Fall 2012 RED
Physics 123 section 2
Final Exam
Colton 2-3669

Please write your CID here __________

No time limit. No notes, no books. Student calculators OK.

Constants and conversión factors which you may or may not need:

$g = 9.8$ m/s^2
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
$k_B = 1.381 \times 10^{-23}$ J/K
$N_A = 6.022 \times 10^{23}$
$R = k_B \cdot N_A = 8.314$ J/mol·K
$\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
$c = 3 \times 10^8$ m/s
$m_{electron} = 9.11 \times 10^{-31}$ kg
Density of water: 1000 kg/m^3
1 inch = 2.54 cm
1 m^3 = 1000 L
1 atm = 1.013×10^5 Pa = 14.7 psi
1 eV = 1.602×10^{-19} J
$T_F = 9/5\, T_C + 32$
$T_K = T_C + 273.15$

Other equations which you may or may not need to know:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$A_{sphere} = 4\pi r^2$
$V_{sphere} = 4/3\ \pi r^3$
$(1 + x)^n \approx 1 + nx$
$\Delta L = \alpha L_0 \Delta T$; $\Delta V = \beta V_0 \Delta T$; $\beta = 3\alpha$

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/k_B T}$$

$v_{most\ probable} = \sqrt{\dfrac{2k_B T}{m}}$

$v_{avg} = \int_0^\infty v f(v)\, dv = \sqrt{\dfrac{8k_B T}{\pi m}}$

$v_{rms} = \sqrt{\int_0^\infty v^2 f(v)\, dv} = \sqrt{\dfrac{3k_B T}{m}}$

Mean free path: $l = \dfrac{1}{\sqrt{2}\pi d^2 n}$

Ave time between collisions: $\tau = l/v_{avg}$

$P = \dfrac{\Delta Q}{\Delta t} = \dfrac{kA\Delta T}{L} = \dfrac{A\Delta T}{R}$; $R = L/k$

$P = \dfrac{\Delta Q}{\Delta t} = e\sigma A T^4$

$e_{Otto} = 1 - \dfrac{1}{r^{\gamma-1}}$; $r = V_{max}/V_{min}$

$S = k_B \ln W$

$\#\,microstates = \begin{pmatrix} N \\ k \end{pmatrix} = \dfrac{N!}{k!(N-k)!}$

$\#\,macrostates = 2^N$

$P = \frac{1}{2}\mu\omega^2 A^2 v$

$r = \dfrac{v_2 - v_1}{v_2 + v_1} = \dfrac{n_1 - n_2}{n_1 + n_2}$; $t = \dfrac{2v_2}{v_2 + v_1} = \dfrac{2n_1}{n_1 + n_2}$

$R = |r|^2$; $T = 1 - R$

$v_{string} = \sqrt{T/\mu}$; $\mu = m/L$

$v_{rod} = \sqrt{Y/\rho}$; $Y = \dfrac{stress}{strain} = \dfrac{F/A}{\Delta L/L}$

$v_{sound} = \sqrt{B/\rho}$

$v_{sound} = 343\,\frac{\text{m}}{\text{s}} \sqrt{\dfrac{T}{293\text{K}}}$

$\beta = 10\log\left(\dfrac{I}{I_0}\right)$; $I = I_0 10^{\beta/10}$; $I_0 = 10^{-12}$ W/m^2

$f' = f\dfrac{v \pm v_o}{v \pm v_s}$

$\sin\theta = 1/\text{Mach\#}$

$\Delta x \Delta k \geq \frac{1}{2}$; $\Delta x \Delta p \geq \hbar/2$

$\Delta t \Delta \omega \geq \frac{1}{2}$; $\Delta t \Delta E \geq \hbar/2$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{L}\right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{2\pi nx}{L}\right)dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{2\pi nx}{L}\right)dx$$

musical half step: $f_2/f_1 = 2^{1/12}$

$\tan\theta_{Brewster} = n_2/n_1$

$f = R/2$

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

(R_1 = pos, R_2 = neg if convex-convex)

$\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$

(p = pos if object in front of surface, q = pos if image in back of surface, R = pos if center of curvature in back of surface)

$\phi = 2\pi \Delta PL/\lambda$

$\Delta PL = d\sin\theta$

$E = E_0\,(e^{i\phi 1} + e^{i\phi 2} + \ldots)$

$I \sim |E|^2$

2 narrow slit: $I = I_0 \cos^2\left(\dfrac{2\pi}{\lambda}\dfrac{d}{2}\sin\theta\right)$

1 wide slit: $I = I_0 \operatorname{sinc}^2\left(\dfrac{\pi a \sin\theta}{\lambda}\right)$

circular: $\theta_{min.resolve} = 1.22\lambda/D$

grating: $R = \lambda_{ave}/\Delta\lambda$ = #slits × m

Bragg: $2d\sin\theta_{bright} = m\lambda$ (θ from horizontal)

