Fall 2012
Physics 123 section 2
Exam 3
Colton 2-3669

RED

Solutions

Please write your CID here ________

No time limit. No notes, no books. Student calculators OK.

Constants and conversion factors which you may or may not need:

$g = 9.8$ m/s^2
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
$k_B = 1.381 \times 10^{-23}$ J/K
$N_A = 6.022 \times 10^{23}$
$R = k_B \cdot N_A = 8.314$ J/mol·K
$\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
$c = 3 \times 10^8$ m/s
$m_{electron} = 9.11 \times 10^{-31}$ kg
Density of water: 1000 kg/m^3
1 inch = 2.54 cm
1 m^3 = 1000 L
1 atm = 1.013×10^5 Pa = 14.7 psi
1 eV = 1.602×10^{-19} J
$T_F = 9/5\, T_C + 32$
$T_K = T_C + 273.15$

Other equations which you may or may not need to know:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A_{sphere} = 4\pi r^2$

$V_{sphere} = 4/3\ \pi r^3$

$(1 + x)^n \approx 1 + nx$

$\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta T$; $\beta = 3\alpha$

$$f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_B T}$$

$$v_{most\ probable} = \sqrt{\frac{2k_B T}{m}}$$

$$v_{avg} = \int_0^\infty v f(v)\, dv = \sqrt{\frac{8k_B T}{\pi m}}$$

$$v_{rms} = \sqrt{\int_0^\infty v^2 f(v)\, dv} = \sqrt{\frac{3k_B T}{m}}$$

Mean free path: $l = \dfrac{1}{\sqrt{2}\pi d^2 n}$

Ave time between collisions: $\tau = l/v_{avg}$

$P = \dfrac{\Delta Q}{\Delta t} = \dfrac{kA\Delta T}{L} = \dfrac{A\Delta T}{R}$; $R = L/k$

$P = \dfrac{\Delta Q}{\Delta t} = e\sigma A T^4$

$e_{Otto} = 1 - \dfrac{1}{r^{\gamma - 1}}$; $r = V_{max}/V_{min}$

$S = k_B \ln W$

$\#microstates\,(k = \#heads) = \dbinom{N}{k} = \dfrac{N!}{k!(N-k)!}$

$\#\,microstates\,(total) = 2^N$

$P = ½\mu\omega^2 A^2 v$

$r = \dfrac{v_2 - v_1}{v_2 + v_1} = \dfrac{n_1 - n_2}{n_1 + n_2}$; $t = \dfrac{2v_2}{v_2 + v_1} = \dfrac{2n_1}{n_1 + n_2}$

$R = |r|^2$; $T = 1 - R$

$v_{string} = \sqrt{T/\mu}$; $\mu = m/L$

$v_{rod} = \sqrt{Y/\rho}$; $Y = \dfrac{stress}{strain} = \dfrac{F/A}{\Delta L/L}$

$v_{sound} = \sqrt{B/\rho}$

$v_{sound} = 343\frac{\text{m}}{\text{s}}\sqrt{\dfrac{T}{293\text{K}}}$

$\beta = 10\log\left(\dfrac{I}{I_0}\right)$; $I = I_0 10^{\beta/10}$; $I_0 = 10^{-12}$ W/m^2

$f' = f\dfrac{v \pm v_o}{v \pm v_s}$

$\sin\theta = 1/\text{Mach\#}$

$\Delta x \Delta k \geq ½$; $\Delta x \Delta p \geq \hbar/2$

$\Delta t \Delta\omega \geq ½$; $\Delta t \Delta E \geq \hbar/2$

$$f(x) = a_0 + \sum_{n=1}^\infty a_n \cos\left(\frac{2\pi nx}{L}\right) + \sum_{n=1}^\infty b_n \sin\left(\frac{2\pi nx}{L}\right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi nx}{L}\right)dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi nx}{L}\right)dx$$

musical half step: $f_2/f_1 = 2^{1/12}$

$\tan\theta_{Brewster} = n_2/n_1$

$f = R/2$

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

(R_1 = pos, R_2 = neg if convex-convex)

