Solutions

(15 pts) Problem 1: Multiple choice conceptual questions. **Fill in your answers on the bubble sheet.**

1.1. You have two balloons, one filled with air and one filled with helium. If you put both balloons into a tub of liquid nitrogen, which one will end up with the largest volume?
   a. the air balloon   *like in demo — the air in "air balloon" will condense*
     b. the helium balloon
     c. they will end up with the same volume

1.2. A pool is filled half with water and half with a light oil (density 600 kg/m³). The oil floats on the water. When a diver comes up from the bottom of the pool, from the water into the oil, she will experience a buoyant force in the oil that is _____ the buoyant force she felt in the water.
     a. greater than
     b. less than $F_B \sim$ *density of fluid*
     c. the same as

1.3. For the next three problems, consider the cyclic process described by the figure. For A to B: is $W_{on\ gas}$ positive, negative, or zero?
     a. Positive
     b. Negative   *Expands : work done by gas*
     c. Zero
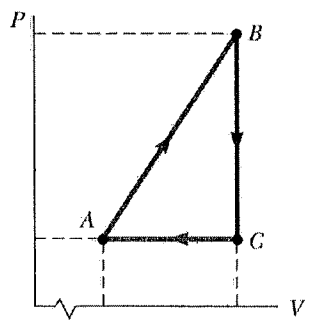

1.4. For B to C: is heat added or taken away from the gas?
     a. Added   *No work, so $\Delta E_{int} = Q$*
     b. Taken away
     c. Neither ($Q_{added} = 0$)   *ΔE_{int} is negative since temp decreases*

1.5. For C to A: does the internal energy increase, decrease, or stay the same?
     a. Increase
     b. Decrease   *Temp decreases (visualize contours)*
     c. Stays the same ($\Delta E_{int} = 0$)

1.6. The figure shows a circular piece of steel with a gap. When the steel is heated, the width of the gap:
     a. decreases
     b. increases   *Like a photographic enlargement*
     c. stays the same
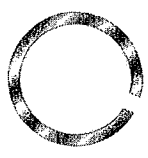
1.7. How many degrees of freedom is a CH_4 molecule likely to have at room temperature? (That is a molecule that has a carbon atom at the middle of a tetrahedron, with 4 hydrogen atoms sticking out from the carbon in four different directions along the points of the tetrahedron.)
     a. 0   f. 5
     b. 1   g. 6   *3 translational*
     c. 2   h. 7   *3 rotational*
     d. 3   i. 8
     e. 4   j. 9

1.8. Bernoulli's Law is a statement of:
     a. conservation of energy
     b. conservation of (regular) momentum
     c. conservation of angular momentum
     d. conservation of mass/volume
     e. probability
     f. none of the above

1.9. The first law of thermodynamics is a statement of:
     a. conservation of energy
     b. conservation of (regular) momentum
     c. conservation of angular momentum
     d. conservation of mass/volume
     e. probability
     f. none of the above

1.10. The second law of thermodynamics is a statement of:
    a.   conservation of energy
    b.   conservation of (regular) momentum
    c.   conservation of angular momentum
    d.   conservation of mass/volume
    (e.) probability
    f.   none of the above

1.11. As an airplane flies horizontally at a constant elevation, the pressure above a wing is _____ the pressure below the wing.
   a.   larger than
    (b.) smaller than   *The difference in pressures generates lift*
    c.   the same as

1.12. A plastic cube and a metal cube of the same size and shape are put into water. The plastic cube floats; the metal cube sinks. On which cube is the bouyant force the largest?
    a.   plastic
    (b.) metal   *B ~ Volume submerged*
    c.   same buoyant force

1.13. This normalized histrogram was created from 50 students in a physics class. If you pick a student at random, what are the chances he/she will be between 61 and 67 inches tall?
   a.   0 – 10%
    b.   10 – 20%
    (c.) 20 – 30%
    d.   30 – 40%
    e.   40 – 50%
    f.   50 – 60%
    g.   60 – 70%
    h.   70 – 80%
    i.   80 – 90%
    j.   90 – 100%