$f' = f\sqrt{\dfrac{1 \pm \beta}{1 \mp \beta}}$

$x_{\text{frame2}} = \gamma x_{\text{frame1}} \pm \gamma\beta(ct)_{\text{frame1}}$

$(ct)_{\text{frame2}} = \pm\gamma\beta x_{\text{frame1}} + \gamma(ct)_{\text{frame1}}$

$\begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame2}} = \begin{pmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}_{\text{frame1}}$

$E^2 = (pc)^2 + (mc^2)^2$; $E = pc$

$1/p'_{photon} - 1/p_{photon} = 2/(m_{electron}c)$

Scores: (for grader to fill in). **100 total points.**

Problem 1	________	Problem 7	________
Problem 2	________	Problem 8	________
Problem 3	________	Problem 9	________
Problem 4	________	Problem 10	________
Problem 5	________	Extra Credit	________
Problem 6	________	**Total**	________

Instructions:

- Record your answers to the multiple choice questions ("Problem 1" on the next page) on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(42 pts) **Problem 1**: Multiple choice conceptual questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. A boat is on a lake. If an anvil (that sinks) is pushed from the boat into the water, will the overall water level of the lake rise, fall or stay the same? (compared to when the anvil was in the boat)
  a. rise
  b. fall
  c. stay the same

When in the boat, buoyant force supports its weight. Displaces volume of water equal to its weight. In water, it displaces volume of water equal to its volume, which is less. Overall water level falls.

1.2. Three cubes of the same size and shape are put in water. They all sink. One is lead, one is steel and one is a dense wood (ironwood). $\rho_{lead} > \rho_{steel} > \rho_{ironwood}$. On which cube is the buoyant force the greatest?
  a. lead
  b. steel
  c. wood
  d. same buoyant force

$B = \rho_{fl} V_{obj} g$ = same for all

1.3. An extremely precise scale is used to measure an iron weight. It is found that in a room with the air sucked out, the mass of the weight is precisely 2.000000 kg. If you add the air back into the room, will the scale reading increase, decrease, or stay the same?
  a. increase
  b. decrease
  c. stay the same

air supports some of its weight

1.4. You have two jars of gas: helium and neon. Both have the same volume, same pressure, same temperature. Which jar contains the greatest number of gas molecules? (The mass of a neon molecule is greater than the mass of a helium molecule.)
  a. jar of helium
  b. jar of neon
  c. same number

$PV = nRT$

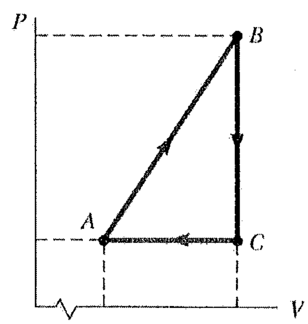
If P, V, and T are same, then n is same

1.5. If one mole of an ideal gas doubles its volume in an isothermal expansion, its pressure is:
  a. quadrupled
  b. doubled
  c. unchanged
  d. halved
  e. quartered

From $PV = nRT$ again

1.6. The figure is a standard P-V diagram for an ideal gas. In which of the three changes shown is there the largest positive change in entropy?
  a. A→B
  b. B→C
  c. C→A
  d. cannot be determined without more information

ΔS requires heat to flow into gas. Only positive for A→B.



1.7. What should the molar heat capacity of all solids be, according to the Dulong-Petit law?

a. R
b. 3/2 R
c. 5/2 R
d. 6/2 R
e. 6 R

6 degrees of freedom, as per the HW problem

1.8. A heat engine performs x joules of work in each cycle and has an efficiency of e. For each cycle of operation, how much heat energy is produced by burning fuel?

a. x
b. x/e
c. xe
d. $(1-x)$
e. $(1-x)/e$
f. $(1-x)e$

$e = \frac{W}{Q_h} \rightarrow Q_h = \frac{W}{e}$

1.9. If you double the number of microstates available to a thermodynamic system, but how much does the entropy change?

a. $S_{new} = 2S_{old}$
b. $S_{new} = ½ S_{old}$
c. $S_{new} = S_{old} \ln 2$
d. $S_{new} = S_{old} / \ln 2$
e. $S_{new} = S_{old} + 2k_B$
f. $S_{new} = S_{old} - 2k_B$
g. $S_{new} = S_{old} + k_B \ln 2$
h. $S_{new} = S_{old} - k_B \ln 2$

