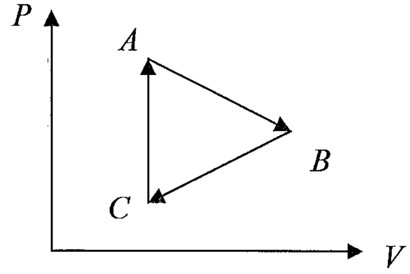
Phys 123 Section 2 Winter 2011 Exam 3 Solutions

(15 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. A gas undergoes the cycle shown in the figure. For A to B, is $W_{on\ gas}$ positive, negative, or zero?
gas expands → $W_{by\ gas}$ = positive and $W_{on\ gas}$ = negative
- a. positive
- b. negative
- c. zero



1.2. Same figure. For the complete cycle, A → B → C → A, how does the net Q compare to net $W_{by\ gas}$? (They are both positive quantities.)
- a. net Q < net $W_{by\ gas}$
- b. net Q > net $W_{by\ gas}$
- c. net Q = net $W_{by\ gas}$

$\Delta E = Q + W_{on}$
= 0 for cycle, so $|Q| = |W_{on}| = W_{by}$

1.3. Consider a transverse traveling wave of the form: $y(x,t) = (3x - 15t)^2$. (You may assume that the numbers have the appropriate units associated with them to make x, y, and t be in standard SI units.) What is the wave's wavelength?
- a. 2π/3 m
- b. 2π/5
- c. 3π/2
- d. 3π
- e. 3/π
- f. 5π/2
- g. 5π
- h. 5/π
- i. 6π
- j. 10π m

$k = 3$

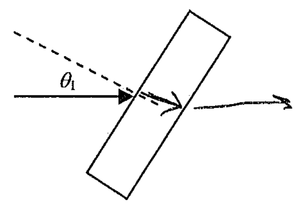
$\lambda = \frac{2\pi}{k} = \frac{2\pi}{3}$

1.4. Same situation. What is the wave's speed? $\omega = 15$
- a. 3 m/s
- b. 5
- c. 6
- d. 15
- e. 30
- f. 45 m/s

$v = \frac{\omega}{k} = \frac{15}{3} = 5$

1.5. A light ray enters a piece of glass from the air at an angle θ_1, measured from the perpendicular. The two sides of the glass piece are parallel to each other (see figure). The ray exits the glass at an angle θ_2, also measured from the perpendicular (not shown). Which of the following is true?
- a. $\theta_1 < \theta_2$
- b. $\theta_1 = \theta_2$
- c. $\theta_1 > \theta_2$

done in a HW assignment, path will be as shown



1.6. Which would allow more light through? (The light is initially polarized horizontally. Assume perfect polarizers.)
- a. A single polarizer set at 20° above the horizontal allows $\cos^2(20°) = 88.3\%$
- b. Two polarizers: one at 10° above the horizontal and one at 20° above the horizontal
- c. Same

allows $\cos^2(10°) \times \cos^2(10°) = 94.1\%$

1.7. Jane and John are trying to look at each other. Jane is in a lake, under the water; John is on the shore. If Jane can see John, can he necessarily see her?
- a. Yes
- b. No

Light ray path works both ways

1.8. Same situation. If John can see Jane, can she necessarily see him?
- a. Yes
- b. No

Same!

1.9. In transparent glass, which travels faster: red light (λ=630 nm) or green light (λ=500 nm)?
- a. red light
- b. green light
- c. both travel at the same speed

lower λ → higher E → more interaction → slower travel
Large λ travels faster!