Add up areas, looks like 24% to me.
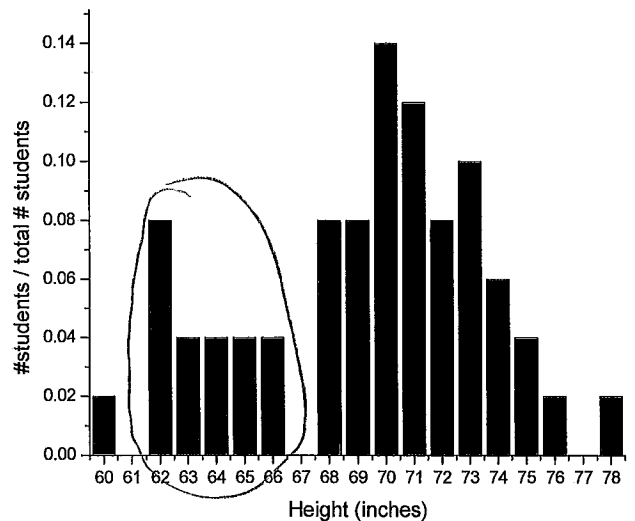
1.14. As I'm taking data in my lab, I typically average data for 1 second per point. Suppose I decide to average the data for 2 seconds per point instead. How much better is my signal-to-noise ratio likely to be?
    a.   the same
    (b.) $\sqrt{2}$ times better
    c.   2 times better
    d.   4 times better
    e.   8 times better

1.15. First, heat is added to a gas while keeping its volume constant, increasing its temperature to 2.5× the original value. Next, the gas is compressed to 40% of its original value while keeping the temperature constant. Which of the following diagrams best represents the two processes on a standard P-V diagram?

a)   b)   c)   d)

e)   f)   g)   (h)   *Const temp.*   *Const volume*
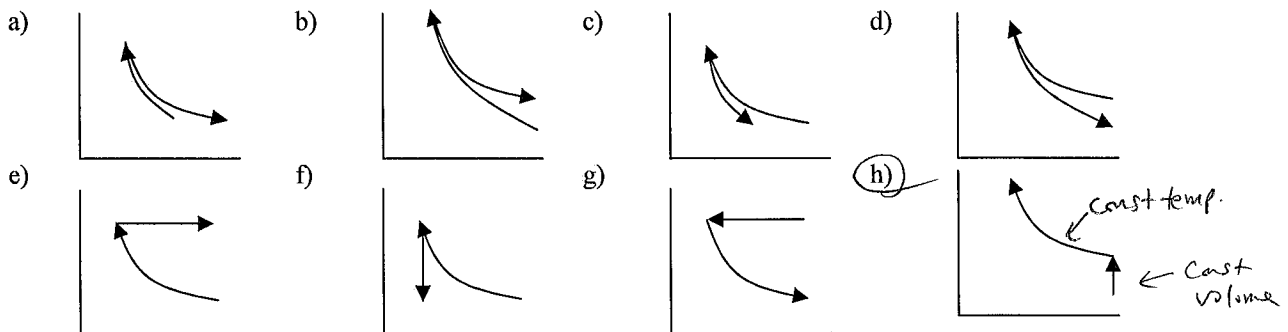
(10 pts) **Problem 2**. (a) You have a 1.003 cm diameter steel ball which you desire to pass through a 1.000 cm inner diameter aluminum ring. Both are at 50° C. If you heat up only the ring, how hot does it need to get (°C)?

$$\Delta L = \alpha L \Delta T \quad (\text{for ring})$$

$$.003 \text{ cm} = \left(24 \cdot 10^{-6} /°C\right)\left(1 \text{ cm}\right) \Delta T$$

$$\Delta T \approx 125 °C$$

$$\boxed{T = 175 °C}$$

(b) A typical 100 W incandescent light bulb has a tungsten filament which is at a temperature of 3000 K. Typically, of the 100 W that goes into the bulb, 97.4 W of heat is conducted or convected away, and only 2.6 W is radiated as light (and most of that is invisible infrared light—now you see why incandescent lights are so inefficient). If you assume the emissivity of the filament to be 0.4, what is the filament's surface area?

