Math Review – Dr. Colton, Winter 2011

1. Calculate the derivative of the following functions:

   a. $y = \sqrt{x}$

   $y = x^{1/2}$

   $\boxed{\frac{dy}{dx} = \frac{1}{2} x^{-1/2}} = \frac{1}{2\sqrt{x}}$

   b. $y = \sqrt{x}\sqrt{3-x}$

   $y = x^{1/2}(3-x)^{1/2}$

   $\frac{dy}{dx} = \left(\frac{1}{2}x^{-\frac{1}{2}}\right)(3-x)^{1/2} + \left(x^{1/2}\right)\left(\frac{1}{2}(3-x)^{-1/2}(-1)\right)$ (Product Rule)

   $= \boxed{\frac{1}{2}x^{-1/2}(3-x)^{1/2} - \frac{1}{2}x^{1/2}(3-x)^{-1/2}}$

   c. $y = \sin(3x)$

   $\frac{dy}{dx} = \cos(3x)\ (3)$

   $= \boxed{3\cos 3x}$

   d. $y = e^{-5x}\sin(3x)$

   $\frac{dy}{dx} = \left(-5e^{-5x}\right)(\sin 3x) + \left(e^{-5x}\right)(3\cos 3x)$ (Product Rule)

   $= \boxed{-5e^{-5x}\sin 3x + 3e^{-5x}\cos 3x}$

   e. $y = \dfrac{1}{(3-2x)^4+1}$ $= \left[(3-2x)^4+1\right]^{-1}$

   $\frac{dy}{dx} = -1\left[(3-2x)^4+1\right]^{-2}\left[4(3-2x)^3\right](-2)$

   $= \boxed{\dfrac{8(3-2x)^3}{\left[(3-2x)^4+1\right]^2}}$

2. Calculate the integrals of the following functions, with x going from 1 to 2:

   a. $y = 3x^2$

$$\int_1^2 3x^2\,dx = x^3\Big|_1^2$$

$$= 2^3 - 1^3$$

$$= \boxed{7}$$

   b. $y = \frac{1}{x}$

$$\int_1^2 \frac{dx}{x} = \ln x\Big|_1^2$$

$$= \ln 2 - \ln 1$$

$$= \boxed{\ln 2} \quad (\text{since } \ln 1 = 0)$$

   c. $y = \sin(5x)$

$$\int_1^2 \sin(5x)\,dx = \frac{-\cos(5x)}{5}\Big|_1^2$$

$$= -\frac{1}{5}\left[\cos(10) - \cos(5)\right)$$

$$= \boxed{\frac{1}{5}(\cos 5 - \cos 10)}$$

these numbers are in radians.

3. Perform the following matrix multiplication:

   a. $\begin{pmatrix} 3 & 5 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + 5y \\ 2x - 5y \end{pmatrix}$

Note: although I will use 2x2 matrices to simplify some of the relativity equations, if you've never seen them before don't be too worried. You will be able to do all of the same calculations with the non-matrix equations given in the book.

Dr. Colton