$\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$

(p = pos if object in front of surface, q = pos if image in back of surface, R = pos if center of curvature in back of surface)

$\phi = 2\pi\Delta PL/\lambda$

$\Delta PL = d\sin\theta$

$E = E_0\,(e^{i\phi_1} + e^{i\phi_2} + \ldots)$

$I \sim |E|^2$

2 narrow slits: $I = I_0 \cos^2\left(\dfrac{2\pi}{\lambda}\dfrac{d}{2}\sin\theta\right)$

1 wide slit: $I = I_0 \text{sinc}^2\left(\dfrac{\pi a \sin\theta}{\lambda}\right)$

circular: $\theta_{min.resolve} = 1.22\lambda/D$

grating: $R = \lambda_{ave}/\Delta\lambda$ = #slits × m

Bragg: $2d\sin\theta_{bright} = m\lambda$ (θ from horizontal)

$f' = f\sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}}$

$x_{\text{frame 2}} = \gamma x_{\text{frame 1}} \pm \gamma\beta(ct)_{\text{frame 1}}$

$(ct)_{\text{frame 2}} = \pm\gamma\beta x_{\text{frame 1}} + \gamma(ct)_{\text{frame 1}}$

$$\begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame 2}} = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame 1}}$$

$E^2 = (pc)^2 + (mc^2)^2$; $E = pc$

$1/p'_{photon} - 1/p_{photon} = 2/(m_{electron}c)$

Scores: (for grader to fill in). **100 total points.**

Problem 1 ________
Problem 2 ________
Problem 3 ________
Problem 4 ________
Problem 5 ________
Problem 6 ________
Problem 7 ________
Problem 8 ________

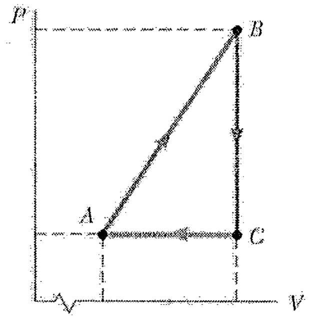
Extra Credit ________

Total ________

Instructions:

- Record your answers to the multiple choice questions ("Problem 1") on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(30 pts) **Problem 1**: Multiple choice conceptual questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.



1.1. Consider the cyclic process described by the figure. For A to B: is $W_{on\ gas}$ positive, negative, or zero?

a. Positive
b. Negative   Gas does work
c. Zero
d. It depends on the temperature of the gas

1.2. Which of the following laser beams would be considered to be "polarized"?

a. The electric field in the beam oscillates up and down as the beam travels through space. Linear pol
b. The electric field in the beam rotates around in a circle as the beam travels through space. Circular pol.
c. The electric field in the beam oscillates in random directions as the beam travels through space.
d. (a) and (b)
e. (a) and (c)
f. (b) and (c)
g. None of the above

1.3. First an *unpolarized* beam of light is sent through a perfect polarizer which is oriented at 45° from the horizontal. Then a *horizontally polarized* beam of light (same brightness) is sent through the same polarizer. How do the intensities of light getting through the polarizer compare?

half of light gets through

Half of light also gets through

a. More of the unpolarized beam gets through.
b. More of the horizontally polarized beam gets through.
c. The same amount of light gets through.

1.4. You are looking across the surface of a very large, very calm lake. You are wearing polarized sunglasses, and you notice that your sunglasses nearly completely cut out the glare from the sun reflecting off of the lake. How far above the horizon is the sun in the sky? The index of refraction for water in the lake is 1.33.

a. 0 - 10° above the horizon (i.e. close to grazing incidence)
b. 10 - 20°
c. 20 - 30°
d. 30 - 40°
e. 40 - 50°
f. 50 - 60°
g. 60 - 70°
h. 70 - 80°
i. 80 - 90° above the horizon (i.e. close to normal incidence)