$S = k \ln W$ ← # microstates

$S_2 - S_1 = k(\ln 2W - \ln W)$

$= k(\ln 2 + \cancel{\ln W} - \cancel{\ln W})$

$= k \ln 2$

1.10. The second law of thermodynamics is a statement of:

a. conservation of energy
b. conservation of linear momentum
c. conservation of angular momentum
d. conservation of mass and/or volume
e. probability

1.11. If a single violin produces a sound level of 60 dB, how loud would you expect two violins to be? (The violins are playing the same note, but not exactly in phase with each other.)

a. 60 dB
b. 62
c. 63
d. 70
e. 80
f. 90
g. 120 dB

factor of 2 in intensity = +3 dB

1.12. A car horn emits a 2000 Hz sound. Will you hear a higher frequency if the car approaches you at 30 mph while you are at rest, or if you approach the car at 30 mph while the car is at rest?

a. Car approaches you
b. You approach car
c. Same frequency shift for both cases

Compare $\frac{343+30}{343}$ to $\frac{343}{343-30}$ ← larger

1.13. A wave packet is constructed by adding together an infinite number of cosine waves, all having amplitude 1:

$$f(x,t) = \cos\left(k_1(x - v_1 t)\right) + \cos\left(k_2(x - v_2 t)\right) + ...$$

The spatial frequencies present in the summation range from k = 5 to 6, and they are spaced infinitely close together. (All numbers in this problem are given in standard SI units.) Due to dispersion in the medium, waves with different k values travel at different velocities. For a given cosine term, the relationship between the velocity and the k-value is given by: $v = 1 + 2k + 2k^2$. What is the phase velocity of the wave packet?

$k_{ave} = 5.5$

i.e. I'm giving the phase velocity eqn

a. Less than 70 m/s
b. 70 - 78
c. 78 - 86
d. 86 - 94
e. 94 - 102
f. 102 - 110
g. 110 - 118
h. 118 - 126
i. More than 126 m/s

$v = 1 + 2(5.5) + 2(5.5)^2$

$= 72.5$ m/s

1.14. Same situation. What is the group velocity of the wave packet?

a. Less than 160 m/s
b. 160 - 190
c. 190 - 220
d. 220 - 250
e. 250 - 280
f. 280 - 310
g. 310 - 340
h. 340 - 370
i. More than 370 m/s

$V_p = \frac{\omega}{k} = 1 + 2k + 2k^2$

$\omega = 1 + 2k^2 + 2k^3$

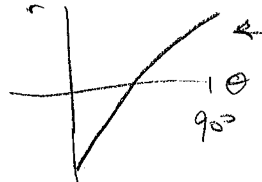
$V_g = \frac{d\omega}{dk} = 4k + 8k^2$

at k_{ave}, $= 4(5.5) + 8(5.5)^2 = 264 \frac{m}{s}$

1.15. What is the phase shift in the reflected beam when a p-polarized light ray goes from air to glass at close to grazing incidence (Snell's law θ near 90°).

a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

For p:

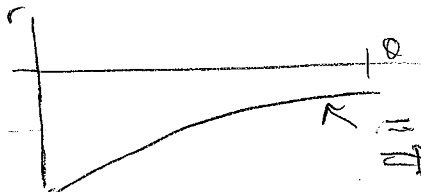


← r = positive ⇒ no phase shift

1.16. What is the phase shift in the reflected beam when an s-polarized light ray goes from air to glass at close to grazing incidence (Snell's law θ near 90°).

a. 0° (i.e. no phase shift)
b. 45°
c. 90°
d. 180°

For s:



↖ = negative ⇒ 180° phase shift

1.17. At a wavelength of 589 nm, a typical diamond has an index of refraction of 2.42. What is the speed of light in the diamond for that wavelength?

a. Less than 1.2×10^8 m/s
b. 1.2 - 1.5
c. 1.5 - 1.8
d. 1.8 - 2.1
e. 2.1 - 2.4
f. 2.4 - 2.7
g. $2.7 - 3.0 \times 10^8$ m/s

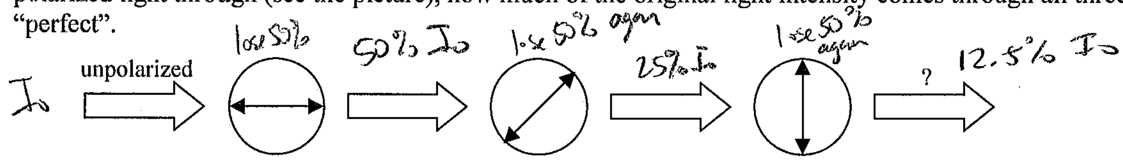
$V = \frac{c}{n} = \frac{3 \cdot 10^8}{2.42} = 1.24 \cdot 10^8$ m/s

