Solutions

Fall 2012 RED
Physics 123 section 2
Exam 2
Colton 2-3669

Please write your CID here __________

No time limit. No notes, no books. Student calculators OK.

Constants and conversion factors which you may or may not need:

$g = 9.8$ m/s^2
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
$k_B = 1.381 \times 10^{-23}$ J/K
$N_A = 6.022 \times 10^{23}$
$R = k_B \cdot N_A = 8.314$ J/mol·K
$\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
$c = 3 \times 10^8$ m/s
$m_{electron} = 9.11 \times 10^{-31}$ kg
Density of water: 1000 kg/m^3
1 inch = 2.54 cm
1 m^3 = 1000 L
1 atm = 1.013×10^5 Pa = 14.7 psi
1 eV = 1.602×10^{-19} J
$T_F = 9/5\, T_C + 32$
$T_K = T_C + 273.15$

Other equations which you may or may not need to know:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$A_{sphere} = 4\pi r^2$
$V_{sphere} = 4/3\, \pi r^3$
$(1+x)^n \approx 1 + nx$
$\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta T$; $\beta = 3\alpha$
$$f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_B T}$$
$$v_{most\ probable} = \sqrt{\frac{2k_B T}{m}}$$
$$v_{avg} = \int_0^\infty v f(v)\,dv = \sqrt{\frac{8k_B T}{\pi m}}$$
$$v_{rms} = \sqrt{\int_0^\infty v^2 f(v)\,dv} = \sqrt{\frac{3k_B T}{m}}$$
Mean free path: $l = \dfrac{1}{\sqrt{2}\pi d^2 n}$
Ave time between collisions: $\tau = l/v_{avg}$
$$P = \frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{L} = \frac{A\Delta T}{R};\ R = L/k$$
$$P = \frac{\Delta Q}{\Delta t} = e\sigma A T^4$$
$$e_{Otto} = 1 - \frac{1}{r^{\gamma-1}};\ r = V_{max}/V_{min}$$
$S = k_B \ln W$
$$\#microstates\,(k = \#heads) = \binom{N}{k} = \frac{N!}{k!(N-k)!}$$
$\#\,microstates\,(total) = 2^N$

$P = \frac{1}{2}\mu\omega^2 A^2 v$
$$r = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2};\ t = \frac{2v_2}{v_2 + v_1} = \frac{2n_1}{n_1 + n_2}$$
$R = |r|^2$; $T = 1 - R$
$v_{string} = \sqrt{T/\mu}$; $\mu = m/L$
$$v_{rod} = \sqrt{Y/\rho};\ Y = \frac{stress}{strain} = \frac{F/A}{\Delta L/L}$$
$v_{sound} = \sqrt{B/\rho}$
$$v_{sound} = 343\,\tfrac{\text{m}}{\text{s}}\sqrt{\frac{T}{293\text{K}}}$$
$$\beta = 10\log\left(\frac{I}{I_0}\right);\ I = I_0 10^{\beta/10};\ I_0 = 10^{-12}\ \text{W/m}^2$$
$$f' = f\frac{v \pm v_o}{v \pm v_s}$$
$\sin\theta = 1/\text{Mach\#}$
$\Delta x \Delta k \geq 1/2$; $\Delta x \Delta p \geq \hbar/2$
$\Delta t \Delta\omega \geq 1/2$; $\Delta t \Delta E \geq \hbar/2$
$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{L}\right)$$
$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx$$
$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi nx}{L}\right)dx$$
$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi nx}{L}\right)dx$$
musical half step: $f_2/f_1 = 2^{1/12}$