$\tan\theta_B = \frac{n_2}{n_1}$

$\theta_B = \tan^{-1}\left(\frac{1.33}{1}\right) = 53.1°$

This is angle from ⊥, so angle from horizon = 90° − 53.1° = 36.9°

1.5. A curved mirror with a radius of curvature of 10 cm is replaced in an optical system with a mirror having radius of curvature of 20 cm. How will the new focal length compare to the old one?

a. $f_{new} = ¼\ f_{old}$
b. $f_{new} = ½\ f_{old}$
c. $f_{new} = f_{old}$
d. $f_{new} = 2\ f_{old}$
e. $f_{new} = 4\ f_{old}$

$f = \frac{r}{2} \rightarrow \frac{f_{new}}{f_{old}} = \frac{r_{new}/\cancel{2}}{r_{old}/\cancel{2}} = \frac{20}{10}$

$f_{new} = 2\ f_{old}$

1.6. What type of image is produced by the objective lens of a regular refracting telescope?

a. Real   object is very far away → real image produced near focus.
b. Virtual

1.7. What type of image is produced by the eyepiece lens of a regular refracting telescope?

a. Real
b. Virtual

Final image = same side of lens as object → virtual

1.8. You are given a choice between two telescopes. Telescope 1 has an objective with focal length of 100 cm and an eyepiece with focal length of 20 cm. Telescope 2 has an objective with focal length of 120 cm and an eyepiece with focal length of 40 cm. Which would produce the larger angular magnification, m?

a. Telescope 1
b. Telescope 2
c. Same

$m = f_o/f_e \rightarrow m_1 = \frac{100}{20} = 5$

$m_2 = \frac{120}{40} = 3$

1.9. You have a choice between two telescopes. Telescope 1 has a diameter of 0.3 m, and is optimized to work with 600 nm wavelength light. Telescope 2 has a diameter of 0.5 m, and is optimized to work with 800 nm wavelength light. Which would provide the best angular resolution?

a. Telescope 1
b. Telescope 2
c. Same

$\theta_{min} = \frac{1.22\lambda}{D} \rightarrow \theta_{min\,1} = 1.22\,\frac{600\,nm}{.3\,m}$

$\theta_{min\,2} = 1.22\,\frac{800\,nm}{.5\,m}$

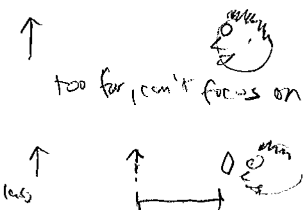
$\frac{\theta_{min1}}{\theta_{min2}} = \frac{6}{8}\cdot\frac{5}{3} = \frac{10}{8}$

Small θ_{min} = better → θ_2

1.10. To correct nearsightedness, you need to choose a lens that will create a ______ image at ______, of an object at ______

a. real; the near point; 25 cm
b. real; the far point; infinity
c. real; infinity; the far point
d. real; infinity; 25 cm
e. virtual; the near point; 25 cm
f. virtual; the far point; infinity
g. virtual; infinity; the far point
h. virtual; infinity; 25 cm

w/o glasses



too far, can't focus on anything past "far point"

w/glass

image produced at far point

1.11. To correct farsightedness, you need to choose a lens that will create a ______ image at ______, of an object at ______

a. real; the near point; 25 cm
b. real; the far point; infinity
c. real; infinity; the far point
d. real; infinity; 25 cm
e. virtual; the near point; 25 cm
f. virtual; the far point; infinity
g. virtual; infinity; the far point
h. virtual; infinity; 25 cm

w/o glasses

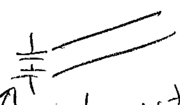
too close when holding book at 25cm (for example), can't focus

w/ glasse

image produced at near point

1.12. In slit problems, when can you make the approximation that the rays come out from the slits parallel to each other?

a. When the slit size/spacing is much smaller than L (the distance to the screen)
b. When the slit size/spacing is much larger than L
c. When the part of the diffraction pattern you are interested in is much smaller than L
d. When the part of the diffraction pattern you are interested in is much larger than L

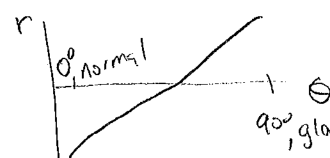


as long as this distance is small, rays are ≈ //

1.13. What is the phase shift in the reflected beam when a p-polarized light ray goes from air to glass at close to grazing incidence (Snell's law θ near 90°).

a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

p-polar:



1.14. What is the phase shift in the reflected beam when a p-polarized light ray goes from air to glass at close to normal incidence (Snell's law θ near 0°).