1.18. Same situation. If the wavelength increases a bit, the speed of light will most likely:

a. increase
b. decrease
c. stay the same

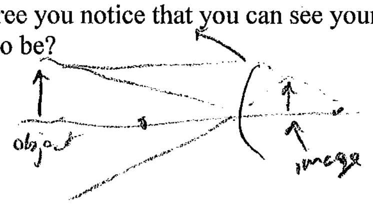
↳ more red = faster

1.19. When unpolarized light is shined at two crossed polarizers (axes 90° apart), no light gets through. However, if a third polarizer is added between the two, some light *can* get through (we did this as a demo in class). If the first polarizer lets horizontally polarized light through, the second lets 45° polarized light through, and the third polarizer lets vertically polarized light through (see the picture), how much of the original light intensity comes through all three? All polarizers are "perfect".



a. 0 - 5% of the original intensity
b. 5 - 10%
c. 10 - 15%
d. 15 - 20%
e. 20 - 25%
f. 25 - 30%
g. 30 - 35%
h. 35 - 40%
i. 40 - 45%
j. 45 - 50% of the original intensity

1.20. While decorating your Christmas tree you notice that you can see yourself in one of the spherical, reflective ornaments. What type of image is it most likely to be?



a. Real, inverted, and enlarged
b. Real, inverted, and reduced
c. Real, upright, and enlarged
d. Real, upright, and reduced
e. Virtual, inverted, and enlarged
f. Virtual, inverted, and reduced
g. Virtual, upright, and enlarged
h. Virtual, upright, and reduced

1.21. When you look down at an object under water, will it appear to be closer to you or farther away than actuality?

a. Closer to you
b. Farther away
c. Same as actual distance

image

1.22. In slit problems, when can you make the approximation that $\sin\theta = y/L$?

Small angle!

a. When the slit size/spacing is much smaller than L
b. When the slit size/spacing is much larger than L
c. When the part of the diffraction pattern you are interested in is much smaller than L
d. When the part of the diffraction pattern you are interested in is much larger than L

θ = L

1.23. Which of the following is the best resolution of the twin paradox, as discussed in class? Speedo = the twin that goes on the rocket; Goslo is the twin that stays home. Speedo goes on a trip to a distant star, then returns to Earth.

a. Speedo will be older than Goslo, because he has a larger proper time during the trip.
b. Speedo will be younger than Goslo, because he accelerates during part of his trip.
c. The two twins end up the same age, because each ages the slowest during half the total trip.

1.24. Which of the following is the best resolution of the barn paradox, as discussed in class and analyzed for homework? (Lee is the one running with the ladder; Cathy is the one at rest relative to the barn.)

a. Cathy sees the ladder fit entirely within the barn, but Lee does not.
b. Lee sees the ladder fit entirely within the barn, but Cathy does not.
c. Each of them sees the ladder fit entirely within the barn.

1.25. A spaceship is heading toward an enemy space station at 0.5c and launches a missile at it that the spaceship crew sees leaving at 0.8c. If the length of the missile is measured by various observers, who would measure the length to be the shortest?

→ largest γ relative to the missile

a. the ship's crew
b. the station's crew
c. an ant living on the missile

1.26. Dr. Colton is throwing a ball forward while riding on a fast train past the class (stationary, on the ground). Suppose he throws the ball at 0.5 c, and the train is moving at 0.01 c in the same direction. Which of the following will be closest to the speed the class observes the ball to be thrown at?

not very relativistic. Close to regular addition

a. 0.003 c
b. 0.20 c
c. 0.49 c
d. 0.51 c
e. 0.80 c
f. 0.997 c

1.27. What is the maximum momentum that a particle with mass *m* and velocity *v* can have?

$\gamma m v$ ← can be arbitrarily large

a. *mv*
b. *mc*
c. *mcv*
d. *2mv*
e. *2mc*
f. *2mcv*
g. There is no maximum

1.28. What is the maximum total energy (kinetic + rest) that a particle with mass *m* and velocity *v* can have?

$\gamma m c^2$ ← same

a. $\frac{1}{2}mv^2$
b. $\frac{1}{2}mv^2 + mc^2$
c. $\frac{1}{2}mc^2$
d. mc^2
e. $3/2\ mc^2$
f. There is no maximum

283 K 293 K

(9 pts) **Problem 2.** A heat pump pumps heat from outside your house (10°C) to inside your house (20°C). It pumps heat into your house at a rate of 2600 J per cycle.

(a) What is the theoretical limit for the heat pump's coefficient of performance?

$$COP = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c}$$

$$COP_{max} = \frac{T_h}{T_h - T_c} = \frac{293}{10} = \boxed{29.3}$$

(b) The actual COP is 3.5. How much work per cycle is required to operate the heat pump?

$$W = \frac{Q_h}{COP} = \frac{2600\ J}{3.5} = \boxed{742.9\ J}$$

(c) How much heat per cycle is removed from the great outdoors?