1.10. Should polarized sunglasses be oriented to block vertically or to block horizontally-polarized light?
- a. block vertically-polarized light
- b. block horizontally-polarized light

At Brewster's angle, only horizontally polarized light reflects.
Near Brewster's angle, light will still be mostly horizontal polarization

1.11. What kind of lenses do near-sighted people need to correct their vision?

a. converging

b. diverging

→ far point = 1 m (for example)
object at p = ∞ needs to be brought to q = −1
$f = (\frac{1}{p} + \frac{1}{q})^{-1} \rightarrow f$ = negative

1.12. An astronomer is using a telescope to look at Mars, which to the unassisted eye has an angular diameter of 1×10^{-5} rad. Her telescope has an objective focal length of 900 mm and an eyepiece focal length of 10 mm. How large will Mars look through the telescope?

a. Less than 0.5×10^{-5} rad

b. $0.5 - 2 \times 10^{-5}$

c. $2 - 5 \times 10^{-5}$

d. $5 - 20 \times 10^{-5}$

e. $20 - 50 \times 10^{-5}$

f. $50 - 200 \times 10^{-5}$

g. More than 200×10^{-5} rad

ang. mag $m = \frac{f_o}{f_e} = \frac{900}{10} = 90.$

$\theta_{new} = m \times \theta = 90 \times 10^{-5}$ rad

1.13. A coating ($n = 1.7$) is put on a glass lens ($n = 1.5$) to reduce reflections when the wavelength is $\lambda = 800$ nm (this is the wavelength of the light in a vacuum). Which of these equations would properly allow you to determine how thick the coating needs to be? The symbol t represents the thickness of the coating.

a. $2t = m\lambda$

b. $2t = (m + ½)\,\lambda$

c. $2t = m\lambda n_{coating}$

d. $2t = (m + ½)\,\lambda n_{coating}$

e. $2t = m\lambda/n_{coating}$

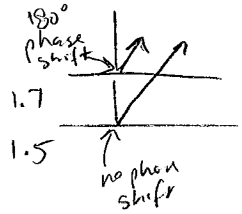
f. $2t = (m + ½)\,\lambda/n_{coating}$

g. $2t = m\lambda n_{glass}$

h. $2t = (m + ½)\,\lambda n_{glass}$

i. $2t = m\lambda/n_{glass}$

j. $2t = (m + ½)\,\lambda/n_{glass}$



reflection has minimum when
$2t = (m+\frac{1}{2})\lambda$ − phase shift
$2t = m \times \lambda_{coating}$

$2t = m \frac{\lambda_{vacuum}}{n_{coating}}$

1.14. The electric field in a light wave is described by the following equation: $\vec{E} = A\hat{z}\cos\left[k\left(\frac{2x-y}{\sqrt{5}}\right) - \omega t\right]$. In what direction is it traveling? (The vectors in the answer choices are all unit vectors.)

a. $\hat{z}$

b. $-\hat{z}$

c. $\frac{2\hat{x}-\hat{y}}{\sqrt{5}}$

d. $\frac{-2\hat{x}+\hat{y}}{\sqrt{5}}$

e. $\frac{\hat{x}+2\hat{y}}{\sqrt{5}}$

f. $\frac{-\hat{x}-2\hat{y}}{\sqrt{5}}$

$\vec{k}$ must be in $\frac{2\hat{x}-\hat{y}}{\sqrt{5}}$ direction

1.15. Same equation. In what direction is the electric field oscillating?

a. From $\hat{z}$ to $-\hat{z}$

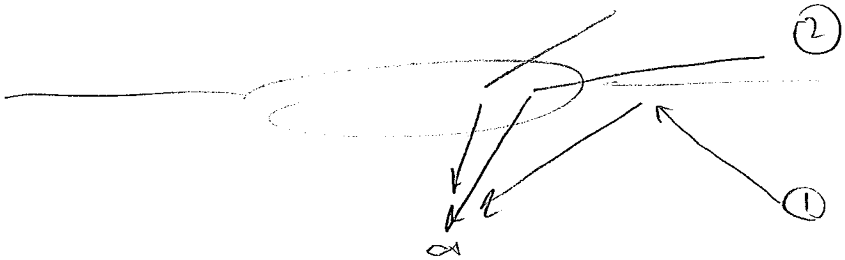
b. From $\frac{2\hat{x}-\hat{y}}{\sqrt{5}}$ to $\frac{-2\hat{x}+\hat{y}}{\sqrt{5}}$

c. From $\frac{\hat{x}+2\hat{y}}{\sqrt{5}}$ to $\frac{-\hat{x}-2\hat{y}}{\sqrt{5}}$

$\vec{E}$ itself is in $\hat{z}$ direction

(9 pts) **Problem 2.**

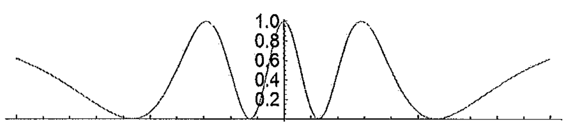
(a) One of the homework problems talked about a fish seeing light enter the water through a "bright circle", rather than through the entire surface of the lake. Why is that what the fish sees?