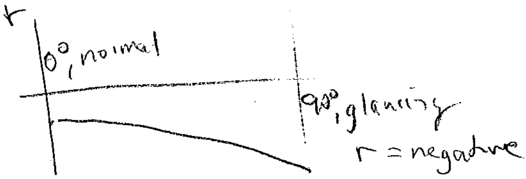
a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

Same picture

r = negative

1.15. What is the phase shift in the reflected beam when an s-polarized light ray goes from air to glass at close to grazing incidence (Snell's law θ near 90°).

S-polar.



a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

1.16. What is the phase shift in the reflected beam when an s-polarized light ray goes from air to glass at close to normal incidence (Snell's law θ near 0°).

a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

same picture, r still negative

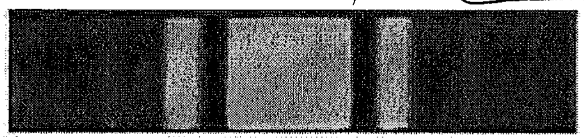
1.17. A coating (n = 1.7) is put on a glass lens (n = 1.5) to reduce reflections when the wavelength is λ = 800 nm (this is the wavelength of the light in a vacuum). Which of these equations would properly allow you to determine how thick the coating needs to be? The symbol t represents the thickness of the coating.

a. $2t = m\lambda$
b. $2t = (m + ½)\,\lambda$
c. $2t = m\lambda n_{coating}$
d. $2t = (m + ½)\,\lambda n_{coating}$
e. $2t = m\lambda/n_{coating}$
f. $2t = (m + ½)\,\lambda/n_{coating}$
g. $2t = m\lambda n_{glass}$
h. $2t = (m + ½)\,\lambda n_{glass}$
i. $2t = m\lambda/n_{glass}$
j. $2t = (m + ½)\,\lambda/n_{glass}$

$\Delta OPL + \text{phase shift} = (m+\frac{1}{2})\lambda$

1.0
1.7 — phase shift, no phase shift
1.5

$2nt + \frac{1}{2}\lambda = (m+\frac{1}{2})\lambda \rightarrow \boxed{2nt = m\lambda}$ n = n coating



1.18. This is the diffraction pattern for a single wide slit. What will happen if the slit is the same, but if green light is used instead of red light?

a. The fringes will get closer together
b. The fringes will get farther apart
c. No change

λ decreases → all distances in pattern decrease

1.19. The electric field in a light wave is described by the following equation: $\vec{E} = A\left(\frac{-\hat{x}+\hat{y}+4\hat{z}}{\sqrt{18}}\right)\cos\left[k\left(\frac{x-y}{\sqrt{2}}\right)-\omega t\right]$. In what direction is the wave traveling?

a. $\frac{\hat{x}+\hat{y}}{\sqrt{2}}$
b. $\frac{\hat{x}-\hat{y}}{\sqrt{2}}$
c. $\frac{-\hat{x}+\hat{y}+4\hat{z}}{\sqrt{18}}$
d. $\frac{\hat{x}-\hat{y}+4\hat{z}}{\sqrt{18}}$
e. $\frac{\hat{x}+\hat{y}-4\hat{z}}{\sqrt{18}}$