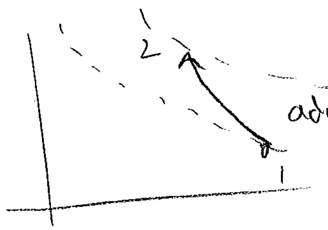
$$Q_c = Q_h - W$$

$$= 2600 - 742.9$$

$$= \boxed{1857.1\ J}$$

(8 pts) **Problem 3**. In the demo I did where a piece of cotton was ignited by compressing air in a piston, I estimate that I compressed the piston to about 0.15 of the original volume before the cotton caught fire. The air started at room temperature, call it 294 K. Assuming this was an adiabatic process, (a) determine the temperature of the air when the cotton ignited, and (b) draw a picture of the process on a P-V diagram. (Hint: treat air as a diatomic gas.)

$\rightarrow \gamma = \frac{7}{5} = 1.4$



combine $P_1 V_1^\gamma = P_2 V_2^\gamma$ and $PV = nRT$

$$\left(\frac{\cancel{nR}T_1}{V_1}\right) V_1^\gamma = \left(\frac{\cancel{nR}T_2}{V_2}\right) V_2^\gamma$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = (294)\left(\frac{\cancel{V_1}}{.15\cancel{V_1}}\right)^{1.4-1}$$

$$= (294)\left(\frac{1}{.15}\right)^{.4}$$

$$= \boxed{627.93\text{ K}}$$

(surprisingly close to some values for the ignition point of cotton that I found on the internet)

(13 pts) **Problem 4.** (problem credit: Dr. Durfee) The high B string on a guitar is tuned such that its fundamental frequency is 246.9 Hz. The string has a diameter of 0.82 mm. This string is strung onto a classical guitar with a "scale length" of 64.8 cm. That's the length between the forced nodes at the ends of the string. According to the manufacturer, the string needs to be at a tension of 51.6 N to play in tune.

f   L

(a) What is the wavelength λ, wavenumber k, and angular frequency ω, of the fundamental mode of the string (1st harmonic)?

fundamental: $\lambda = 2L$ $\qquad \lambda = 2(64.8) = \boxed{129.6\text{ cm}} = 1.296\text{ m}$

$k = 2\pi/\lambda = 2\pi/1.296 = \boxed{4.848\text{ rad/m}}$

$\omega = 2\pi f = 2\pi \cdot 246.9\text{ Hz} = \boxed{1551.3\text{ rad/s}}$

(b) What is the speed at which waves travel on the string?

$v = \omega/k$ or $= f \cdot \lambda$

$= (246.9)(1.296)$

$= \boxed{319.98\text{ m/s}}$

(c) What is the linear mass density μ, of the string?

$v = \sqrt{T/\mu} \rightarrow v^2 = \frac{T}{\mu} \rightarrow \mu = T/v^2$

$\mu = \frac{51.6}{(319.98)^2} = \boxed{5.040 \cdot 10^{-4}\text{ kg/m}}$

(d) If you install the string on a child's guitar with a scale length of only 45.7 cm, what tension will it need in order to play in tune? (The linear mass density remains unchanged.)

$f_1 = f_2$

$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$

$\frac{\sqrt{T_1/\cancel{\mu_1}}}{\lambda_1} = \frac{\sqrt{T_2/\cancel{\mu_2}}}{\lambda_2}$

$\frac{T_1}{\lambda_1^2} = \frac{T_2}{\lambda_2^2}$

$\rightarrow T_2 = T_1 \left(\frac{\lambda_2}{\lambda_1}\right)^2$

$= (51.6\text{ N}) \left(\frac{\cancel{2}\cdot 45.7}{\cancel{2}\cdot 64.8}\right)^2$

$= \boxed{25.66\text{ N}}$

(8 pts) **Problem 5.**

(a) A transverse wave undergoes a transition from rope 1 to rope 2. The wave travels half as fast on rope 2 as on rope 1. How do the reflected and transmitted amplitudes compare to the incident amplitude?

reflected $r = \frac{v_2 - v_1}{v_2 + v_1} = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = \frac{-1/2}{3/2} = \boxed{-\frac{1}{3}}$ (180° phase shift, ampl. reduced to 1/3 original)

transmitted $t = \frac{2 v_2}{v_2 + v_1} = \frac{2 \cdot \frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{3/2} = \boxed{\frac{2}{3}}$ (no phase shift, ampl. reduced to 2/3 original)

(b) A light wave strikes a boundary between air and glass ($n = 1.5$) at normal incidence. How do the reflected and transmitted *powers* compare to the incident power?