$\tan\theta_{Brewster} = n_2/n_1$
$f = R/2$
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
(R_1 = pos, R_2 = neg if convex-convex)
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$
(p = pos if object in front of surface, q = pos if image in back of surface, R = pos if center of curvature in back of surface)
$\phi = 2\pi\Delta PL/\lambda$
$\Delta PL = d\sin\theta$
$E = E_0\,(e^{i\phi 1} + e^{i\phi 2} + \ldots)$
$I \sim |E|^2$
2 narrow slits: $I = I_0 \cos^2\left(\dfrac{2\pi}{\lambda}\dfrac{d}{2}\sin\theta\right)$
1 wide slit: $I = I_0 \text{sinc}^2\left(\dfrac{\pi a \sin\theta}{\lambda}\right)$
circular: $\theta_{min.resolve} = 1.22\lambda/D$
grating: $R = \lambda_{ave}/\Delta\lambda = \#\text{slits} \times m$
Bragg: $2d\sin\theta_{bright} = m\lambda$ (θ from horizontal)
$$f' = f\sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$
$x_{frame2} = \gamma x_{frame1} \pm \gamma\beta(ct)_{frame1}$
$(ct)_{frame2} = \pm\gamma\beta x_{frame1} + \gamma(ct)_{frame1}$
$$\begin{pmatrix} x \\ ct \end{pmatrix}_{frame2} = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}_{frame1}$$
$E^2 = (pc)^2 + (mc^2)^2$; $E = pc$
$1/p'_{photon} - 1/p_{photon} = 2/(m_{electron}c)$

Scores: (for grader to fill in). **100 total points.**

Problem 1	______	Problem 7	______
Problem 2	______	Problem 8	______
Problem 3	______	Extra Credit	______
Problem 4	______	**Total**	______
Problem 5	______		
Problem 6	______		

Instructions:

- Record your answers to the multiple choice questions ("Problem 1" on the next page) on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(30 pts) **Problem 1**: Multiple choice conceptual questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. Water (no viscosity, incompressible) flows from a little pipe into a big pipe while also decreasing in height. That is, the water is flowing downhill. The flow speed (m/s) in the little pipe will be _______ in the big pipe.

a. greater than ✓
b. the same as
c. less than
d. cannot be determined from the information given

$A_1 v_1 = A_2 v_2$

smaller, so v_2 must be bigger

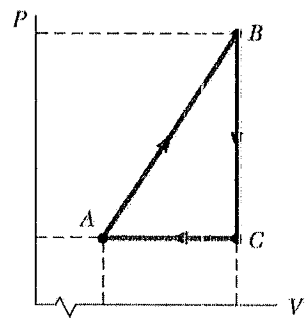
1.2. You have two balloons that are the same size, one filled with air and one filled with helium. If you put both balloons into a tub of liquid nitrogen, which one will end up with the largest volume? Hint: we did this as a demo.

a. the air balloon
b. the helium balloon
c. they will end up with very close to the same volume

1.3. Consider the cyclic process described by the figure. For A to B: is $W_{on\ gas}$ positive, negative, or zero?

a. Positive
b. Negative
c. Zero
d. Can't tell without more details

Expanding → $W_{on\ gas}$ = negative



1.4. As a wave travels into a medium in which its speed decreases, its frequency _____.

a. decreases
b. increases
c. remains the same

1.5. As a wave travels into a medium in which its speed decreases, its wavelength _____.

a. decreases
b. increases
c. remains the same

$v = \lambda f$ → if v decreases + f stays the same, then λ decreases

1.6. Martha is holding one end of a clothesline which stretches across her yard. When she flicks the line, the pulse she generates travels down the line then gets completely absorbed by the flexible pole the line is tied to. She wiggles her end for 1 minute with an amplitude of 5 cm. Then she wiggles her end for 30 seconds, with an amplitude of 10 cm. How much energy does the pole absorb in the last 30 seconds compared to the first minute?

a. ¼ as much
b. ½ as much
c. the same
d. 2x as much
e. 4x as much

$P \sim A^2$ and $E = P \times t$

$$\frac{E_2}{E_1} = \frac{A_2^2 t_2}{A_1^2 t_1} = \left(\frac{10}{5}\right)^2 \cdot \left(\frac{30}{60}\right) = 2^2 \cdot \frac{1}{2}$$