$\vec{k} = k\left(\frac{\hat{x}-\hat{y}}{\sqrt{2}}\right)$

1.20. Same equation. In what direction is the electric field oscillating?

a. From $\frac{\hat{x}-\hat{y}}{\sqrt{2}}$ to $\frac{-\hat{x}+\hat{y}}{\sqrt{2}}$
b. From $\frac{\hat{x}+\hat{y}}{\sqrt{2}}$ to $\frac{-\hat{x}-\hat{y}}{\sqrt{2}}$
c. From $\frac{-\hat{x}+\hat{y}+4\hat{z}}{\sqrt{18}}$ to $\frac{\hat{x}-\hat{y}-4\hat{z}}{\sqrt{18}}$
d. From $\frac{\hat{x}-\hat{y}+4\hat{z}}{\sqrt{18}}$ to $\frac{-\hat{x}+\hat{y}-4\hat{z}}{\sqrt{18}}$
e. From $\frac{\hat{x}+\hat{y}-4\hat{z}}{\sqrt{18}}$ to $\frac{-\hat{x}-\hat{y}+4\hat{z}}{\sqrt{18}}$

direction of oscillation $= \pm\left(\frac{-\hat{x}+\hat{y}+4\hat{z}}{\sqrt{18}}\right)$

(10 pts) **Problem 2.** Give short answers/explanations to the following questions:

(a) All answers to this part will be accepted for full credit. On the last two exams, I had a multiple choice problem that said this:

> You have two balloons that are the same size, one filled with air and one filled with helium. If you put both balloons into a tub of liquid nitrogen, which one will end up with the largest volume?
> a. the air balloon
> b. the helium balloon
> c. they will end up with very close to the same volume

Only 54% of the class got the question correct on the first exam, and only 58% of the class got it correct on the second exam. Which answer did you choose, and what were your main reasons for choosing that answer? (I'm not interested in making half the class look stupid here, I'm just genuinely curious about what wasn't clear when I talked about this in class.)

Correct answer: helium ends up with more volume.

reason: The air inside the air balloon liquifies, because the liquid nitrogen is (by definition) cold enough to turn the nitrogen in the air into a liquid.

The helium, by contrast, remains an ideal gas.

(b) A sound wave is measured to have a certain pressure amplitude, P_1, and a certain intensity, I_1, at a distance of 1 m from the source (a directionless speaker). In terms of P_1 and I_1, respectively, what will the pressure and intensity be at a distance of 2 m away?

$I \sim$ inverse square $\rightarrow$ $\boxed{I_2 = \frac{1}{4} I_1}$

$I \sim \text{amplitude}^2$, and pressure is the amplitude of a sound wave,

so $P \sim$ inverse

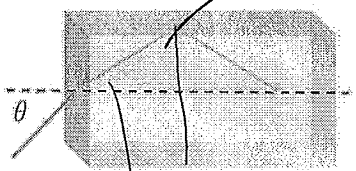
$\boxed{P_2 = \frac{1}{2} P_1}$

(c) Why do I call the single slit diffraction pattern the pattern from a "wide slit", where the textbook calls it the pattern from a "narrow slit"? What does "wide" mean in my context? What does "narrow" mean in the book's context?

wide = wide enough that you get effects from finite width of slit; i.e. sinc^2 function superimposed on double slit pattern.

narrow = narrow enough that you get diffraction effects at all! (i.e. beam doesn't just travel through undisturbed)

(10 pts) **Problem 3**. A rectangular block of clear plastic is sitting on the ground. A beam of light enters the left face of the plastic at an angle θ from the perpendicular, as shown in the figure. The transmitted beam then strikes the top of the block. What is the maximum angle θ which will result in total internal reflection off of the top surface? The index of refraction for the plastic is 1.4.



$\theta_{critical}$ from Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1.4)\sin\theta_1 = (1)\sin 90^\circ$

$\theta_1 = \sin^{-1}\left(\frac{1}{1.4}\right) = 45.58^\circ$

this angle $= 90 -$ that angle (from triangle)

call it θ_2

$\theta_2 = 90^\circ - 45.58^\circ = 44.42^\circ$

Finally, this angle from Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1)\sin\theta = (1.4)(\sin 44.42^\circ)$

$\theta = \sin^{-1}(\quad)$

$\boxed{\theta = 78.46^\circ}$

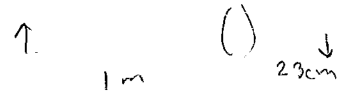
(10 pts) **Problem 4.** You shine red light on a penny. You place a lens exactly 1 meter from the penny, and a red image of the penny forms at a distance of 23.00 cm from the lens, on the opposite side of the lens. You then shine blue light on the penny. How far from the lens will the blue image form? The index of refraction for this particular glass is 1.600 for the red light and 1.690 for the blue light.