$r = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - 1.5}{1 + 1.5} = \frac{-.5}{2.5} = -.2$

$R = |r|^2 = .04 \longrightarrow$ $\boxed{\text{4\% reflected}}$

$T = 1 - R = \boxed{96\%}$ transmitted

(12 pts) **Problem 6**. Give short answers/explanations to the following questions:

(a) What general kind of wave would result for each of these situations?

(1) A wave is created by adding together a large number of sine waves at the same frequency, ω_0, but with the sine waves all having possibly different amplitudes and phases.

Get a sine wave at W_0, with an unknown amplitude & phase

(2) A wave is created by adding together two traveling waves at the same frequency, ω_0, but with the waves passing through each other traveling in opposite directions.

Get a standing wave, at same frequency W_0

(3) A wave is created by adding together an infinite number of sine waves at integer multiples of the same frequency, ω_0, with the sine waves all having the same phase but possibly different amplitudes.

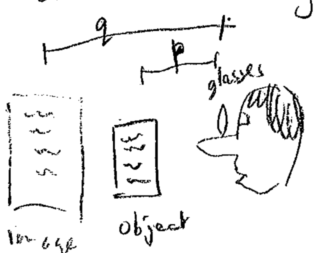
This is a Fourier series! Fundamental is W_0.

Get a nonsinusoidal shape wave with period $2\pi/W_0$

(b) A near sighted person is wearing glasses to improve their vision while reading a book. These glasses are essentially like magnifying glasses held close to their eyes. Qualitatively, what would "p" be? What would "q" be? (Specify the distance from what to what, and whether they are positive or negative.)

p = distance from book to glasses, positive

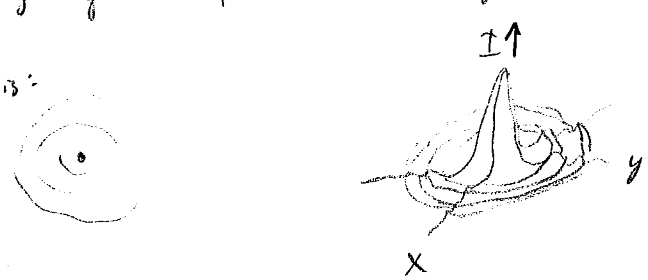
q = distance from glasses to image of book (on same side as book), negative



(c) If I try to use a lens to produce an image of a point source of light, the image will unfortunately not be exactly a point. Why is that, and what will the image look like? Make a rough sketch of what the image will be like.

Due to diffraction through the circular aperture of the lens, you get a pattern with rings.

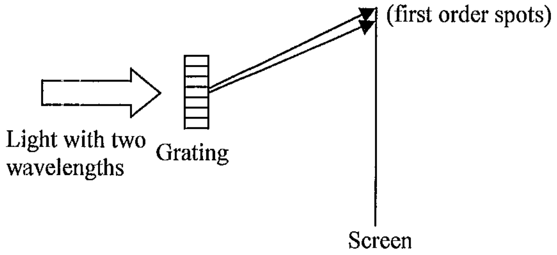
Like this:



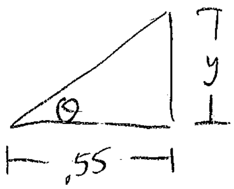
(12 pts) **Problem 7**. A diffraction grating contains 1200 lines/mm. You want to use the grating to separate two colors of light as shown in the figure: one with λ = 817.6 nm and the other with λ = 818.0 nm. How far apart physically will the two first order spots appear on a screen that is set up 0.55 m away from the grating?

λ_1 λ_2

$d = \frac{1\,mm}{1200} = 8.333 \cdot 10^{-7} m$



grating eqn: $d \sin\theta = m\lambda$



Also $\tan\theta = y/L$

① $\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)$

$= \sin^{-1}\left(\frac{1 \cdot 817.6 \cdot 10^{-9}}{8.333 \cdot 10^{-7}}\right) = 78.849°$

$y_1 = .55 \tan\theta_1 = .55 \tan(78.966°)$

$= 2.79016\,m$

② $\theta_2 = \sin^{-1}\left(\frac{818.0 \cdot 10^{-9}}{8.333 \cdot 10^{-7}}\right) = 78.992°$

$y_2 = .55 \tan(78.992°) = 2.8274\,m$

$\Delta y = y_2 - y_1$

$= \boxed{.0372\,m}$

3.72 cm

(9 pts) **Problem 8**. The planet Zog is at rest with respect to the Earth, 10 light years away in both Zog's and Earth's frames of reference. Henry travels from Earth to Zog at a speed of 0.99 c (relative to both Zog and the Earth) to make his bread delivery and pick up the famous "Zoggy cakes".

