(15 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. An extremely precise scale is used to measure an iron weight. It is found that in a room with the air sucked out, the mass of the weight is precisely 1.000000 kg. If you add the air back into the room, will the scale reading increase, decrease, or stay the same?

a. increase
b. decrease
c. stay the same

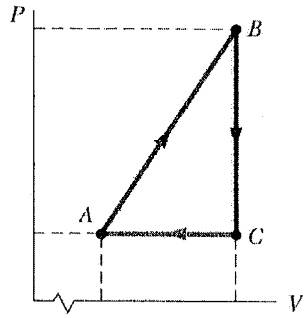
The buoyant force from the air helps support some of the weight.

1.2. Water flows from a little pipe into a big pipe with no friction or height change. No other pipes are connected. How many kg/s flow in the little pipe as compared to the big pipe? (kg/s is called the "mass flow rate")

a. more kg/s flow in the little pipe
b. more kg/s flow in the big pipe
c. the same kg/s flow in both pipes

This is simply from conservation of mass, also the foundation of the "garden hose" eqn.

1.3. A gas undergoes the cycle shown in the figure. For A to B, is $W_{on\ gas}$ positive, negative, or zero?



a. positive
b. negative
c. zero

gas expands → W_{on} = negative.

1.4. Same figure. For the complete cycle, A → B → C → A, how does the net Q compare to net $W_{by\ gas}$? (They are both positive quantities.)

a. net Q < net $W_{by\ gas}$
b. net Q > net $W_{by\ gas}$
c. net Q = net $W_{by\ gas}$

Full cycle → $\Delta U = 0$
therefore $|Q| = |W_{by}|$

1.5. If you flip 13 coins, what is the probability of getting exactly 3 heads?

a. $\frac{2^{13}}{3!\,10!\,13!}$
b. $\frac{2^{13}3!}{10!\,13!}$
c. $\frac{2^{13}10!}{3!\,13!}$
d. $\frac{3!}{10!\,13!\,2^{13}}$
e. $\frac{3!}{13!\,2^{13}}$
f. $\frac{10!}{3!\,13!\,2^{13}}$
g. $\frac{13!}{3!3!\,2^{13}}$
h. $\frac{13!}{3!\,10!\,2^{13}}$
i. $\frac{13!}{10!\,10!\,2^{10}}$
j. $\frac{13!}{10!10!\,2^{13}}$

$\frac{\#\text{microstates}}{\text{total }\#} = \frac{\binom{13}{3}}{2^{13}} = \boxed{\frac{13!}{10!\,3!\,2^{13}}}$

1.6. Consider a transverse traveling wave of the form: $y(x,t) = (2x - 10t)^4$. (You may assume that the numbers have the appropriate units associated with them to make *x*, *y*, and *t* be in standard SI units.) Is the wave moving in the +*x* or −*x* direction?

a. +*x*
b. −*x*
c. it cannot be determined

$2(x - vt)$
traveling in +x direction
$v = 5$

1.7. Same situation. What is the wave's speed?

a. 2 m/s
b. 4
c. 5
d. 8
e. 10
f. 12
g. 20
h. 40 m/s

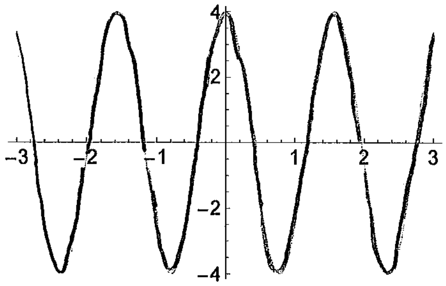
1.8. A car horn emits a 2000 Hz sound. Will you hear a higher frequency if the car approaches you at 30 mph while you are at rest, or if you approach the car at 30 mph while the car is at rest?

a. Car approaches you
b. You approach car
c. Same shift for both cases

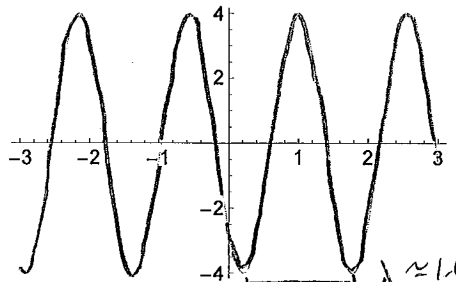
$f' = f \frac{v \pm v_o}{v \pm v_s}$. Need to compare $\frac{v + 30\text{mph}}{v}$ to $\frac{v}{v - 30\text{mph}}$