$$P = e \sigma A \left(T^4 - \cancel{T_0^4}\right) \quad \text{can be neglected since it's much smaller than } T^4$$

$$A = \frac{P}{e \sigma T^4} = \frac{2.6 \text{ W}}{(.4)\left(5.67 \cdot 10^{-8} \frac{W}{m^2 k^4}\right)\left(3000 K\right)^4}$$

$$= \boxed{1.415 \; 10^{-6} \; m^2}$$

(10 pts) **Problem 3**. A heat pump pumps heat from outside your house (10°C) to inside your house (20°C). It pumps heat into your house at a rate of 2000 J per cycle.
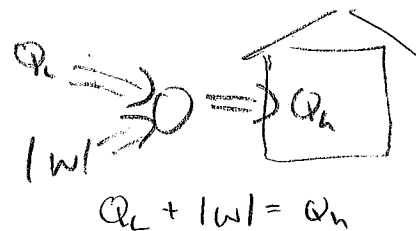
4 pts (a) What is the theoretical limit for the heat pump's coefficient of performance?

$$COP = \frac{Q_h}{|W|} = \frac{Q_h}{Q_h - Q_L}$$

$$COP_{max} = \frac{T_h}{T_h - T_L}$$

$$\approx \frac{293.15 K}{10 K}$$

$$= \boxed{29.3}$$

Q_h

$Q_L + |W| = Q_h$

3 pts (b) The actual COP is 3.5. How much work per cycle is required to operate the heat pump?

$$|W| = \frac{Q_h}{COP} = \frac{2000 J}{3.5} = \boxed{571 J}$$

3 pts (c) How much heat per cycle is removed from the great outdoors?

$$Q_L = Q_h - |W|$$

$$= 2000 J - 571 J$$

$$= \boxed{1429 J}$$

(12 pts) **Problem 4.** You add 200 g of copper at 150°C to 300 g of water at 30°C. The water is in a nifty foam container which completely insulates the water and copper from any outside heat and doesn't absorb any heat itself. What is the final temperature of the water/copper mixture?

$$\left| Q_{\text{lost by Cu}} \right| = \left| Q_{\text{gained by water}} \right|$$

$$(mc\Delta T)_{Cu} = (mc\Delta T)_w$$

$$m_{Cu} C_{Cu} (150 - T_f) = m_w C_w (T_f - 30)$$

$$(200\,g)(387 \tfrac{J}{kg°C})(150°C - T_f) = (300\,g)(4186 \tfrac{J}{kg°C})(T_f - 30°C)$$

$$11610000°C - 774\,T_f = 12558\,T_f - 376740°C$$

$$492840°C = 13332\,T_f$$

$$\boxed{T_f = 36.97°C}$$

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$\Rightarrow \gamma = 5/3$

(20 pts) **Problem 5.** An engine using 0.2 moles of a (monatomic) ideal gas is driven by this cycle: starting from state A (150 kPa, 350 K), the gas is compressed isothermally until it reaches state B at 400 kPa. Then, the gas is heated at constant volume until it reaches state C. Finally, the gas is expanded adiabatically back to the original state.

