 \times .1)\text{ kg}$

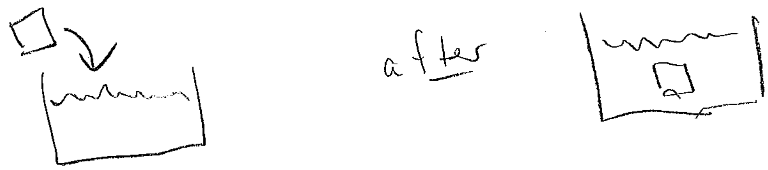
$m_p + 1 = 105\text{ kg}$

$\boxed{m_p = 104\text{ kg}}$

(10 pts) **Problem 4**. My wife sometimes adds an ice cube or two to a hot bowl of soup so that it doesn't burn her mouth. Suppose she has 1 cup (= 0.237 L = 0.237 kg) of soup in a bowl. How much –15°C ice (in grams) will she have to add to the soup in order to lower its temperature from a scalding 96°C to a more manageable 71°C? Assume no heat is lost to the bowl or the surrounding air, and that the soup/ice mixture reaches a new thermal equilibrium essentially instantaneously as soon as she puts the ice in.

Materials constants that you will need: $c_{soup} = c_{water} = 4186$ J/kg·°C, $c_{ice} = 2080$ J/kg·°C; $L_{f\ \text{for water-ice}} = 333000$ J/kg

before



after

$$Q_{\text{lost by soup}} = Q_{\text{gained by ice}}$$

$$(mc\Delta T)_{soup} = (mc\Delta T)_{ice} + (mL)_{ice} + (mc\Delta T)_{\substack{\text{water that used} \\ \text{to be ice}}}$$

$$(.237)(4186)(25) = m\left[(2080)(15) + (333000) + (4186)(71)\right]$$

$$m = .0375 \text{ kg}$$

$$\boxed{37.5\text{g}}$$

(= about 2 standard ice cubes)

(8 pts) **Problem 5**. In my lab, my best vacuum pump can get my cryostat to a vacuum as low as 0.4 milliPascal at room temperature (300 K). The vacuum chamber has a volume of about 20 L. (a) How many gas molecules are still inside the chamber?

$$PV = Nk_B T$$

$$N = \frac{(.4 \times 10^{-3}\,Pa)(.020\,m^3)}{(1.38 \cdot 10^{-23}\,J/K)(300\,K)}$$

$$\boxed{N = 1.93 \cdot 10^{15}}$$

(b) What is the mean free path for those molecules? (Remember $\ell = \frac{1}{\sqrt{2}\pi d^2 n}$, where $d \approx 10^{-10}\,m$ for air molecules and n is the number of molecules per cubic meter.)

$$n = \frac{N}{\text{volume}} = \frac{1.93 \cdot 10^{15}}{.020\,m^3} = 9.662 \cdot 10^{16} \text{ per } m^3$$

$$\ell = \frac{1}{\sqrt{2}\pi d^2 n}$$

$$= \frac{1}{(\sqrt{2})(\pi)(10^{-10})^2(9.662 \cdot 10^{16})}$$

$$= \boxed{233.0\,m}$$

(far longer than the actual size of the cryostat)

(12 pts) **Problem 6**. The efficiencies of air conditioners sold in the United States are usually stated in terms of the "energy efficiency ratio" (EER):

$$EER = \frac{\text{"Cooling power"}(BTU/hr)}{\text{Electric input}(Watts)}$$

It's Q_c/time, just with the wrong units

The "cooling power" is a measure of how much heat can be removed from the refrigerated compartment per hour. The electric input is how much energy per time it takes to run the air conditioner. The conversion factor between BTU and joules is: 1 BTU = 1055 J.

(a) How does the EER relate to the $COP_{refrigerator}$ that we discussed in class/learned from the textbook? Specifically, if my current air conditioner has an EER rating of 6 (towards the low end of the range), what would its COP be?

$$\frac{Q_c}{time} = \text{"Cooling power"}\left(\frac{BTU}{hr}\right) \times \frac{1055\ J}{1\ BTU} \times \frac{1\ hr}{3600\ sec.}$$

$$\frac{Q_c}{time} = \text{"cooling power"} \times .293 \times \frac{J/s}{BTU/hr}$$

So $\boxed{COP = EER \times .293}$

For EER = 6, $\boxed{COP = 1.76}$

(b) If a particular air conditioner has a high enough EER value it gets an "Energy Star" certification. How much electricity (in joules) and how much money (in dollars) would I save annually by using an Energy Star model with an EER of 10 instead of a model with an EER of 6? Info you will need: my electricity costs about 12.7 cents per kW·hr (= 3.53×10^{-6} cents per joule). I run the air conditioner primarily in the three summer months (92 days), and with my current A.C. in those months it takes an average of 52 kW·hr (= 1.87×10^8 J) per day to run it.