The second one is bigger. That's when $v_s = 30$

f(x, t) plotted as f(x) for t=0



f(x, t) plotted as f(x) for t=1



Shifts to the right by one $\boxed{V=1}$

$\boxed{A=4}$

$\lambda \approx 1.6 \Rightarrow \boxed{k = \frac{2\pi}{\lambda} \approx 4}$

1.9. Which wave function f(x,t) is represented by the two graphs displayed? The left-hand graph is the wave function plotted for t=0; the right hand graph is the wave function plotted for t=1.

a. f(x,t) = 2 cos(3x - 3t)
b. f(x,t) = 2 cos(3x - 5t)
c. f(x,t) = 2 cos(4x - 4t)
d. f(x,t) = 2 cos(5x - 3t)
e. f(x,t) = 2 cos(5x - 5t)
f. f(x,t) = 4 cos(3x - 3t)
g. f(x,t) = 4 cos(3x - 5t)
h. f(x,t) = 4 cos(4x - 4t) ← this is the only one that fits.
i. f(x,t) = 4 cos(5x - 3t)
j. f(x,t) = 4 cos(5x - 5t)

1.10. If a transverse pulse travels down your slinky and reflects off of the end which is being held fixed by a friend, will the reflected pulse be inverted or non-inverted?

a. inverted
b. non-inverted
c. it depends on the magnitude of the amplitude of the initial pulse

Fixed end → inverted

1.11. If you hang your slinky vertically and send a transverse pulse from the top towards the bottom, will the reflected pulse be inverted or non-inverted?

a. inverted
b. non-inverted
c. it depends on the magnitude of the amplitude of the initial pulse

Loose end → not inverted

1.12. If a single violin produces a sound level of 60 dB, how loud would you expect two violins to be? (The violins are not playing in phase with each other.)

a. 60 dB
b. 62
c. 63
d. 70
e. 80
f. 90
g. 120 dB

Two times as loud → 3 dB increase

1.13. In the "ladies belt demo" (the belt was like a "closed-closed" string), suppose the fundamental frequency is seen at 200 Hz. What frequency will have three antinodes?

a. 200 Hz
b. 230
c. 300
d. 500
e. 600
f. 1800 Hz

$f_3 = 3f_1$

1.14. Suppose a cellist tunes one of his strings to middle C. (That's not commonly done by cellists, but then again inclined planes aren't usually frictionless either, and since when has that stopped physics professors? ☺) He runs his bow across the string and a middle C sounds. The cellist then plays the same string while touching the middle of the string to force a node there. What note will the audience hear?

a. The same note: a middle C
b. One octave lower: the C below middle C
c. Two octaves lower: the C below that
d. One octave higher: the C above middle C
e. Two octaves higher: the C above that → this could be answer if cellist also forced a node 1/4 of the way. But problem didn't say that.

node in middle → second harmonic

1.15. The sound of a trumpet playing a sustained note is qualitatively different than the sound of a flute playing the same note. Why is that?

a. The two notes have different amplitudes.
b. The two notes have different durations.
c. The two notes have different fundamental frequencies.
d. The two notes have different phases.
e. The two notes have different strengths of harmonics.

Different wave shape → different strengths of harmonics.

(12 pts) **Problem 2.** Suppose you are watching sinusoidal waves travel across a swimming pool. When you look at the water right in front of you, you see it go up and down ten times in 3 seconds. At the peaks of the wave, the water is 1.0 cm below the edge of the pool. At the lowest points of the wave the water is 6.0 cm below the edge of the pool. At one particular moment in time you notice that although the water right in front of you is at its maximum height, at a distance 2 m away the water is at its minimum height. (This is the closest minimum to you.)

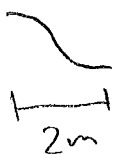
(a) What is the frequency f for this wave?

$\frac{10 \text{ times}}{3 \text{ seconds}} \rightarrow \boxed{f = \frac{10}{3} \text{ Hz}} = \boxed{3.33 \text{ Hz}}$

(b) What is ω for this wave (rad/s)?