$= 2$

1.7. How does the speed of transverse waves on your Slinky change as you stretch out the Slinky's length?

a. v is independent of L
b. v ~ sqrt(L)
c. v ~ L
d. v ~ L^2

As done in a HW problem

1.8. What's the complex conjugate of $\frac{7+8i}{5+6i}$?

a. $\frac{7+8i}{5+6i}$
b. $\frac{7-8i}{5+6i}$
c. $\frac{7+8i}{5-6i}$
d. $\frac{7-8i}{5-6i}$
e. More than one of the above

just replace i with $-i$

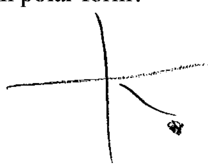
1.9. Using my notation for the polar form of complex numbers, what is $\frac{8\angle 30°}{2\angle 10°}$?

$= \frac{8}{2} \angle 30-10 = 4 \angle 20°$

a. $2\angle 3°$
b. $2\angle 20°$
c. $4\angle 3°$
d. $4\angle 20°$
e. $6\angle 3°$
f. $6\angle 20°$

1.10. What is the number $3-3i$ in polar form?

a. $3\angle 45°$
b. $3\angle 135°$
c. $3\angle 225°$
d. $3\angle 315°$
e. $3\sqrt{2}\angle 45°$
f. $3\sqrt{2}\angle 135°$
g. $3\sqrt{2}\angle 225°$
h. $3\sqrt{2}\angle 315°$



$= 3\sqrt{2} \angle -45°$

1.11. A light wave goes from a piece of glass where its speed is 2×10^8 m/s into some oil where its speed is *also* 2×10^8 m/s. What fraction of the wave's energy reflects at the interface? (The light is traveling perpendicularly to the boundary surface.)

a. 0%
b. 20%
c. 50%
d. 80%
e. 100%

no speed change → no reflection

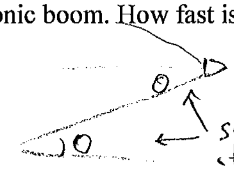
1.12. A certain sound undergoes a 23 dB increase in volume. By how much has the intensity of the sound increased? I_{new}/I_{old} = ________

a. Less than 30
b. 30 - 50
c. 50 - 70
d. 70 - 90
e. 90 - 110
f. 110 - 130
g. 130 - 150
h. 150 - 170
i. 170 - 190
j. More than 190

20 dB = 100×
3 dB ≃ 2×
→ 23 dB ≈ 200×

1.13. A supersonic jet flies overhead. After the jet passes over you, you notice that its position makes a 35° angle with the horizon when you finally hear the sonic boom. How fast is the plane flying?

a. Slower than Mach 1
b. Mach 1 - 1.2
c. Mach 1.2 - 1.4
d. Mach 1.4 - 1.6
e. Mach 1.6 - 1.8
f. Mach 1.8 – 2
g. Faster than Mach 2



same as angle of shock wave cone, $\sin\theta = \frac{1}{\text{Mach\#}}$ → Mach# $= \frac{1}{\sin 35°} = 1.74$

1.14. In the "ladies belt demo" (the belt was like a "closed-closed" string), suppose the fundamental frequency is seen at 200 Hz. What frequency will have four antinodes?

a. 200 Hz
b. 225
c. 300
d. 400
e. 500
f. 600
g. 800
h. 1600 Hz

$f_4 = 4f_1$

1.15. Repeated periodic vibrations impact an object. If they are at the object's natural vibrating frequency, the amplitude of its own vibrations will increase. This phenomenon is known as _____.

a. beats
b. dispersion
c. interference
d. overtones
e. resonance
f. standing waves
g. transmission

1.16. If a wave is traveling through space, the wave's amplitude must always have units of meters.

a. True
b. False

Examples – light wave has units of electric field
sound wave " " " pressure

Common musical intervals for use in the next question

Second = 2 half-steps	Sixth = 9 half-steps
Minor third = 3 half-steps	Minor seventh = 10 half-steps
Major third = 4 half-steps	Major seventh = 11 half-steps
Fourth = 5 half-steps	Octave = 12 half-steps
Fifth = 7 half-steps	