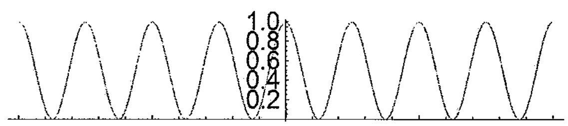


Light rays coming from ① are a result of T.I.R. and will be dark (coming from underwater)

Light rays coming from ② are coming from sky and will be bright

(b) In the lecture that Dr. Peatross taught, he pointed out that a two slit diffraction situation could potentially result in either of the following two patterns. What is the difference? When would you get one instead of the other?





get this if screen is close

get this if screen is far (can use $\sin\theta \approx \theta$)

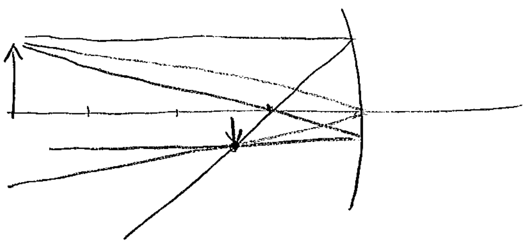
(c) If I try to use a lens to produce an image of a point source of light, the image will unfortunately not be exactly a point. Why is that, and what will the image look like?

Diffraction through the circular aperture of the lens will cause a point of light to turn into  type of pattern.

has rings around a central bright spot.

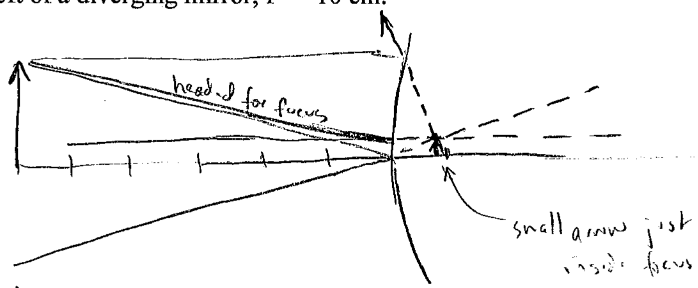
(12 pts) **Problem 3.** Draw accurate ray diagrams for the following situations to indicate where each image will be formed, how large it will be, and whether it will be real or virtual. Use at least three rays for each diagram. Use dashed lines for virtual rays, if present. No equations are necessary, but you are of course free to use equations to double check your final image position if you wish.

(a) An object 40 cm to the left of a converging mirror, f = +10 cm.



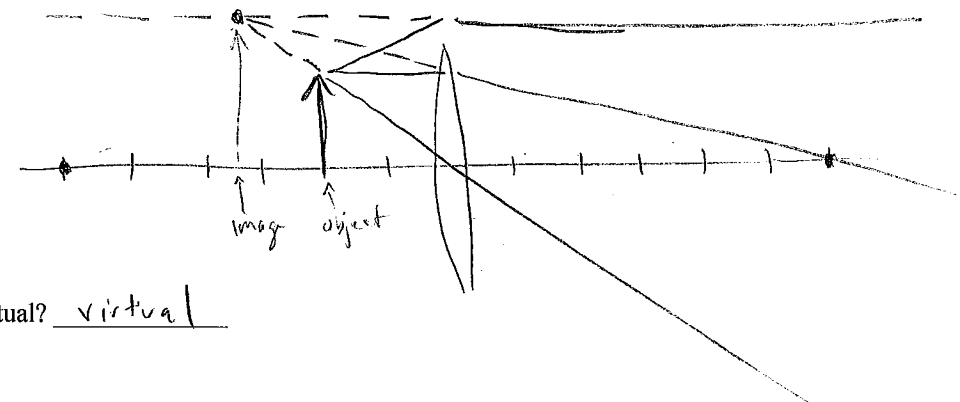
Real or virtual? real

(b) An object 60 cm to the left of a diverging mirror, f = –10 cm.