$\omega = 2\pi f = \frac{10}{3} \cdot 2\pi = \boxed{\frac{20\pi}{3} \frac{\text{rad}}{\text{s}}} = \boxed{20.9 \frac{\text{rad}}{\text{s}}}$

(c) What is λ for this wave?



max to min = 2 m

$\rightarrow$ max to max = $\boxed{\lambda = 4 \text{ m}}$

(d) What is k for this wave (rad/m)?

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2} \frac{\text{rad}}{\text{m}}} = \boxed{1.57 \frac{\text{rad}}{\text{m}}}$

(e) What is the amplitude A of this wave?

max height = -1 cm

min height = -6 cm

peak to peak = 5 cm

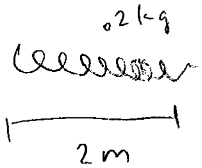
$\rightarrow$ amplitude = $\boxed{2.5 \text{ cm}}$

(f) What is the speed of water waves in this pool?

$v = \frac{\omega}{k} = \frac{20\cancel{\pi}/3}{\cancel{\pi}/2} = \boxed{\frac{40}{3} \frac{\text{m}}{\text{s}}} = \boxed{13.3 \frac{\text{m}}{\text{s}}}$

or $v = \lambda f = 4 \cdot \frac{10}{3} = \frac{40}{3} \frac{\text{m}}{\text{s}}$ same answer!

(10 pts) **Problem 3.** (a) A Slinky is stretched to 2 m long. It has a mass of 0.2 kg. A transverse wave pulse is produced by plucking one end of the Slinky. That pulse makes four round trips (down and back) along the cord in 5 seconds. What is the tension in the Slinky?



$$\mu = \frac{m}{L} = \frac{.2\ \text{kg}}{2\,\text{m}} = \underline{\underline{.1\ \frac{\text{kg}}{\text{m}}}}$$

$$v = \frac{4 \times 2 \times 2\,\text{m}}{5\,\text{sec}} = \underline{\underline{3.2\ \text{m/s}}}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2$$

$$= (.1)(3.2)^2 = \boxed{1.024\ \text{N}}$$

(b) How will the wave speed change if you stretch the Slinky out to 6 m?

$L \to$ increase by $\times 3$ $\Rightarrow$ T will increase by $\times 3$ (Spring force $F = k\Delta x$)

$\mu \to$ decrease by $\times 3$

$$\frac{v_{new}}{v_{old}} = \frac{\sqrt{\dfrac{3T_{old}}{\frac{1}{3}\mu_{old}}}}{\sqrt{\dfrac{T_{old}}{\mu_{old}}}} = \sqrt{9} = 3$$

$$\boxed{v_{new} = 3\,v_{old}}$$

(10 pts) **Problem 4.** Add these cosine functions together and give the amplitude and phase of the resulting function (it will also have a frequency of 11 rad/s):

$$f_1(t) = 3\cos(11t + 3)$$
$$f_2(t) = 2\cos(11t + 1)$$

$3 \angle 3\text{ rad} + 2 \angle 1\text{ rad}$ Just like adding vectors!

$x:\quad x_{tot} = 3\cos 3 + 2\cos 1 = \underline{-1.8894}$

$y:\quad y_{tot} = 3\sin 3 + 2\sin 1 = \underline{+2.1063}$

$\phi = 180 - \theta$

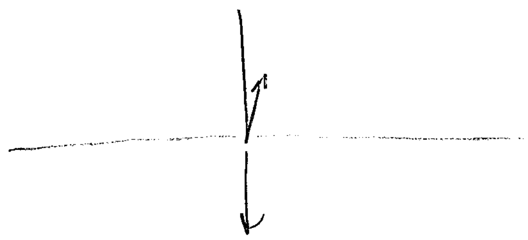
$\theta = \tan^{-1}\left(\frac{2.1063}{1.8894}\right) = 48.11°$

$\underline{\phi = 131.9°} = \underline{2.302\text{ rad}}$

$r = \sqrt{x^2 + y^2} = \sqrt{(1.8894)^2 + (2.1063)^2} = \underline{2.830}$

$$\boxed{f_{combined} = 2.83\cos(11t + 2.302)}$$

(6 pts) **Problem 5.** Suppose you have a light ray going from air into glass, with the light beam hitting the surface of the glass perpendicularly. Light travels at 3.00×10^8 m/s in the air and at 1.90×10^8 m/s in the glass. What percent of the incident light power will reflect off of the surface of the glass? (This is the same as the problem where a wave on a string reflects off an interface where the velocity abruptly changes from 3 to 1.9 m/s.)