1.17. A piano is tuned to an equal temperament scale and two piano keys are played at random. The fundamental frequencies of the two notes are found to be 440 Hz and 523.3 Hz. What musical interval relates the two notes?

a. a second
b. a minor third
c. a major third
d. a fourth
e. a fifth
f. a sixth
g. a minor seventh
h. a major seventh
i. an octave

$\frac{523.3}{440} = \left(2^{1/12}\right)^x$ ← # of halfsteps; $2^{1/12}$ ← one half step

$x = \frac{\log(523.3/440)}{\log(2^{1/12})} = 3$ half steps

1.18. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.

a. A sound wave is a transverse wave. No, longitudinal
b. A sound wave is a pressure wave, which can be thought of as oscillations in pressure with respect to time. True!
c. To hear the sound wave produced by a tuning fork, air molecules must travel from the fork to one's ear. No, just like slinky wave
d. The intensity of a sound wave typically follows an inverse relationship with distance from the source: I ~ 1/r
No, $I \sim 1/r^2$

1.19. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.

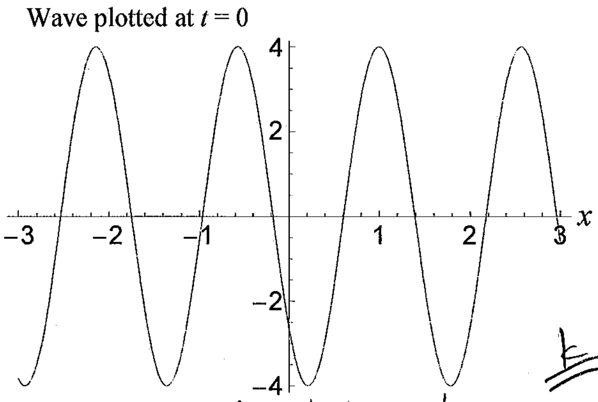
a. Sound waves tend to travel faster through solids than they do through gases. True!
b. Sound waves tend to travel faster through warm air than cold air. True! $v \sim \sqrt{T}$
c. Sound waves tend to travel faster through high pressure air than low pressure air. No, v doesn't depend strongly on P
d. High frequency sound waves tend to travel faster than low frequency sound waves. No, v " " " " f

1.20. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.

a. If a guitar string is touched directly in the middle, the third harmonic is forced to sound. No, would be 2nd harm.
b. The second harmonic of a guitar string corresponds to a complete wavelength fitting in the length of the string. True!
c. The fundamental frequency of a guitar string is the highest frequency at which the string can vibrate. No, lowest freq.
d. The frequency of the fourth harmonic of a string is two octaves above the frequency of the first harmonic.

True! $f_4 = 4f_1$
$= 2 \times (2f_1)$
(↑ one octave; ↑ a second octave)

(8 pts) **Problem 2**



A

$A = 4$

k

$\lambda \approx 1.6 \rightarrow k = \frac{2\pi}{\lambda} \approx 3.9$

The above wave moves to the right at a rate of 13 m/s. Determine the wave equation for the wave, in the form $f(x,t) = A\cos(kx - \omega t + \phi)$. That is, determine A, k, ω, and ϕ.

ϕ wave starts $\approx \frac{.6}{1.6} = 38\%$ of λ to the left of origin

so $\phi = 38\%$ of $2\pi \approx 2.3$

ω $v = \frac{\omega}{k} \rightarrow \omega = vk = 13 \times 3.9 = 52$

Piece together:

$$\boxed{f(x,t) = 4\cos\left(3.9x - 52t + 2.3\right)}$$

(8 pts) **Problem 3.** To remind you, this is the wave equation:

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

(The funny d's refer to "partial derivatives", meaning you take the derivative assuming that everything else is constant.)

In class I mentioned that *any* function—not just sinusoidal waves—that has position and time dependence of the form $x - Ct$ will satisfy the wave equation, with velocity given by the constant C that multiplies t.