(a) According to Henry's twin brother Albert (on Earth), how long does the journey take, and how far does Henry travel?

distance : $\boxed{10 \text{ ly}}$ (given)

time : $v = d/t \rightarrow t = d/v = \dfrac{10 \text{ ly}}{.99c} = \boxed{10.10 \text{ yrs}}$

(b) According to Henry himself, how long does the journey take, and how far does he travel?

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-.99^2}} = 7.089$$

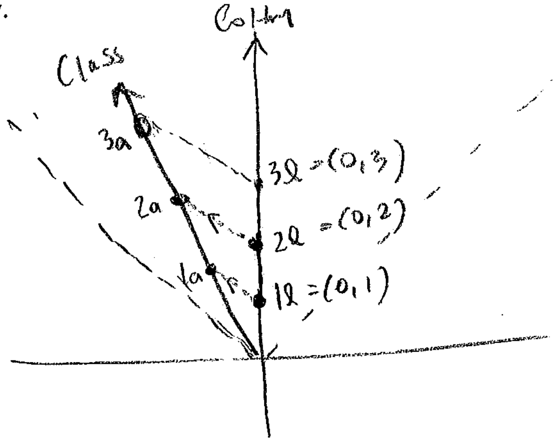
d = Lorentz contracted $= \dfrac{10 \text{ ly}}{7.089} = \boxed{1.411 \text{ ly}}$

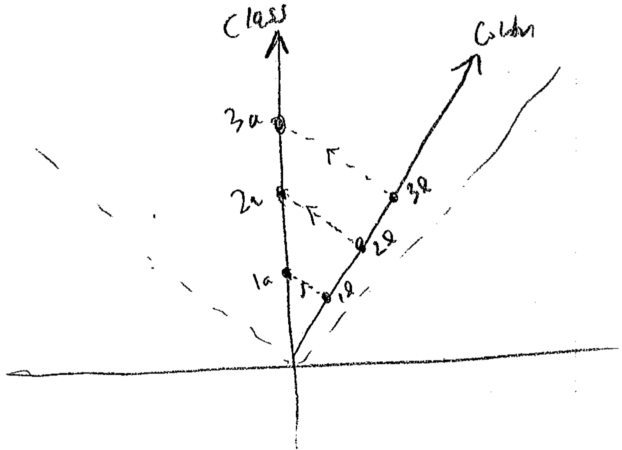
t = also reduced by $\gamma = \dfrac{10.10 \text{ yrs}}{7.089} = \boxed{1.425 \text{ yrs}}$

(or $t = d/v$ again, in Henry's frame

$= \dfrac{1.411 \text{ lyr}}{.99 \cancel{c}} = 1.425$ yrs)

(15 pts) **Problem 9**. Suppose Dr. Colton is on a high speed train, traveling at 0.5 c. The class is standing still next to the train tracks, watching him. Dr. Colton has a digital stopwatch that the class can see, which ticks off seconds (in his frame). The stopwatch starts at 0 precisely when he passes the class. Consider the first few "ticks" on the stopwatch. When would the class see each new digit appear on the stopwatch display? By "see", I specifically mean: when would the light from the appearing digits 1, 2, and 3, reach the class (in their frame)? Draw fairly accurate space-time diagrams for both Dr. Colton's and the class's frames of reference. On each diagram, label the world lines for Dr. Colton and the class, the "light leaves Dr. Colton" and "light arrives at students" events for each of the digits 1, 2, and 3 (labeled as 1l, 2l, 3l, and 1a, 2a, 3a), along with the world lines for the three light waves themselves. Hint: I found it easier to draw the diagrams first, because with their assistance I was able to visualize how to solve the problem numerically.





Plan: (a) Transform 1l, 2l, and 3l to class frame.
(b) Then use 45° trick.

(a) $\beta = .5 \rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.1547$

$\gamma\beta = .57735$

$\begin{pmatrix} x \\ ct \end{pmatrix}_{1l} = \begin{pmatrix} 1.1547 & .57735 \\ .57735 & 1.1547 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} .57735 \\ 1.1547 \end{pmatrix}$

$\begin{pmatrix} x \\ ct \end{pmatrix}_{2l} = (\quad '' \quad)\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1.1547 \\ 2.3094 \end{pmatrix}$

$\begin{pmatrix} x \\ ct \end{pmatrix}_{3l} = (\quad '' \quad)\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1.73205 \\ 3.4641 \end{pmatrix}$

(b) 45° trick

(1) From 1l to 1a → Up .57735 and left .57735 → $\begin{pmatrix} 0 \\ 1.7291 \end{pmatrix}$

(2) From 2l to 2a → up 1.1547 and left same → $\begin{pmatrix} 0 \\ 3.4641 \end{pmatrix}$

(3) From 3l to 3a → up 1.73205 and left same → $\begin{pmatrix} 0 \\ 5.1962 \end{pmatrix}$

Class sees ticks at $\boxed{1.7291\,s, \quad 3.4641\,s, \text{ and } 5.1962\,s}$

$\gamma = 1.1547$

(12 pts) **Problem 10**. Miraculously, you manage to stick two balls of clay together in a high speed collision: a 1 kg mass traveling at 0.5 c collides with and sticks to a 1 kg stationary mass. (All speeds are relative to your laboratory.) Due to the lost kinetic energy being turned into additional mass,* the final combined mass is actually 2.0759 kg rather than 2 kg.