Wave on string:

(or light ray $\perp$ to surface)

$$r = \frac{v_2 - v_1}{v_1 + v_2} = \frac{1.9 - 3}{1.9 + 3} = -.2245$$

(ratio of amplitudes)

$$R = |r|^2 = \boxed{5.04\%}$$

ratio of powers

(8 pts) **Problem 6.** In the "mesmerizing" animated gif that I created to show an example of phase and group velocities going in different directions, this is the actual equation I programmed into Mathematica:

$$f(x,t) = \cos\left(kx + k^{-2}t\right)$$

I then added up a bunch of different waves having k values equally spaced between 0.9 and 1.1, and plotted the sum at various times.

(a) What was the "dispersion relation" of this wave, ω as a function of k?

$f = \cos(kx - \omega t)$

by inspection, $\boxed{\omega = -\frac{1}{k^2}}$

(b) What was the phase velocity? (You can assume all numbers are in SI units.)

$$V_p = \left.\frac{\omega}{k}\right|_{k=k_{ave}} = \left.\frac{-1/k^2}{k}\right|_{k=1} = \left.-\frac{1}{k^3}\right|_{k=1} = \boxed{-1\ \frac{m}{s}}$$

(c) What was the group velocity?

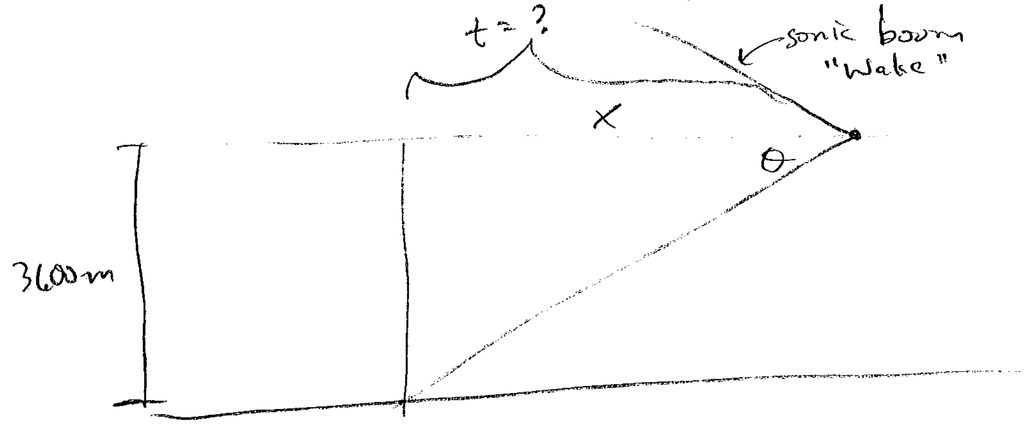
$$V_g = \left.\frac{d\omega}{dk}\right|_{k=k_{ave}} = \left.-(-2)k^{-3}\right|_{k=1} = \left.\frac{2}{k^3}\right|_{k=1} = \boxed{2\ \frac{m}{s}}$$

(d) At what speed was the peak of the wave traveling at?

envelope travels at group velocity

$\boxed{2\ m/s}$

(9 pts) **Problem 7.** A jet airplane flies with a speed of 500 m/s at a constant altitude that is 3600 m above where I'm standing. It passes directly over my head. How soon after I see it pass directly above me will I hear the sonic boom?



sonic boom: $\sin\theta = \frac{1}{\text{Mach \#}} = \frac{343}{500}$

$\theta = \underline{\underline{43.31°}}$

Find x: $\tan\theta = \frac{3600}{x} \Rightarrow x = \frac{3600}{\tan(43.31)} = \underline{\underline{3818\text{ m}}}$

Find t: $v = \frac{x}{t} \rightarrow t = \frac{x}{v}$

$= \frac{3818\text{ m}}{500\text{ m/s}} = \boxed{7.64\text{ sec}}$

(10 pts) **Problem 8.** A pipe open at both ends has a fundamental frequency of 500 Hz when the temperature is 0°C. (a) What is the length of the pipe? (b) What is the fundamental frequency at a temperature of 30°C? Assume that the displacement antinodes occur exactly at the ends of the pipe. Neglect thermal expansion of the pipe. Hint: If you don't have the equation for the speed of sound vs. temperature written down, you can derive it by remembering that the speed is inversely proportional to the square root of the density, remembering what the ideal gas law can tell you about how density relates to temperature, and noting the speed of sound at the standard temperature of 293K must come out to be 343 m/s.