Consider this wave function: $f(x,t) = 5(x-3t)^2$. Show that it does in fact satisfy the wave equation with $v = 3$ by working out both the left hand side of the wave equation and the right hand side independently, then setting them equal to each other.

$$\frac{\partial f}{\partial t} = 10(x-3t)(-3)$$

$$= \underline{-30(x-3t)}$$

$$\frac{\partial^2 f}{\partial t^2} = \underline{90}$$

$$\frac{\partial f}{\partial x} = \underline{10(x-3t)}$$

$$\frac{\partial^2 f}{\partial x^2} = \underline{10}$$

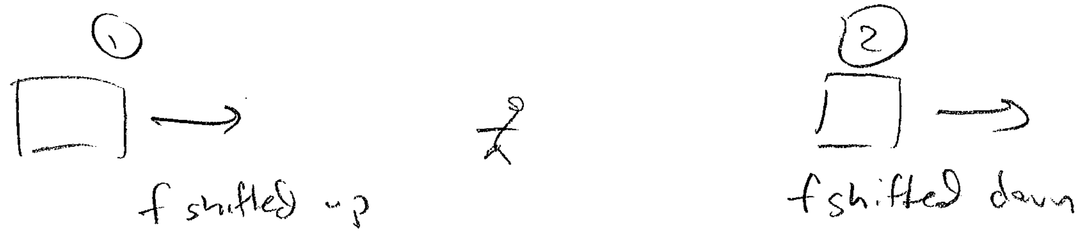
$$\boxed{\text{LHS} : 90}$$

$$\text{RHS} = 3^2 \times 10$$

$$\boxed{= 90}$$

Yes! Wave eqn is satisfied, as long as $v = 3$ (or -3)

(11 pts) **Problem 4**. As a car drives past Steve, the driver sounds the horn. Steve has perfect pitch, and notices that as the car approached, the horn sounded the A above middle C (440.0 Hz). After the car passed, the pitch dropped down two half steps to a G. How fast was the car going? (Use 343 m/s as the speed of sound.)



main Doppler Eqn: $f' = f_0 \dfrac{v \pm v_o}{v \mp v_s}$

Applied to 1 $\quad f_1 = f_0 \dfrac{343}{343 - v_s}$

Applied to 2 $\quad f_2 = f_0 \dfrac{343}{343 + v_s}$

Take ratio: $\quad \dfrac{f_1}{f_2} = \dfrac{\cancel{f_0}\ 343/343-v_s}{\cancel{f_0}\ \cancel{343}/343+v_s} = \dfrac{343 + v_s}{343 - v_s}$

use given "2 half steps" info $\rightarrow f_1/f_2 = 2^{2/12}$

$$2^{2/12} = \frac{343 + v_s}{343 - v_s}$$

Solve for v_s:

$$2^{2/12}(343 - v_s) = 343 + v_s$$

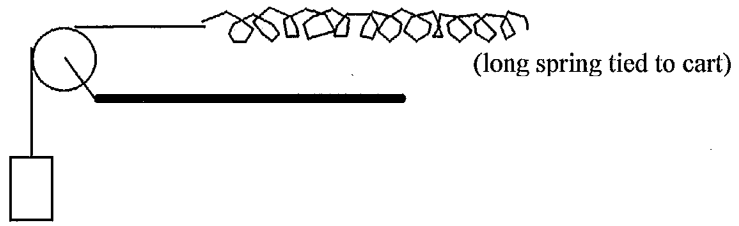
$$343 \cdot 2^{2/12} - 2^{2/12} v_s = 343 + v_s$$

$$343(2^{2/12} - 1) = v_s(2^{2/12} + 1)$$

$$v_s = 343 \cdot \frac{2^{2/12} - 1}{2^{2/12} + 1}$$

$$\boxed{v_s = 19.79 \text{ m/s}}$$

(8 pts) **Problem 5**. In class we did a demo where we predicted the time it would take for a wave to travel down a long spring. The setup was something like this:



