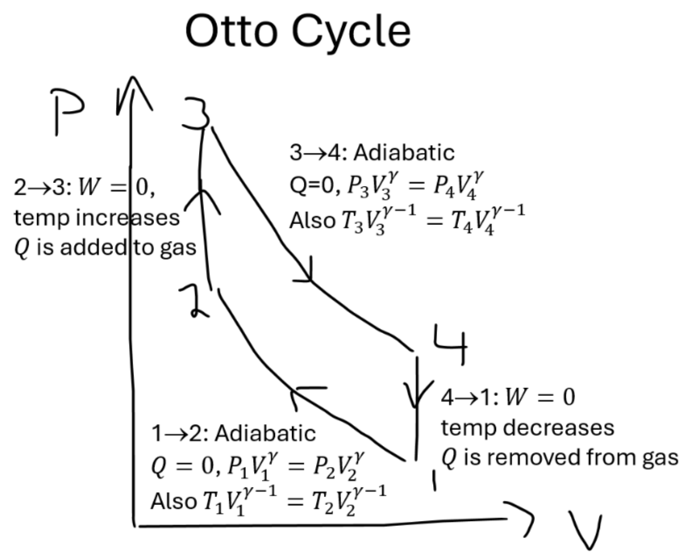
Otto Cycle Efficiency

by Dr. Colton, Physics 123 (last updated: Winter 2026)

Derivation



To calculate the cycle efficiency, use the definition:

$$e = 1 - \frac{Q_c}{Q_h}$$

Q_c occurs during 4→1, which is const volume:

$$Q_c = nC_V\Delta T = nC_V(T_4 - T_1)$$

Q_h occurs during 2→3, which is also const volume:

$$Q_h = nC_V\Delta T = nC_V(T_3 - T_2)$$

$$e = 1 - \frac{nC_V(T_4 - T_1)}{nC_V(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Multiply numerator and denominator by $1/T_1$, and the first term in denominator by T_4/T_4:

$$e = 1 - \frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_4}\frac{T_4}{T_1} - \frac{T_2}{T_1}}$$

Then note that from the T-V adiabatic equation for 1→2, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$ where r is the "compression ratio," which is the max volume divided by the min volume (e.g. $r = V_1/V_2$). Also from the T-V adiabatic equation for 3→4, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1}$.

So the equation becomes:

$$e = 1 - \frac{\frac{T_4}{T_1} - 1}{r^{\gamma-1}\frac{T_4}{T_1} - r^{\gamma-1}}$$

Factoring out $r^{\gamma-1}$ in the denominator and cancelling terms gives us the answer:

$$e = 1 - \frac{\frac{T_4}{T_1} - 1}{r^{\gamma-1}\left(\frac{T_4}{T_1} - 1\right)}$$

$$\boxed{e = 1 - \frac{1}{r^{\gamma-1}}}$$