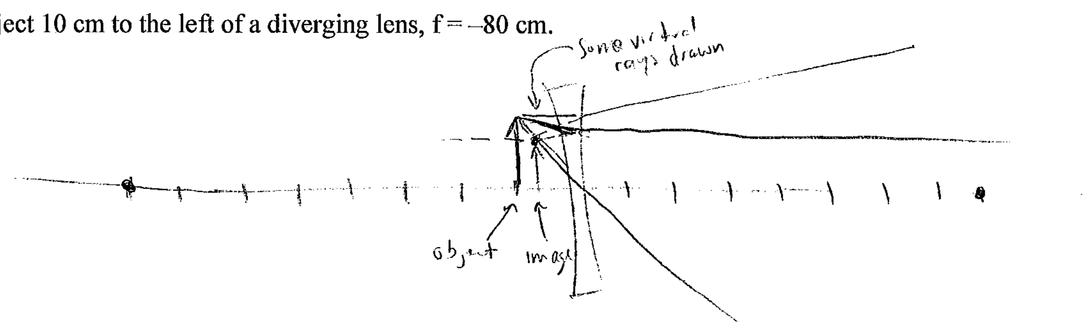
Real or virtual? virtual

(c) An object 20 cm to the left of a converging lens, f = +60 cm.



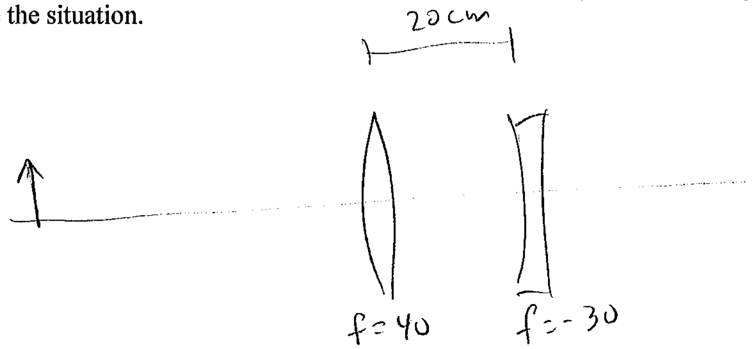
Real or virtual? virtual

(d) An object 10 cm to the left of a diverging lens, f = –80 cm.



Real or virtual? virtual

(10 pts) **Problem 4.** An object is placed 50 cm to the left of lens 1 (converging, f = +40 cm). Lens 1 is placed 20 cm to the left of lens 2 (diverging, f = –30 cm). Where will the final image be formed? Will it be real or virtual? What will the magnification be? You do not have to provide ray diagrams for this problem, although you are certainly welcome to draw them if that will help you visualize the situation.



Lens 1: $q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{40} - \frac{1}{50}\right)^{-1} = 200 \text{ cm}$

$M_1 = -\frac{q}{p} = -\frac{200}{50} = -4$

Lens 2: $p = -180$ (180 cm to the right of lens 2)

$q = \left(\frac{1}{-30} - \frac{1}{-180}\right)^{-1} = \boxed{-36 \text{ cm}}$

$M_2 = -\frac{q}{p} = -\frac{(-36)}{(-180)} = -.2$

$M_{tot} = M_1 \times M_2$

$= (-4)(-.2)$

$= \boxed{+.8}$

q = -36 cm relative to lens 2 (use a negative sign if to the left)

real vs. virtual: virtual (on "wrong side" of lens 2)

M_{tot} = +.8

(8 pts) **Problem 5.**

(a) A particular lens has a focal length of 20 cm. It is "bi-convex", meaning that it is curved outward on both sides. Each side of the lens has the same radius of curvature: 30 cm (in magnitude). What is the index of refraction of the glass from which the lens is made?