Predict the time it would take for a wave pulse to travel to the end of the spring, if we had made the following measurements:

Mass of spring = 290 g
Length of spring = 2.7 m
Hanging mass = 350 g

$$v = \sqrt{\frac{T}{\mu}} \qquad T = m_{hanging} \times g$$

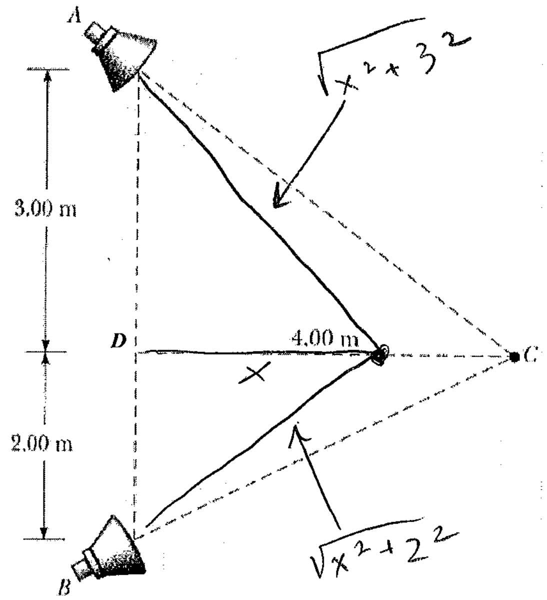
$$\mu = \frac{m_{spring}}{L_{spring}}$$

$$v = \sqrt{\frac{(.35)(9.8)}{(.29)/2.7}} = \underline{5.65 \text{ m/s}}$$

$$x = vt \rightarrow t = \frac{x}{v}$$

$$t = \frac{2.7 \text{ m}}{5.65 \text{ m/s}} = \boxed{0.478 \text{ s}}$$

(13 pts) **Problem 6.** Two speakers are placed as shown in the figure. Both broadcast a signal from the same 1500 Hz source, in phase with each other. A student starts from point C and walks to the left along the dotted line shown, stopping when he hits point D. How many maxima will he encounter along the way, and where will they be? (Take the speed of sound to be 343 m/s.)



$$v = \lambda f \rightarrow \lambda = v/f = \frac{343}{1500} = \underline{0.2287\,m}$$

constructive interference: $\Delta PL = n\lambda$

$$\sqrt{x^2+3^2} - \sqrt{x^2+2^2} = n\lambda$$

$$\sqrt{x^2+9} = n\lambda + \sqrt{x^2+4}$$

$$\cancel{x^2} + 9 = n^2\lambda^2 + 2n\lambda\sqrt{x^2+4} + (\cancel{x^2}+4)$$

$$\frac{5 - n^2\lambda^2}{2n\lambda} = \cancel{2n\lambda}\sqrt{x^2+4}$$

$$x^2+4 = \left(\frac{5-n^2\lambda^2}{2n\lambda}\right)^2$$

$$x = \sqrt{\left(\frac{5-n^2\lambda^2}{2n\lambda}\right)^2 - 4}$$

Try it out for different n's!

Alternatively, I could have solved for n and plugged in the min and max x-values (0 and 4) to see the range of possible n's directly

n	x
0	impossible
1	10.6 → too big
2	4.84 → too big
3	2.63 ✓
4	1.09 ✓
5 and higher	impossible

$$\boxed{\text{Answer: 2 maxima, at } x = 2.63\,m \text{ and } x = 1.09\,m}$$

(9 pts) **Problem 7.** My trumpet, for a particular configuration of values, acts very similarly to an "open-open" pipe with a length of 1.0 meter. Suppose I have been playing minutes so that my breath has warmed up the air inside the trumpet to 32°C. Disregard the thermal expansion of the metal, but do not disregard the change of the speed of sound with temperature.