Lensmaker: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; R_1 and R_2 = constant

so $\frac{f_{red}}{f_{blue}} = \frac{n_{blue}-1}{n_{red}-1}$

$f_{blue} = f_{red} \times \frac{(n_{red}-1)}{(n_{blue}-1)} = \underline{f_{red} \times \frac{.6}{.69}}$ (hold that thought)

Red focus:



$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \longrightarrow f = \left(\frac{1}{100} + \frac{1}{23}\right)^{-1} = \underline{18,70 \text{ cm}}$

Therefore blue focus is

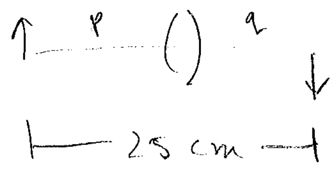
$f_{blue} = 18,70\left(\frac{.6}{.69}\right) = \underline{16.26 \text{ cm}}$

Lens Eqn for blue light

$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} \longrightarrow q = \left(\frac{1}{16.26} - \frac{1}{100}\right)^{-1}$

$\boxed{q = 19.42 \text{ cm}}$

(10 pts) **Problem 5**. You want to produce a real image of a rose at a point in space that is 25 cm away from the rose. Your lens must be placed between the rose and that image point. You must use a lens that has a focal length of 5 cm. (a) Show that there are two possible positions for the lens which will produce an image at the desired spot. Find p and q for both cases. (b) Which position will produce the larger magnitude of magnification $|M|$, and what $|M|$ will it produce?



$p + q = 25$ cm $\rightarrow q = 25 - p$

(a) lens eqn: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$$\frac{1}{5} = \frac{1}{p} + \frac{1}{25-p}$$

multiply by $5p(25-p)$ on both sides

$$p(25-p) = 5(25-p) + 5(p)$$

$$25p - p^2 = 125 - \cancel{5p} + \cancel{5p}$$

$$p^2 - 25p + 125 = 0$$

quadratic formula:

$$p = \frac{+25 \pm \sqrt{625 - 4(125)}}{2}$$

$$p = \frac{25 \pm 11.18}{2}$$

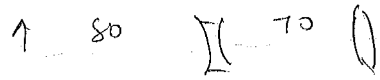
valid possibilities: $\boxed{p = 18.09 \text{ cm}}$ or $\boxed{p = 6.91 \text{ cm}}$

in which case $q = 6.91$ | in which case $q = 18.09$

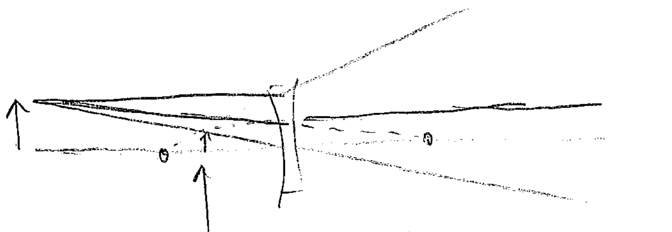
(b) $M = -q/p$, So largest $|M|$ is for this one,

$$|M| = \frac{18.09}{6.91} = \boxed{2.618}$$

(12 pts) **Problem 6**. An object is placed 80 cm to the left of lens 1 (diverging, f = –40 cm). Lens 1 is placed 70 cm to the left of lens 2 (converging, f = 70 cm).



(a) Draw the ray diagram for the image formed by lens 1.



where the three outgoing rays appear to have been coming from.

(b) How far (in magnitude) from lens 2 will the final image be formed? Will the image be to the left or the right of lens 2? Will it be real or virtual? What will be the total magnification? You do not have to provide any ray diagrams for lens 2.