	P (kPa)	V (m³)	T (K)
A	150		350
B	400		350
C			
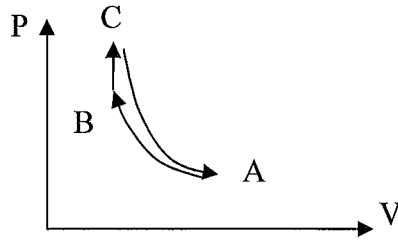
8 pts (a) Find the unknown P's and T's for all three states. (Hint: it's probably easiest to find them in the order listed.)

state A volume:

$$PV = nRT \rightarrow V_A = \frac{nRT_A}{P_A} = \frac{(.2 \, mol)(8.31 \, J/mol\,k)(350k)}{150 \cdot 10^3 \, Pa}$$

$$V_A = \boxed{3.878 \cdot 10^{-3} \, m^3}$$

state B volume (= state C volume):

$$V_B = \frac{nRT_B}{P_B} = \frac{(.2 \, mol)(8.31 \, J/mol\,k)(350k)}{400 \cdot 10^3 \, Pa}$$

$$V_B = \boxed{1.454 \cdot 10^{-3} \, m^3} = V_C \text{ also}$$

state C temperature: (Hint: figure out/use the adiabatic relation between T and V to connect states A and C. Warning: be careful! If you get this part wrong, you will likely miss three of the next five questions.)

$$P_A V_A^\gamma = P_C V_C^\gamma$$

divide $\left(\rightarrow\right.$ $\dfrac{P_A V_A^\gamma = P_C V_C^\gamma}{\dfrac{P_A V_A}{T_A} = \dfrac{P_C V_C}{T_C}}$

$$T_A V_A^{\gamma-1} = T_C V_C^{\gamma-1}$$

$$T_C = T_A \left(\frac{V_A}{V_C}\right)^{\gamma-1} = 350k \left(\frac{3.878 \cdot 10^{-3}}{1.454 \cdot 10^{-3}}\right)^{2/3} = \boxed{673.05 \, K}$$

state C pressure: (Hint: after you get this answer, it's probably worth taking a minute or two and using it to double-check your answer to the state C temperature. If you did things right then, your answer here should satisfy the normal adiabatic relation between P and V that connects states A and C.)

$$PV = nRT \rightarrow P_C = \frac{nRT_C}{V_C} = \frac{(.2 \, mol)(8.31 \, J/mol\,k)(673.05k)}{1.454 \cdot 10^{-3} \, m^3}$$

$$= \boxed{769.3 \, kPa}$$

check $P_C V_C^\gamma \stackrel{?}{=} P_A V_A^\gamma$

$769.2 \, (1.45)^{5/3} \stackrel{?}{=} (150)(3.89)^{5/3}$ ✓

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Problem 5, cont.

8 pts   (b) Find the heat added to the gas during each of the three legs.

<u>A-B</u>   isothermal

$$\Delta E_{int} = W_{on} + Q_{added}$$

$$Q = -W_{on} = + \int P\,dV = \int \frac{nRT}{V}\,dV = nRT \ln V_f/V_0$$

$$= (.2\,mol)(8.31\ J/mol\,K)(350\,K)\ln\left(\frac{1.454 \cdot 10^{-3}}{3.878 \cdot 10^{-3}}\right)$$

$$= \boxed{-570.55\ J}$$

<u>B-C</u>

const V

$$\Delta E_{int} = \cancel{W_{on}} + Q_{added}$$

$$Q = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(.2\,mol)(8.31\ J/mol\,K)(673.05\,K - 350\,K)$$

$$\underset{\text{because}}{\downarrow} \quad = \boxed{805.36\ J}$$
monatomic

<u>C-A</u>

adiabatic

$$\boxed{Q = 0}$$

4 pts   (c) What is the efficiency of an engine using this cycle? Hint: Use the heats you just found in part (b) to identify Q_h and Q_c.

$$Q_h = 805.36\ J$$
$$Q_c = 570.55\ J$$
$$|w| = Q_h - Q_c = 234.81\ J$$

$$e = \frac{|w|}{Q_h} = \frac{234.81\ J}{805.36\ J} = \boxed{29.16\%}$$

(11 pts) Problem 6. (a) Use the ideal gas law to figure out the density of air at 400K at 1 atm. (The average molar mass of air molecules is 0.029 kg/mol.)