(a) If I play the fifth harmonic, what frequency will it have?

first harm: $L = \frac{1}{2}\lambda \rightarrow \lambda = 2L \rightarrow \frac{v}{f} = 2L \rightarrow f = \frac{v}{2L}$

$$v = 343\sqrt{T/293K} = 343\sqrt{305/293} = \underline{350 \text{ m/s}}$$

$$f_1 = \frac{350}{2 \cdot 1} = \underline{175 \text{ Hz}}$$

for "open-open", $f_5 = 5f_1$

$$= \boxed{875 \text{ Hz}}$$

(b) A fellow trumpet player tries to play the same note on his trumpet (an exact duplicate of mine). However, because he has not warmed up his instrument, the air inside the trumpet is 20°C. How many beats per second occur between the tones made by his trumpet and mine? Both of us are playing the fifth harmonics. (Note: I believe this is in fact the dominant source of brass instruments playing out of tune when not warmed up first.)

New $v = 343$ m/s

New $f_1 = \frac{343}{2 \cdot 1} = \underline{171.5 \text{ Hz}}$

New $f_5 = 5f_1 = \underline{857.5 \text{ Hz}}$

$$f_{beat} = |f_1 - f_2| = |875 - 857.5|$$

$$\boxed{f_{beat} = 17.5 \text{ Hz}}$$

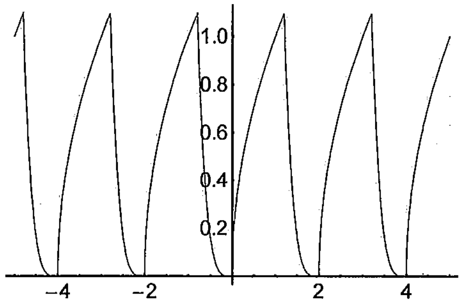
or 17.5 beats/second

(13 pts) **Problem 8**. The function f(x), graphed on the right, is defined as follows:

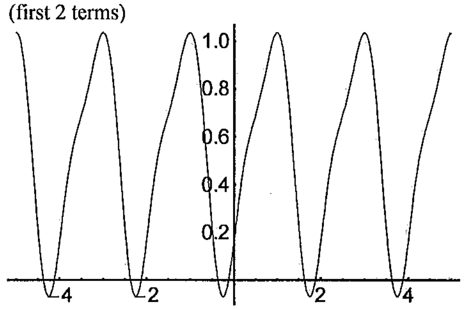
$$f(x) = \begin{cases} 2.67443x^4, \text{ for } x \text{ between } -0.8 \text{ and } 0 \\ \sqrt{x}, \text{ for } x \text{ between } 0 \text{ and } 1.2 \end{cases}$$

(repeated with a period of $L = 2$)

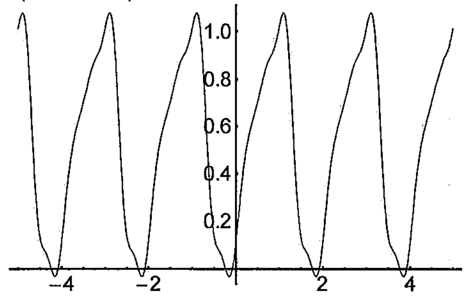
As with any periodic function, this can be turned into a Fourier series. Plots of f(x) for increasing numbers of terms in the Fourier summation are also shown.



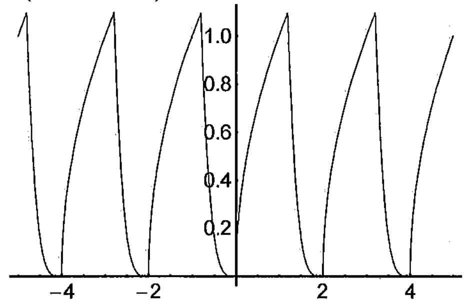
(first 2 terms)



(first 5 terms)



(first 100 terms)



(a) Calculate the constant term of the series. Hint: do the integral from -0.8 to 1.2.

$$a_0 = \frac{1}{L}\int_{\text{period}} f(x)\,dx$$

$$= \frac{1}{2}\left[\int_{-.8}^{0} 2.67443x^4\,dx + \int_0^{1.2} x^{1/2}\,dx\right]$$

$$= \frac{1}{2}\left[\frac{2.67443x^5}{5}\Bigg|_{-.8}^{0} + \frac{x^{3/2}}{3/2}\Bigg|_0^{1.2}\right]$$

$$= \frac{1}{2}\left[-\frac{2.67443\cdot(.8)^5}{5} + \frac{(1.2)^{3/2}}{3/2}\right] = \boxed{.5258}$$

(b) Will there be sine terms in the series, cosine terms, or both? Justify your answer.