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = (n-1)\underbrace{\left(\frac{1}{30} - \frac{1}{-30}\right)}_{\frac{1}{15}}$$

$$n - 1 = \frac{15}{20}$$

$$n = 1 + \frac{15}{20} = \boxed{1.75}$$

(b) Suppose an unnamed physics professor foolishly decides to replace this lens with a lens made out of ice, $n = 1.31$, also bi-convex. (For some reason, it seemed like a good idea at the time...) What would the radius of curvature of each side of the ice lens need to be in order to have the same focal length as the glass lens that it is replacing?

$$\frac{1}{f} = (n-1)\left(\frac{2}{R}\right)$$

$$R = (n-1)(2)(f)$$

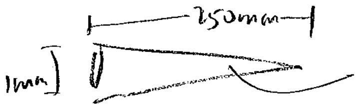
$$= (0.31)(2)(20)$$

$$= \boxed{12.4 \text{ cm}}$$

small R means it → must be curved much more than the glass lens!

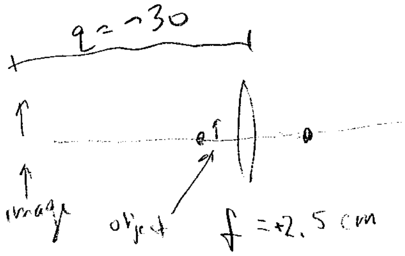
(10 pts) **Problem 6.** In order to better view an ant, height $h = 1$ mm, you use a magnifying glass with f = +2.5 cm. You place the magnifying glass close to your eye, then you adjust both the magnifying glass and your head until you have a clear view of the image of the ant, which has formed 30 cm behind the lens. What angular magnification have you obtained via the magnifying glass? You may assume that the ant was initially viewed at the standard 25 cm near point.

initially



$\theta_0 = \frac{1}{250}$ radians

with lens



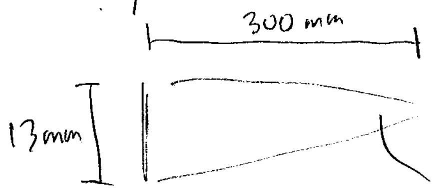
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \longrightarrow p = \left(\frac{1}{f} - \frac{1}{q}\right)^{-1}$$

$$p = \left(\frac{1}{2.5} - \frac{1}{-30}\right)^{-1} = 2.308 \text{ cm}$$

$$M = -\frac{q}{p} = \frac{-(-30)}{2.308} = 13$$

↳ height of ant image ≈ 13 mm

New angle set by this picture:



$\theta = \frac{13}{300}$ radians

$$m = \frac{13/300}{1/250} = \boxed{10.83}$$

(11 pts) **Problem 7.** Two very narrow slits separated by 0.5 mm are illuminated with 633 nm light. The peak intensity on a screen 4 m away is 0.1 W/cm². What is the intensity at a distance 3 mm from the center of the central peak?

$\theta \approx \tan\theta \approx \sin\theta = \frac{3}{4000}$ rad

If you know the equation, then:

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

$$I(y = 3\,\text{mm}) = .1\,\frac{W}{cm^2}\cos^2\left(\frac{\pi \cdot .5 \cdot 10^{-3}\left(\frac{3}{4000}\right)}{633 \cdot 10^{-9}}\right)$$

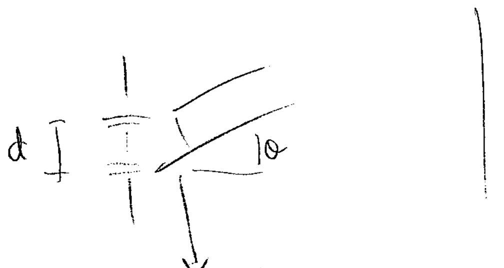
in radians!