5 pts

$$PV = nRT$$

$$PV = \left(\frac{m}{MM}\right)RT$$

$$\frac{m}{V} = \frac{P \times MM}{RT} = \frac{(1.01 \cdot 10^5 \, Pa)(.029 \, ^{kg}/_{mol})}{(8.31 \, ^{J}/_{mol\,K})(400 \, K)}$$

$$= \boxed{.881 \, ^{kg}/_{m^3}}$$

(b) In Lab 1, you used a contraption like this to measure the density of an unknown liquid. Suppose after pressurizing the bottle you measure $h = 75$ cm and the <u>gauge</u> pressure to be 1.5 psi (<u>not</u> the absolute pressure P). What was the density of the liquid?

6 pts

$$\text{Pressure here} = P = P_0 + P_{gauge}$$

$$\text{Pressure also} = P_0 + \rho g h$$

$$\cancel{P_0} + P_{gauge} = \cancel{P_0} + \rho g h$$

$$\rho = \frac{P_{gauge}}{g h}$$

$$= \frac{1.5 \, psi \times \dfrac{1.01 \cdot 10^5 \, Pa}{14.7 \, psi}}{(9.8 \, ^{m}/_{s^2})(.75 m)}$$

$$= \boxed{1402 \, ^{kg}/_{m^3}}$$
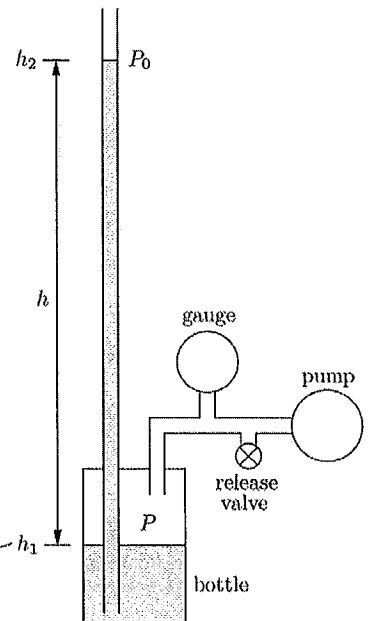
(11 pts) **Problem 8.** (a) A 3 kg block of metal is suspended from a scale and immersed in water as in the figure. The dimensions of the block are 12 cm × 10 cm × 8 cm. What will be the reading of the spring scale? (I.e. what will be the tension in the string holding up the block?)
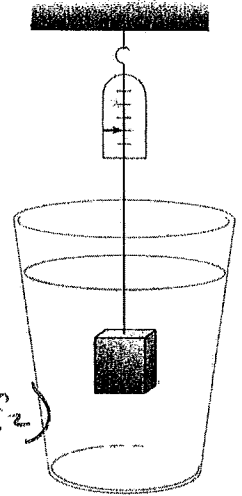
5 pts

$B \uparrow \uparrow \uparrow T$

$\downarrow mg$

$\Sigma F = 0 \longrightarrow T + B = mg$

$T = mg - B$

$T = mg - \rho V g$

$= (3 kg)(9.8 \frac{m}{s^2}) - 1000 \frac{kg}{m^3}(.12 \times .10 \times .08 m^3)(9.8 \frac{m}{s^2})$

$= \boxed{19.992 N}$

(b) What gauge pressure (not absolute pressure) must a pump generate to get a jet of water to leave its nozzle with a speed of 10 m/s at a height of 3 m above the pump? Assume that the area of the nozzle is *very small* compared to that of the pipe near the pump.

6 pts

3m
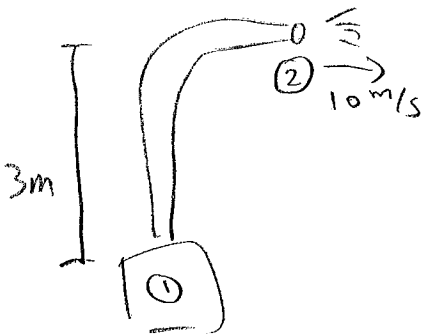
Bernoulli:

$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

$(\cancel{1 atm} + P_{gauge}) + 0 + 0 = \cancel{1 atm} + \rho g h_2 + \frac{1}{2}\rho v_2^2$