First: you need this eqn $\boxed{v_{sound} = 343\frac{m}{s}\sqrt{\frac{T}{293K}}}$

if you don't have that on your card, here's how the hint says to derive it:

$v \sim \sqrt{\frac{1}{\rho}}$

$PV = nRT$

$PV = \left(\frac{m}{MM}\right)RT$

$P = \frac{\rho}{MM}RT$

$T \sim \frac{1}{\rho}$

$v \sim \sqrt{T}$

if $v = \text{Const} \times \sqrt{T}$, then you can find constant by plugging in $343 = \text{const} \times \sqrt{293}$

$\text{Const} = \frac{343}{\sqrt{293}}$

or $v = 343 \cdot \sqrt{\frac{T}{293}}$ ✓

$\longrightarrow v(T=273) = 331\frac{m}{s}$

$v(T=303) = 348\frac{m}{s}$

(a) With that knowledge, we have (from $f_1 = \frac{v}{2L}$)

$L = \frac{v}{2f_1} = \frac{331\ m/s}{2(500\ Hz)} = \boxed{.331\ m}$

(b) New temp

$f_1 = \frac{v}{2L} = \frac{348\ m/s}{2(.331\ m)} = \boxed{526.8\ Hz}$

(10 pts) **Problem 9.** A speaker at the front of a room and an identical speaker at the rear of the room are being driven at 400 Hz by the same sound source. A student walks at a very uniform rate of 1.5 m/s away from one speaker and toward the other. How many beats per second does the student hear?

Doppler: $f' = f \dfrac{v \pm v_o}{v \mp v_s}$

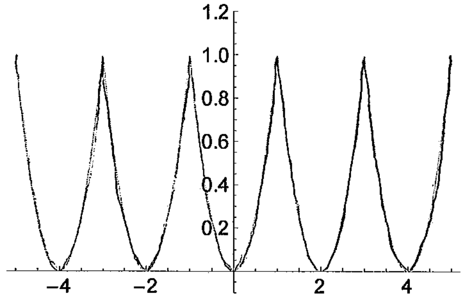
$$f_{front} = 400\left(\frac{343+1.5}{343}\right) = \underline{\underline{401.75}}$$

$$f_{back} = 400\left(\frac{343-1.5}{343}\right) = \underline{\underline{398.25}}$$

$$f_{beat} = |\Delta f| = \boxed{3.50\ \text{Hz}}$$

(10 pts) **Problem 10.** The function f(x), graphed on the right, is defined as follows:

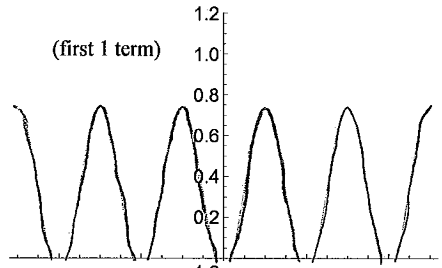
$f(x) = x^2$, for x between -1 and $+1$
(repeated with a period of $L = 2$)

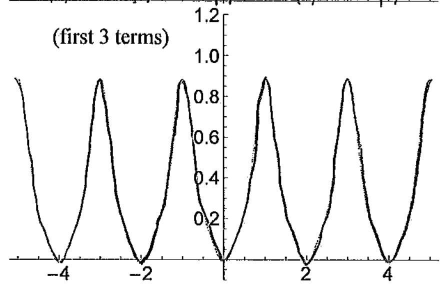


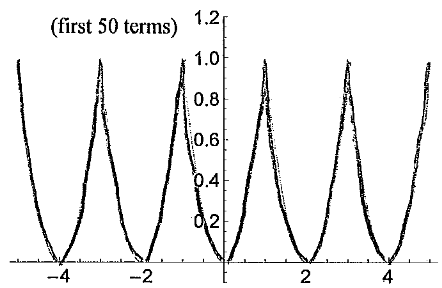
I worked out the Fourier coefficients for this function, and found the following:

$$f(x) = 0.33 - 0.41\cos(\pi x) + 0.10\cos(2\pi x) - 0.045\cos(3\pi x) + 0.025\cos(4\pi x) + \ldots$$

I've rounded all the numbers a bit. Plots of f(x) for increasing numbers of terms in the summation are shown on the right.