New AC $COP = 10 \times .293 = \underline{2.93}$ I'll use that later

$\boxed{\text{Original AC}}$: $92\ days \times \frac{1.87 \cdot 10^8\ J}{1\ day} = \underline{1.72 \cdot 10^{10}\ J}$ this is the work required

$COP = \frac{Q_c}{W} \rightarrow Q_c = W \times COP = (1.72 \cdot 10^{10}\ J)(1.76) = \underline{3.028 \cdot 10^{10}\ J}$

Q_c will be the same w/ new AC.

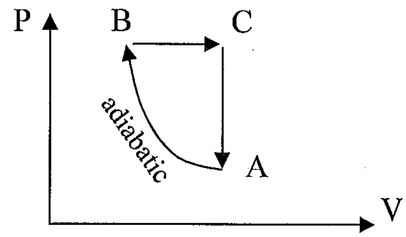
$\boxed{\text{New AC}}$: $W = \frac{Q_c}{COP} = \frac{3.028 \cdot 10^{10}\ J}{2.93} = \underline{1.033 \cdot 10^{10}\ J}$

The work (electricity) that's saved is $1.72 \cdot 10^{10} - 1.033 \cdot 10^{10} = \boxed{6.87 \cdot 10^9\ J}$

Money saved $= 6.87 \cdot 10^9\ J \times \frac{3.53 \cdot 10^{-6}\ cents}{Joule} \times \frac{1\ dollar}{100\ cents} = \boxed{\$242}$

$\gamma = \frac{7}{5} = 1.4$, $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$

(14 pts) **Problem 7**. An engine using 0.5 moles of a diatomic ideal gas is driven by this cycle: starting from state A (100 kPa, 300 K), the gas is compressed adiabatically until it reaches state B at 800 kPa. Then, the gas is heated at constant pressure until it reaches state C. Finally, the gas is cooled at constant volume back to the original state.



	P (kPa)	V (m^3)	T (K)
A	100	0.01247	300
B	800	0.002822	543.4
C	800	(= V_A)	2400

(a) Find the unknown V's and T's for all three states. Fill your answers in on the table. (Warning: be careful! If you get these wrong, you will likely miss many of the subsequent questions.)

A. $PV = nRT \rightarrow V = \frac{nRT}{P} = \frac{(.5)(8.31)(300)}{100\,000} = \boxed{.01247\ m^3}$

B. $P_A V_A^\gamma = P_B V_B^\gamma \rightarrow V_B = V_A \left(\frac{P_A}{P_B}\right)^{1/\gamma} = (.01247)\left(\frac{1}{8}\right)^{1/1.4} = \boxed{.002822\ m^3}$

$PV = nRT \rightarrow T_B = \frac{P_B V_B}{nR} = \frac{(800000)(.002822)}{(.5)(8.31)} = \boxed{543.4\ K}$

C. $V_A = V_C \rightarrow \frac{\cancel{nR}T_A}{P_A} = \frac{\cancel{nR}T_C}{P_C} \rightarrow T_C = T_A\left(\frac{P_C}{P_A}\right) = 8T_A = \boxed{2400\ K}$

(b) Find the heat added to the gas during each of the three legs.

A-B $\boxed{Q = 0}$ (adiabatic)

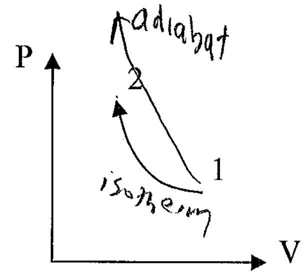
B-C $Q = nC_p\Delta T = (.5)\left(\frac{7}{2}\cdot 8.31\right)(2400 - 543.4) = \boxed{27000\ J}$

C-A $Q = nC_v\Delta T = (.5)\left(\frac{5}{2}\cdot 8.31\right)(300 - 2400) = \boxed{-21814\ J}$

(c) What is the efficiency of an engine using this cycle? Hint: Use the heats you just found in part (b).

$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{21814}{27000} = \boxed{19.2\%}$

(10 pts) **Problem 8**. A diatomic ideal gas is compressed isothermally from state 1 (100 kPa, 0.008 m^3, 300 K) to state 2 (200 kPa, 0.004 m^3, 300 K).