1.8611 rad = 106.6°

$$= \left(.1\,\frac{W}{cm^2}\right)(.08195)$$

$$= \boxed{8.195\,\frac{mW}{cm^2}}$$

If you don't know the equation, then to derive it you do this:



$\Delta PL = d\sin\theta$

$\phi = \left(\frac{\Delta PL}{\lambda}\right) \cdot 2\pi = \frac{2\pi d \sin\theta}{\lambda}$

Relative to middle, phase of each slit is $e^{i\phi/2}$ and $e^{-i\phi/2}$

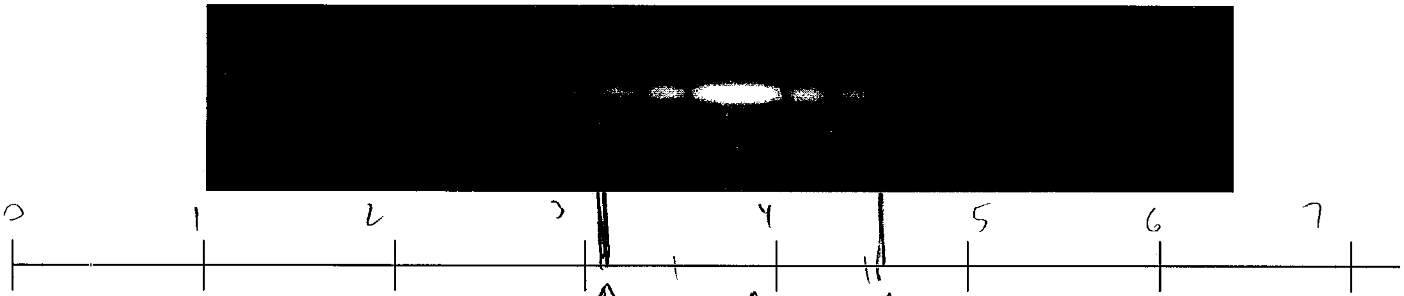
Then $E = E_0\big(\underbrace{e^{i\phi/2} + e^{-i\phi/2}}_{2\cos\phi/2}\big)$

$I \sim E^2$

$I = I_0 \cos^2 \phi/2$

$$\boxed{I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

(11 pts) **Problem 8.** Suppose you want to measure the width of one of your hairs. You use a high quality green laser pointer ($\lambda = 532$ nm) to create a diffraction pattern from the hair on a screen that is 1.2 m away from you. This is the actual pattern you see, drawn to scale. What is the diameter of the hair? (I have provided a ruler of sorts below, with tick marks in inches. Be as accurate as you can given the limitations of the ruler.)



$m=-3$
at $\approx 3.1''$

$m=+3$
at $\simeq 4.6''$

center therefore
at $\simeq 3.85''$

distance to $m=+3$
$= .75''$
$= 1.905 \cdot 10^{-2}$ m

y = small,
so $\sin\theta \simeq \theta \simeq \tan\theta$

$m\lambda = a \sin\theta$ for dark spots

$$(3)(532 \cdot 10^{-9}) = a \left(\frac{1.905 \cdot 10^{-2}\text{ m}}{1.2\text{ m}} \right)$$

$$a = 1.005 \cdot 10^{-4}\text{ m}$$

$$\boxed{100.5\ \mu\text{m}}$$

(10 pts) **Problem 9.**

(a) In one of the spectrometers in my lab I have a diffraction grating with 1200 lines/mm. It is about 3.5 inch × 3.5 inch square. If I'm using the spectrometer to study fluorescence being emitted by a material with a wavelength around 820 nm, what separation between wavelengths will I be able to resolve? (The spectrometer is designed to use the first order diffracted beam.)