Both – the function is neither odd nor even, not even if you subtract off the constant term.

(c) What k-value will all the spatial frequencies in the series be multiples of?

$$k_0 = \frac{2\pi}{L} = \frac{2\pi}{2} = \boxed{\pi}$$

(d) Write down an integral that you could use to determine the coefficient of the 1st sine term of the series, assuming it is non-zero and you had enough patience to work out the integral. Please don't actually try to solve the integral, but it should be simplified enough that you could (for example) just type it into Mathematica to obtain its numerical value. Specifically, if the function you are integrating changes over the domain, separate the integral into appropriate pieces.

$$b_1 = \frac{2}{2}\int_0^2 f \sin(k_0 x)\,dx$$

$$= \boxed{\int_{-.8}^{0} 2.67443\,x^4 \sin \pi x\,dx + \int_0^{1.2} x^{1/2} \sin \pi x\,dx}$$

(5 pts, <u>no partial credit</u>) **Problem 9. Extra credit.** You may choose one of these two problems to do for extra credit. (If you give answers for both, only the first one will be graded.)

$k_{ave} = \frac{2\pi}{.6} = 10.47$

(a) One of the Wikipedia animations on group velocity depicted a situation where the phase velocity and group velocity were in the same direction but different in magnitude. Specifically, on my own monitor I measured the speed of the envelope to be 0.46 cm/s and the speed of the ripples to be 0.92 cm/s. The main wavelength of the ripples was 0.6 cm. Figure out the dispersion relationship for this animation, $\omega(k)$, assuming the numbers I measured on my monitor to be precisely correct.

$$V_g = \frac{d\omega}{dk} = .46$$

$$V_p = \frac{\omega}{k} = .92$$

$$\frac{d\omega}{dk} = \frac{1}{2}\frac{\omega}{k}$$

$$2\frac{d\omega}{\omega} = \frac{dk}{k}$$

$$2\int\frac{d\omega}{\omega} = \int\frac{dk}{k}$$

$$2\ln\omega = \ln k + C$$

$$\ln\omega^2 = \ln(k \cdot C')$$

$$\omega^2 = C'k$$

$$\omega = C'' k^{1/2}$$

find C'' by V_g or V_p numbers

$$\left.\frac{\omega}{k}\right|_{k_{ave}} = .92 \rightarrow \left.\frac{C'' k^{1/2}}{k}\right|_{k_{ave}} = .92$$

$$\frac{C''}{\sqrt{10.47}} = .92$$

$$C'' = 2.977$$

Piece together,

$$\boxed{\omega = 2.977 k^{1/2}}$$

(in units of cm and s)

(b) (topic from exam 1) Prove that ΔS of the universe will always increase for calorimetry-type situations where hot object 1 gives heat to cold object 2 until both are at the same equilibrium temperature.

$$\Delta S = \int\frac{dQ}{T} \quad \text{for each one}$$

$$\Delta S_{total} = \left(\int\frac{dQ}{T}\right)_1 + \left(\int\frac{dQ}{T}\right)_2$$

estimate integrals w/ average value

$$= \frac{Q_1}{T_{ave1}} + \frac{Q_2}{T_{ave2}}$$

$$Q_1 = -Q_2$$

$$\Delta S_{tot} = Q_2\left(-\frac{1}{T_{ave1}} + \frac{1}{T_{ave2}}\right)$$

$$\Delta S_{tot} = Q_2\left(-\frac{1}{\text{large}} + \frac{1}{\text{small}}\right)$$

positive

always > 0!

proved.