(a) Why are there no sin(nπx/L) terms in the series?

f(x) is even

(b) Prove that the constant term in my expression is correct.

$$a_0 = \frac{1}{L}\int f(x)\,dx$$

$$= \frac{1}{2}\underbrace{\int_{-1}^{1} x^2\,dx}_{\frac{1}{3}x^3\big|_{-1}^{1} = \frac{2}{3}}$$

$$= \frac{1}{2} \times \frac{2}{3} = \boxed{\frac{1}{3}}$$

(c) Why are all of the cosine function arguments multiples of πx?

The fundamental period is 2

therefore the fundamental frequency will be $\frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

Therefore all frequency components will be multiples of π.

(d) Set up an integral that you could, for example, plug into Mathematica to solve for the fourth cosine term of the series ($n = 4$).

$$a_n = \frac{2}{L}\int_{-1}^{1} f(x)\cos\frac{2n\pi x}{L}\,dx$$

for $n = 4$: $a_4 = \frac{2}{2}\int_{-1}^{1} x^2 \cos\frac{2(4)\pi x}{2}\,dx$

$$\boxed{a_4 = \int_{-1}^{1} x^2 \cos 4\pi x\,dx}$$

(12 pts) **Problem 11.** Suppose a string is tuned to exactly 220 Hz, the standard frequency for the A below middle C. The string is then forced to vibrate in the third harmonic (2nd overtone), which is used to tune another string (the "second string" referred to below).

(a) What frequency (in Hz) is the second string being tuned to?

3rd harmonic $= 3 \times 220\,\text{Hz} = \boxed{660\,\text{Hz}}$

(b) What musical note is the second string being tuned to? (If you are not confident with musical scales, at least put how many half-steps higher than the A this note is.)

method (a): from playing trumpet I know the overtone pattern goes $\overset{f_1}{C} - \overset{f_2}{C} - \overset{f_3}{G} - \overset{f_4}{C}$ (with f_1 depending on valves)

f_3 is therefore an octave and a fifth higher than f_1

So, an octave and a fifth higher than that A is $\boxed{\text{an E that's a little more than one octave above middle C}}$

method (b) $f_3 = 3f_1$. How many half steps is that?

Each half step is $2^{1/12}$, so Eqn is $2^{x/12} = 3$

$$\frac{x}{12} \log 2 = \log 3$$

$$x = 12 \frac{\log 3}{\log 2} = \boxed{19 \text{ half steps}}$$

19.02, if you want to be precise.

these do in fact agree!

(c) If another string (the "third string") were tuned to that same note using an equal temperament scale, what frequency would it be vibrating at?

Exactly 19 half steps would get us

$$f = 220 \cdot 2^{19/12} = \boxed{659.25\,\text{Hz}}$$

(d) How many beats would occur per second if the second string and the third string were played simultaneously?

$$f_{beat} = |\Delta f| = 660 - 659.25 = \boxed{0.75\ \frac{\text{beats}}{\text{sec}}}$$

(4 pts, no partial credit) **Problem 12.** According to the Dulong-Petit law, what should the specific heat of copper be? Note that the molar heat capacity C, in J/mol·°C, is related to the specific heat c, in J/kg°C, via the molar mass. Hint: You can compare your answer to the measured specific heat of copper, given on pg 1 of this exam.

MM = .06355 kg/mole

6 degrees of freedom

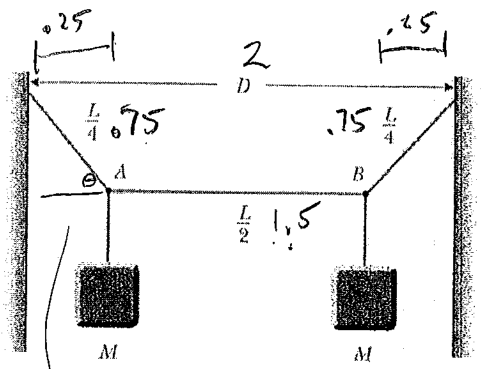
$\rightarrow U = 6 \cdot \left(\frac{kT}{2}\right) \cdot N$

$\rightarrow C = 3R$ heat cap/mole

$3 \cdot 8.31 \frac{J}{mole\,°C} \times \frac{1\ mole}{.06355\ kg} = \boxed{392.3 \frac{J}{kg\,°C}}$

Compares well with measured value of 387 $\frac{J}{kg\,°C}$ ✓

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)



(a) A light string of mass 15 g and length $L = 3$ m has its ends tied to two walls that are separated by the distance $D = 2$ m. Two objects, each of mass $M = 2$ kg, are suspended from the string as in the figure. If a wave pulse is sent from point A, how long does it take to travel to point B?