Resolution $= \frac{\lambda}{\Delta\lambda}$ also $= N \cdot m$, $m = 1$

N → # slits illuminated

$= 3.5 \text{ inch} \times \frac{25.4 \text{ mm}}{1 \text{ inch}} \times \frac{1200 \text{ lines}}{1 \text{ mm}}$

$= 106{,}680$

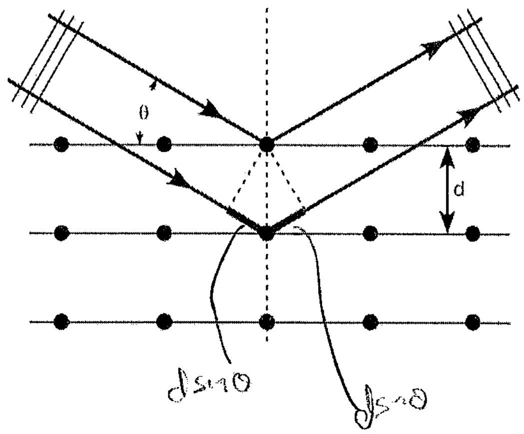
$$\Delta\lambda = \frac{\lambda}{R} = \frac{\lambda}{Nm}$$

$$= \frac{820 \text{ nm}}{(106680)(1)}$$

$$= \boxed{.00769 \text{ nm}}$$

This is, in fact, basically right on the rated resolution of the spectrometer

(b) A beam of x-rays ($\lambda = 0.3$ nm) strikes a crystal and diffractions as shown. (The circles represent atoms inside the crystal.) The x-rays reflect off of the planes of atoms connected with horizontal lines, as shown. If the incident and reflected beams are both at an angle of 35° with respect to the horizontal, what is the spacing d between the planes of atoms that are shown? (This is the $m = 1$ diffraction order.)



$\Delta PL = 2d\sin\theta$

For constructive interference

$OPL = m\lambda$ (and $m = 1$)

$2d\sin\theta = \lambda$

$$d = \frac{\lambda}{2\sin\theta} = \frac{.3 \text{ nm}}{2\sin 35°} = \boxed{.262 \text{ nm}}$$

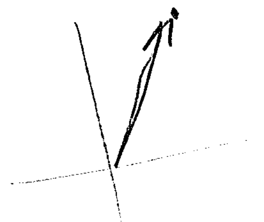
(4 pts, no partial credit) **Problem 10.** Add these cosine functions together and give the amplitude and phase of the resulting function (it will also have a frequency of 5 rad/s):