(a) In the process heat flowed out of the gas. How can you tell that without using any equations?

Adiabats are steeper than isotherms, so if no heat had gone out, it would have ended up at a higher temperature.

(b) How much heat flowed out of the gas?

$$\Delta E = Q + W_{on} \rightarrow Q = -W_{on} = +\underbrace{nRT}_{=PV} \ln \frac{V_f}{V_0}$$

$\Delta E = 0$ because isothermal

if not memorized, then need to do integral: $\int p\,dV = \int \frac{nRT}{V} dV = nRT \ln V_f/V_0$

$$Q = (100000)(.008) \ln\left(\frac{1}{2}\right)$$

$$\boxed{Q = -554.5\text{ J}}$$

i.e. 554.5 J flowed out

(c) What was the change in entropy of the gas during this process? Did the entropy of the gas increase or decrease?

$$\Delta S = \int \frac{dQ}{T} = \frac{1}{T}\int dQ \quad \text{since } T = \text{constant}$$

$$= \frac{Q}{T}$$

$$= \frac{-554.5\text{ J}}{300\text{ K}} = \boxed{-1.848\text{ J/K}}$$

entropy decreased

(d) If the heat went from the gas to surroundings which were at a constant 290 K, what was the net entropy change of the universe during the process? (That's the entropy change of gas + entropy change of surroundings. Hint: the net change must be positive!)

Surroundings are also isothermal. Q gained by surroundings = + 554.5 J

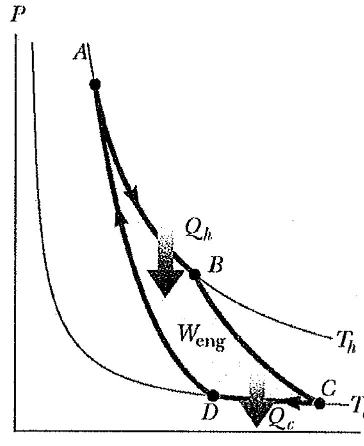
$$\Delta S = \frac{Q}{T} = \frac{554.5\text{ J}}{290\text{ K}} = +1.912\text{ J/K}$$

$$\Delta S_{tot} = \Delta S_{gas} + \Delta S_{surroundings}$$

$$= -1.848 + 1.912 = \boxed{+.0637\text{ J/K}}$$

positive! :)

(5 pts, no partial credit) **Problem 9. Extra credit.** Derive the efficiency of this specific cycle (A-B = isotherm at T_h; B-C = adiabat; C-D = isotherm at T_c; D-A = adiabat) by calculating Q_h and Q_c, and show that the result equals the Carnot efficiency formula as expected. Note: you must be very clear with your work. Since this is an "all or nothing" problem, and since you should know the answer you are trying to obtain, I am not going to hunt around very much trying to determine whether what you have done is correct. If I can't *easily* follow your logic and your work, I will likely give you no credit even if correct work is there someplace.



A-B: $\underbrace{\Delta E}_{=0} = Q + W_{on}$ (isothermal)

$Q = -W_{on} = +nRT_h \ln \frac{V_B}{V_A}$ this is Q_h! as in problem 8b

B-C $Q=0$ (adiabatic)

C-D Just like A-B, but lower temp

$Q = nRT_c \ln \frac{V_D}{V_C} = -\left(nRT_c \ln \frac{V_C}{V_D}\right)$ this is Q_c

D-A $Q=0$ (adiabatic)

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{\cancel{nR}T_c \ln(V_C/V_D)}{\cancel{nR}T_h \ln(V_B/V_A)}$$

almost there! How does $\frac{V_B}{V_A}$ compare to $\frac{V_C}{V_D}$?

B-C is adiabatic so $P_B V_B^{\gamma} = P_C V_C^{\gamma} \rightarrow T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$

same with A-D $T_A V_A^{\gamma-1} = T_D V_D^{\gamma-1}$

divide ... $\frac{\cancel{T_B} V_B^{\gamma-1}}{\cancel{T_A} V_A^{\gamma-1}} = \frac{\cancel{T_C} V_C^{\gamma-1}}{\cancel{T_D} V_D^{\gamma-1}}$

raise to the $\frac{1}{\gamma-1}$ power

$\frac{V_B}{V_A} = \frac{V_C}{V_D}$

Therefore

$\ln\left(\frac{V_B}{V_A}\right) = \ln\left(\frac{V_C}{V_D}\right)$

and

$\boxed{e = 1 - \frac{T_c}{T_h}}$