$P_{gauge} = 1000 \frac{kg}{m^3}(9.8 \frac{m}{s^2})(3m) + \frac{1}{2}(1000 \frac{kg}{m^3})(10 \frac{m}{s})^2$

$= \boxed{79400 Pa}$

(11 pts) **Problem 8** Heat is added to 4 moles of a <u>diatomic</u> ideal gas at 300K, while keeping its volume constant as shown in the diagram. This causes the temperature to increase to 900K. (a) What is the change in entropy of the gas?
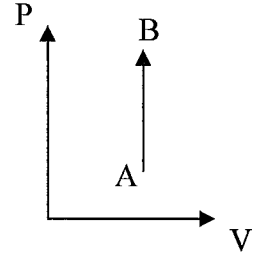
5 pts

$$\Delta S = \int \frac{dQ}{T}$$

const. volume: $Q = n C_v \Delta T$

$dQ = n C_v dT$

$$= \int \frac{n C_v \, dT}{T}$$

$$= n C_v \ln \frac{T_f}{T_0}$$

$$= \left(4 \, mol\right)\left(\frac{5}{2} \cdot 8.31 \, J/mol\,k\right) \ln 3$$

$$= \boxed{91.29 \, J/k}$$

(P-V diagram: vertical axis P, horizontal axis V, with point A at bottom and B above it, arrow pointing up from A to B)

(b) If the heat came in from a reservoir kept at 950K, what was the change in entropy of the universe? Hint 1: It's the sum of ΔS_{gas} and $\Delta S_{reservoir}$... and it had better end up positive! Hint 2: Finding $\Delta S_{reservoir}$ is a constant temperature problem.

6 pts

$$\Delta S = \int \frac{dQ}{T} \qquad const. \, temp$$

$$= \frac{1}{T} \int dQ$$

$$= \frac{Q}{T} \longrightarrow |Q| = Q_{gained \, by \, gas}$$

$|Q| = n C_v \Delta T = (4 \, mol)(\frac{5}{2} \cdot 8.31 \frac{J}{mol\,k})(600 k)$

$Q = -49860 \, J$

→ negative, since heat flows out of reservoir

$$\Delta S = \frac{-49860 \, J}{950 \, k} = -52.48 \, J/k$$

$$\Delta S_{universe} = \Delta S_{gas} + \Delta S_{reservoir}$$

$$= 91.29 \, J/k - 52.48 \, J/k \quad = \boxed{+38.81 \, J/k}$$

(4 pts; no partial credit) **Extra Credit 1.** If you flip 40 coins, what is the probability of getting exactly 12 heads?

Pascal's triangle :

ways to get 0 heads
ways to get 1 head
ways to get 2 heads

1 flip → 1 1
2 flips → 1 2 1
3 flips → 1 3 3 1
← etc →

# of ways of getting 12 heads = 12th element (starting counting at 0) of 40th row

$$= \binom{40}{12} = \frac{40!}{28! \, 12!}$$

total # possibilities = 2^{40}

$$\text{Answer} = \frac{40!}{28! \, 12! \, 2^{40}} = 5.08 \cdot 10^{-3}$$

$$\boxed{.508\%}$$

(4 pts; no partial credit) **Extra Credit 2.** Suppose an atom has only three available energy levels, which are at these energies: state 1 = 0 eV; state 2 = 0.2 eV; state 3 = 0.5 eV. (The conversion between eV and joules is found on page 1 of the exam.) If the temperature is 1000 K, what is the probability that the atom is in the lowest energy state?

$\underline{}$ .5eV = $8.01 \cdot 10^{-20}$ J

$\underline{}$ .2eV = $3.204 \cdot 10^{-20}$ J

$\underline{}$ 0 eV = 0 J

$BF = e^{-E/k_B T}$

.003014

.09810

1

Ground State Prob $= \dfrac{1}{1 + .0981 + .003014} \approx$ $\boxed{90.82\%}$