Lens 1: $q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{-40} - \frac{1}{80}\right)^{-1} = -26.67 \text{ cm}$

$M_1 = -q/p = \frac{+26.67}{80} = \underline{+.333}$

Lens 2: $p = 70 + 26.67$

$= 96.67$

$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{70} - \frac{1}{96.67}\right)^{-1} = \underline{253.75 \text{ cm}}$ ← real, since this is +

$M_2 = -\frac{q}{p} = \frac{-253.75}{96.67} = \underline{-2.625}$

$M_{tot} = M_1 \times M_2 = (.333)(-2.625)$

$= \underline{-.875}$

|q| = 253.75 cm (away from lens 2)

Final image will be to the right (left/right) of lens 2

real vs. virtual: real

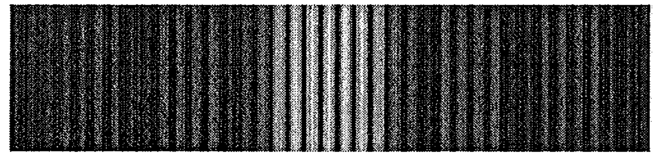
total magnification: –.875

small angles! $\sin\theta \simeq \theta = Y/L$

(8 pts) **Problem 7.** The following diffraction pattern is obtained from two wide slits, using a laser with $\lambda = 633$ nm, on a screen that is 5.2 m away from the slits. This is the actual pattern you see, drawn with accurate dimensions. What is the width of each slit, and what is their separation? (I have provided a ruler of sorts below, with tick marks in inches. Be as accurate as you can given the limitations of the ruler. Answers within about 10% of the correct answer will receive full credit.)

from high frequency doubleslit pattern

from sinc^2 modulation (envelope)



from center to first minimum $\simeq .5''$ $= \underline{1.27\text{cm}}$ ← this is y

Single slit minima: $a\sin\theta = m\lambda$

for $m=1$: $a\left(\frac{Y}{L}\right) = \lambda$

$$a = \frac{\lambda L}{y}$$

$$a = \frac{(633\cdot10^{-9}\text{m})(5.2\text{m})}{(.0127\text{m})}$$

$$a = 2.59\cdot10^{-4}\text{m}$$

$$\boxed{a = 259\,\mu\text{m}}$$

width of slits

high freq: I count ~23 fringes in 2 inches

→ from center to 1st maximum is $\frac{2}{23}$ inch $= \underline{.221\text{ cm}}$ ← this is y

double slit maxima: $d\sin\theta = m\lambda$

for $m=1$: $d\left(\frac{Y}{L}\right) = \lambda$

$$d = \frac{\lambda L}{y}$$

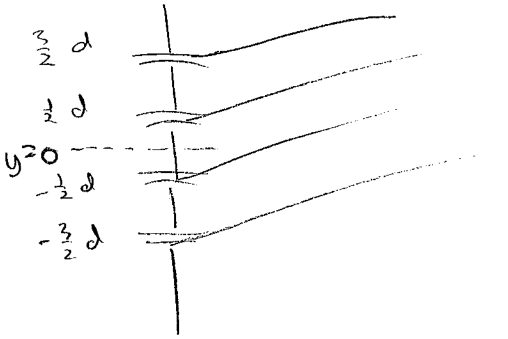
$$d = \frac{(633\cdot10^{-9}\text{m})(5.2\text{m})}{(.00221\text{m})}$$

$$d = 1.49\cdot10^{-3}\text{m}$$

$$\boxed{d = 1.49\text{ mm}}$$

separation of slits

(10 pts) **Problem 8**. Find the intensity formula $I(\theta)$ for the diffraction pattern for four equally spaced slits, each separated from its neighbors by a distance d. Put it in terms of sine and/or cosine functions as appropriate, using $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$ and/or $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$. Hint: You will need to add up phase-shifted sinusoidal waves from each of the slits, the phase shifts arising from path length differences. It's easiest if you reference each phase to the center of the four slits.