$$f_1(t) = 4\cos(5t+6)$$
$$f_2(t) = 7\cos(5t+8)$$

$4\angle 6\,\text{rad} + 7\angle 8\,\text{rad} = ?$

x components $4\cos 6 + 7\cos 8 = 2.8222$

y components $4\sin 6 + 7\sin 8 = 5.8078$



$r = \sqrt{2.8222^2 + 5.8078^2} = \underline{6.457}$

$\theta = \tan^{-1}\left(\frac{5.8078}{2.8222}\right) = \underline{64.08^\circ}$

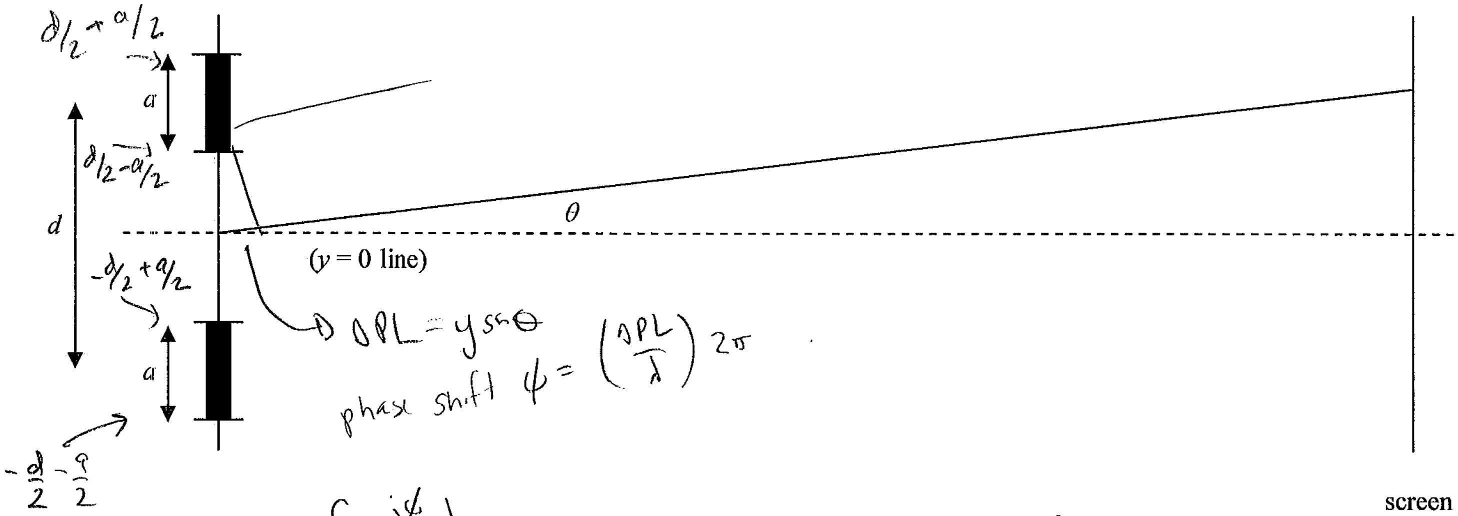
$= \underline{1.118\text{ rad}}$

$f_1 + f_2 = \boxed{6.457\cos(5t + 1.118)}$

amplitude

phase

(5 pts, no partial credit) **Extra Credit.** In class I told you that the diffraction pattern (intensity vs angle θ) of two finite width slits is the same as the diffraction pattern of one finite width slit, times that of two infinitely narrow slits. Derive that result mathematically by adding up the relevant phases (using $y = 0$ as the reference phase) and computing the diffraction pattern of the following situation: two slits each of width a, with center-to-center separation of d. (The dark areas are the slit openings.)



$$E \sim \int e^{i\phi}\, dy$$

$$= \int_{-d/2-a/2}^{-d/2+a/2} e^{i y \sin\theta\, 2\pi/\lambda}\, dy + \int_{d/2-a/2}^{d/2+a/2} e^{i y \sin\theta\, 2\pi/\lambda}\, dy$$

$$= \left.\frac{e^{i y \sin\theta\, 2\pi/\lambda}}{\sin\theta\, 2\pi/\lambda}\right|_{y=-d/2-a/2}^{-d/2+a/2} + \left.\frac{e^{i y \sin\theta\, 2\pi/\lambda}}{\sin\theta\, 2\pi/\lambda}\right|_{y=d/2-a/2}^{d/2+a/2}$$

$$= \frac{1}{\frac{2\pi \sin\theta}{\lambda}} \left[e^{i(-d/2+a/2)\sin\theta\, 2\pi/\lambda} - e^{i(-d/2-a/2)\sin\theta\, 2\pi/\lambda} + e^{i(d/2+a/2)\sin\theta\, 2\pi/\lambda} - e^{i(d/2-a/2)\sin\theta\, 2\pi/\lambda} \right]$$

these combine

these combine

$$= \frac{2i}{\frac{2\pi \sin\theta}{\lambda}} \left[\sin\left(\left(\frac{d}{2} + \frac{a}{2}\right)\sin\theta \frac{2\pi}{\lambda}\right) - \sin\left(\left(\frac{d}{2} - \frac{a}{2}\right)\sin\theta \frac{2\pi}{\lambda}\right) \right]$$

we have $\sin(a+b) - \sin(a-b) = \cancel{\sin a \cos b} + \cos a \sin b - (\cancel{\sin a \cos b} - \cos a \sin b)$
$= 2 \cos a \sin b$

$$= \frac{\frac{a}{2} 4i}{\frac{a}{2}\frac{2\pi \sin\theta}{\lambda}} \cos\left(\frac{d}{2}\sin\theta \frac{2\pi}{\lambda}\right) \sin\left(\frac{a}{2}\sin\theta \frac{2\pi}{\lambda}\right)$$

becomes "sinc"

$I \sim E^2$ so...

$$\boxed{I = I_0 \cos\left(\frac{\pi d}{\lambda}\sin\theta\right) \mathrm{sinc}\left(\frac{\pi a}{\lambda}\sin\theta\right)}$$

This is exactly the single wide slit (width a) formula times the two infinitely narrow slit (separation d) formula!