(a) With what speed does the combined mass move after the collision?

1 $\rightarrow$ 1 2.0759 $\rightarrow$

$$\sum \vec{p}_{bef} = \sum \vec{p}_{aft}$$

$$\gamma m v/c + \gamma m v/c = \gamma m v/c$$

$$(1.1547)(1)(.5) + 0 = \gamma\beta\, 2.0759$$

$$\gamma\beta = .27812$$

$$\frac{\beta}{\sqrt{1-\beta^2}} = .278$$

$$\beta = .278\sqrt{1-\beta^2}$$

$$\beta^2 = .278^2(1-\beta^2)$$

$$\beta^2(1+.278^2) = .278^2$$

$$\beta = \frac{.278}{\sqrt{1+.278^2}} = \boxed{.26795}$$

(b) Find the total energy before the collision, and the total energy after the collision.

bef

$$\gamma mc^2 + \gamma mc^2$$

$$\left((1.1547)(1) + (1)(1)\right)c^2$$

$$\boxed{2.1547\ \text{kg}\cdot c^2}$$

$$\approx 1.939\cdot 10^{17}\ \text{J}$$

aft

$$\gamma mc^2$$

$$\left(\frac{1}{\sqrt{1-.26795^2}}\right)(2.0759)\,c^2$$

$$= \boxed{2.1547\ \text{kg}\cdot c^2}$$

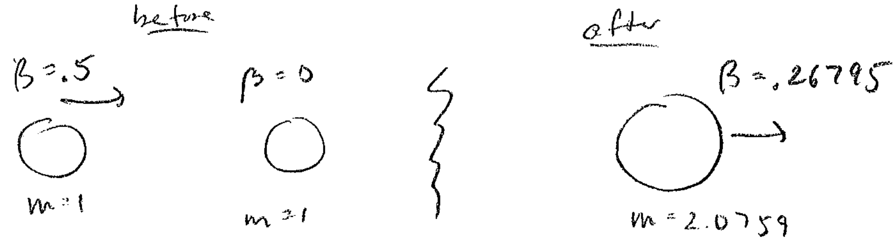
$$= 1.939\cdot 10^{17}\ \text{J}$$

The same!

* Equivalently, you could think of this as meaning that the combined energies "lost" in the collision (the energies of heat, sound, etc.) are equal to 0.0759 kg × c^2 joules.

(5 pts, <u>no partial credit</u>) **Problem 11. Extra credit.** Return to problem 10. Consider the situation from the frame of reference that is moving to the right at the speed you found in part (a). In that reference frame, determine the momentum before and after the collision, and the energy before and after the collision.

From problem 10:

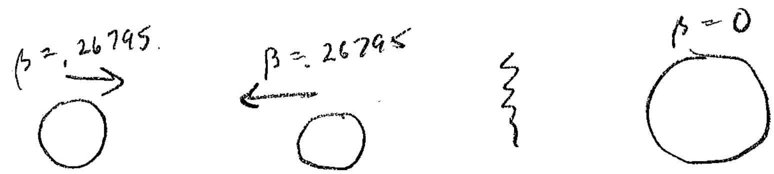


In reference frame of ball after collision...

Need to find speed of ball 1

$$\beta_{13} = \frac{\beta_{12} + \beta_{23}}{1 + \beta_{12}\beta_{23}} = \frac{.5 - .26795}{1 + (.5)(-.26795)} = \underline{.26795} \quad \text{cool!}$$

situation is



<u>momentum</u>: bef $\gamma m v + \gamma m v = \boxed{0}$ (same γ, same β, but one is negative)

aft $\gamma m v = \boxed{0}$ (v=0)

<u>energy</u>: <u>bef</u> $\gamma m c^2 + \gamma m c^2 = \frac{1}{\sqrt{1 - .26795^2}} \cdot c^2 + \text{same} = \boxed{2.0759 \text{ kg } c^2}$

$(= 1.87 \cdot 10^{17}\text{ J})$

<u>aft</u> $\gamma m c^2 = 1 \cdot 2.0759 \cdot c^2 = \boxed{2.0759 \text{ kg} \cdot c^2}$

same!

Both momentum and energy are conserved in this reference frame as well