$\cos\theta = \frac{.25}{.75} \rightarrow \theta = 70.53°$

FBD for pt A

$T_1 \sin\theta$, $T_1 \cos\theta$, T_2, Mg

$\Sigma F_y = 0 \rightarrow T_1 \sin\theta = Mg$

$\Sigma F_x = 0 \rightarrow T_1 \cos\theta = T_2$

divide: $\tan\theta = \frac{Mg}{T_2}$

$T_2 = \frac{Mg}{\tan\theta}$

$T_2 = \frac{2(9.8)}{\tan(70.53°)}$

$T_2 = 6.929\,N$

in section 2: $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{6.929}{.015/3}} = 37.23\,m/s$

also $v = \frac{x}{t} \rightarrow t = \frac{x}{v} = \frac{1.5\,m}{37.23\,m/s} = \boxed{.0403\ s}$

(b) Suppose that your shower stall is rectangular: 2.4 m tall, 1.2 m wide, and 1.0 m deep. As you sing in the shower, what is the lowest frequency (in Hz) that will resonate? Do not ignore side-to-side sound waves. Hint: As discussed in the HW extra credit "Dr. Durfee milkshake problem", the fact that the waves propagate three-dimensionally means that the wavenumber k is really the sum of three components $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$, where $k_x = 2\pi/(\text{x-wavelength})$ and similarly for k_y and k_z. This results in an overall k for the fundamental mode which is larger than it would be if the side-to-side standing waves could be neglected.

Lowest mode: $\lambda = 2L$

$k_x = \frac{2\pi}{\lambda_x} = \frac{2\pi}{2(2.4)} = \frac{\pi}{2.4}$

$k_y = \frac{2\pi}{\lambda_y} = \frac{\pi}{1.2}$

$k_z = \frac{2\pi}{\lambda_z} = \frac{\pi}{1.0}$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = 4.29\ \text{rad/m}$

$v = \frac{\omega}{k} \rightarrow \omega = vk \rightarrow 2\pi f = vk \rightarrow f = \frac{vk}{2\pi}$

$f = \frac{343(4.29)}{2\pi} = \boxed{234.4\ Hz}$

Extra Credit, cont.

(c) Suppose you want to create your own animated wave image with Mathematica, illustrating a situation where the group velocity is 5 m/s and the phase velocity is 20 m/s for your desired center k value of $k = 3$ rad/m. What is the equation for the wave function you would need to use? Solving for ω as a function of k is sufficient, because then (as in problem 6) your equation can be $\cos(kx - \text{function\_of\_k} \cdot t)$, summed over a bunch of different k values.

$V_{phase} = 4\, V_{group}$

$\frac{\omega}{k} = 4 \frac{d\omega}{dk}$ (when $k = 3$)

$4\int \frac{d\omega}{\omega} = \int \frac{dk}{k}$

$4 \ln \omega = \ln k + C$

$\omega^4 = \underbrace{C}_{\text{(a different } C)} k$

$\omega = \underbrace{C}_{\text{another } C} k^{1/4}$

When $k = 3$, $\frac{\omega}{k} = 20$, $\omega = 60$

$60 = C\, 3^{1/4}$

$C = 45.59$

Answer:

$\boxed{\omega = 45.59\, k^{1/4}}$

Wave function is

$\boxed{\cos\left(kx - 45.59 k^{1/4} t\right)}$

(d) Work out the integral involved in the Fourier transform problem and find a general expression for the cosine coefficients.

I sure wouldn't want to do this by hand!

But Mathematica tells me the integral gives:

$$\boxed{a_n = \frac{4 n\pi \cos n\pi + 2\left(-2 + n^2\pi^2\right)\sin n\pi}{n^3\pi^3}}$$

If you notice $\sin(n\pi) = 0$

and $\cos(n\pi) = (-1)^n$, then you can further simplify:

$$\boxed{a_n = \frac{4(-1)^n}{n^2\pi^2}}$$