$$E = E_0 \left(e^{i\phi_1} + e^{i\phi_2} + e^{i\phi_3} + e^{i\phi_4} \right)$$

each $\phi = \dfrac{2\pi \Delta PL}{\lambda}$ where $\Delta PL = y \sin\theta$

↑ distance from center of slits to each slit.

$$E = E_0 \left(e^{-i \frac{2\pi}{\lambda} \frac{3}{2} d \sin\theta} + e^{-i \frac{2\pi}{\lambda} \frac{1}{2} d \sin\theta} + e^{+i \frac{2\pi}{\lambda} \frac{1}{2} d \sin\theta} + e^{+i \frac{2\pi}{\lambda} \frac{3}{2} d \sin\theta} \right)$$

combine to make $2\cos\left(\frac{2\pi}{\lambda} \frac{1}{2} d \sin\theta\right)$

combine to make $2\cos\left(\frac{2\pi}{\lambda} \frac{3}{2} d \sin\theta\right)$

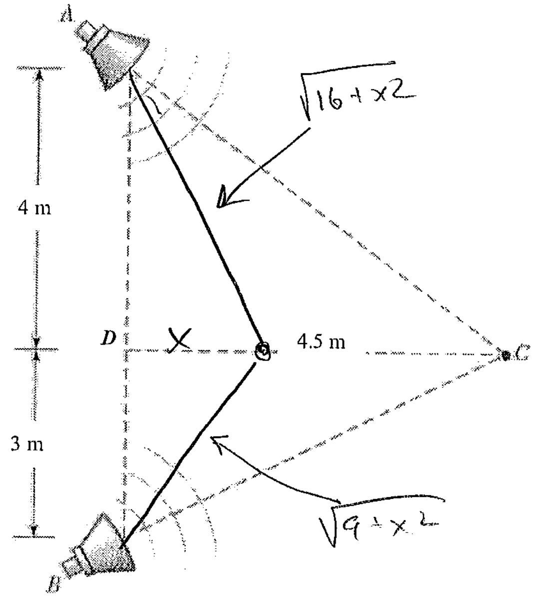
$$E = 2E_0 \left(\cos \frac{\pi d \sin\theta}{\lambda} + \cos \frac{3\pi d \sin\theta}{\lambda} \right)$$

$I \sim E^2$, so

$$\boxed{I = I_0 \left(\cos \frac{\pi d \sin\theta}{\lambda} + \cos \frac{3\pi d \sin\theta}{\lambda} \right)^2}$$

(5 pts, no partial credit) **Problem 9. Extra credit.** Only four students got this problem entirely correct on the last exam, so here is your chance for redemption. The numbers have been changed.

Two speakers are placed as shown in the figure. Both broadcast a signal from the same 1600 Hz source, in phase with each other. A student starts from point D and walks to the right along the dotted line shown, stopping when he hits point C. Find the positions of all of the maxima he will encounter between D and C. (Take the speed of sound to be 343 m/s.)



max: $\Delta PL = m\lambda$

$$\sqrt{16+x^2} - \sqrt{9+x^2} = m\lambda$$

$$\sqrt{16+x^2} = \sqrt{9+x^2} + m\lambda$$

$$16+\cancel{x^2} = (9+\cancel{x^2}) + 2m\lambda\sqrt{9+x^2} + m^2\lambda^2$$

$$\frac{7-m^2\lambda^2}{2m\lambda} = \cancel{2m\lambda}\sqrt{9+x^2}$$

$$x^2 = \left(\frac{7-m^2\lambda^2}{2m\lambda}\right)^2 - 9$$

$$x = \sqrt{\left(\frac{7-m^2\lambda^2}{2m\lambda}\right)^2 - 9}$$

with $\lambda = \frac{v}{f} = \frac{343}{1600} = .2144\text{ m}$

x must be between 0 and 4.5 m

Try some values of m

m	resulting x		
0	infinity	X	
1	15.94	X	too high
2	7.36	X	too high
3	4.15	✓	works
4	2.08	✓	works
5 and higher	complex	X	

} 2 maxima, at

$\boxed{\text{2.08 m and 4.15 m away from pt D}}$