Phys123 Exam 1 Solutions

Instructions:

- Record your answers to the multiple choice questions ("Problem 1") on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly. Draw a picture if possible. Be clear about what equations you are using, and why. Prove that you understand what is going on in the problem. It's generally a good idea to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Double-check your calculator work. Think about whether your answer makes sense; if not, go over your work again or try working the problem a different way to double-check things.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(36 pts) **Problem 1**: Multiple choice questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper for your own reference.

1.1. A ballet dancer (mass *m*) stands on her toes during a performance with area *A* in contact with the floor. What is the pressure exerted by the floor over the area of contact if the dancer is jumping upwards with an acceleration of *a*?

a. *mg*
b. *m*(*g*+*a*)
c. *m*(*g*–*a*)
d. *mg*/*A*
e. *m*(*g*+*a*)/*A*
f. *m*(*g*–*a*)/*A*

$\uparrow N$, $\downarrow mg$

$\Sigma \vec{F} = m\vec{a}$

$N - mg = ma$

$N = mg + ma$

$= m(g+a)$

$P = \frac{N}{\text{area}}$

$\boxed{P = \frac{m(g+a)}{A}}$

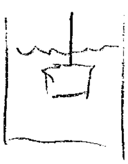
1.2. A plastic cube and a metal cube of the same size and shape are put into water. The plastic cube floats; the metal cube sinks. On which cube is the buoyant force the largest?

a. plastic
b. metal
c. same buoyant force

has more displaced volume

1.3. A metal block is suspended in an empty tank from a scale indicating a weight of *W*. The tank is then filled with water until the block is covered. If the density of the metal is three times the density of the water, what apparent weight of the block will the scale read?

a. 1/2 *W*
b. 2/3 *W*
c. *W*
d. 3/2 *W*
e. 3 *W*



$\uparrow T \uparrow B$

$\downarrow W = mg$

$B = \rho_{fl} \cdot g \cdot V_{obj} = \left(\frac{1}{3}\rho_{metal}\right) g\, V_{obj}$

$= \frac{1}{3} \cdot g \cdot m_{metal}$

$T = W - B = \frac{2}{3} m_{metal}\, g$

1.4. Water (no viscosity, incompressible) flows from a little pipe into a big pipe while also increasing in height. That is, the water is flowing uphill. The flow speed (m/s) in the little pipe will be ______ in the big pipe.

a. greater than
b. the same as
c. less than
d. cannot be determined

$A_1 v_1 = A_2 v_2 \longrightarrow v$ will decrease if A increases

So little pipe is faster

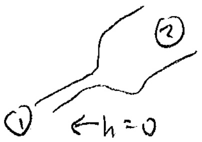
1.5. Same situation. The volume flow rate (m^3/s) in the little pipe will be ______ in the big pipe.

a. greater than
b. the same as
c. less than
d. cannot be determined

$VFR = A \cdot v$ = constant if the water is not compressible

1.6. Same situation. The pressure of the water in the small tube will be _________ the pressure in the large tube.

a. greater than
b. less than
c. equal to
d. cannot be determined



$P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

$P_1 = P_2 + \rho g h_2 - \frac{1}{2}\rho(v_1^2 - v_2^2)$

height change tends to increase P_1

speed change tends to decrease P_1

Can't say which effect "wins".

1.7. As an airplane flies horizontally at a constant elevation, the pressure above a wing is ___________ the pressure below the wing.

a. larger than
b. smaller than
c. the same as

That's why a plane gets "sucked up" into the air.

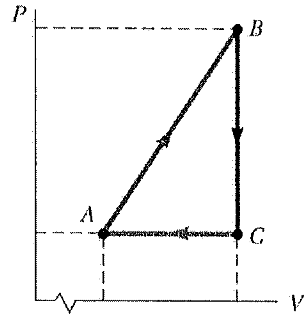
1.8. Gas A is composed of molecules which are twice as massive as the molecules in Gas B. Gas A is also at twice the temperature (kelvin) as gas B. Which is true about the rms speed of molecules in gas A compared to gas B?

a. $v_{rms,A} = \frac{1}{4} v_{rms,B}$
b. $v_{rms,A} = \frac{1}{2} v_{rms,B}$
c. $v_{rms,A} = \frac{1}{\sqrt{2}} v_{rms,B}$
d. $v_{rms,A} = v_{rms,B}$
e. $v_{rms,A} = \sqrt{2}\, v_{rms,B}$
f. $v_{rms,A} = 2 v_{rms,B}$
g. $v_{rms,A} = 4 v_{rms,B}$

$\frac{1}{2} m v^2 = \frac{3}{2} k_B T \rightarrow v = \sqrt{\frac{3 k_B T}{m}}$

if T and m both double, v stays the same.

1.9. For the next four problems, consider the cyclic process described by the figure. For A to B: does the internal energy increase, decrease, or stay the same?



a. Increase
b. Decrease
c. Stays the same ($\Delta E_{int} = 0$)
d. Cannot be determined

B is "up hill" from A, so $T_B > T_A$ and hence $E_{int\,B} > E_{int\,A}$

1.10. For B to C: is heat added or taken away from the gas?

a. Added
b. Taken away
c. Neither ($Q_{added} = 0$)
d. Cannot be determined

$\Delta E_{int} = Q + W_{on}$
$W_{on} \rightarrow 0$ because const vol.
$Q = \Delta E_{int}$ = negative because $T_C < T_B$

1.11. For C to A: is $W_{on\ gas}$ positive, negative, or zero?

a. Positive
b. Negative
c. Zero
d. Cannot be determined

Volume decreases, so W_{on} = positive

1.12. In which of the three changes is there the largest positive change in entropy?

a. A→B
b. B→C
c. C→A

only A→B has positive change in entropy, since Q is negative for both B→C and C→A

1.13. One mole of *atoms* exist at 300K. Suppose they could either form a monatomic gas, a diatomic gas, or a solid. In which case would they likely have the highest internal energy?

a. Monatomic gas
b. Diatomic gas
c. Solid
d. Same for all three

dof = 3 — $E_{int} = \frac{3}{2} nRT$ ↑ 1 mole
dof = 5 — $\frac{5}{2} nRT$ ↑ $\frac{1}{2}$ mole
dof = 6 — $\frac{6}{2} nRT$ ↑ 1 mole Largest!

1.14. If one mole of an ideal gas doubles its volume and undergoes an isothermal expansion, its pressure is:

a. quadrupled
b. doubled
c. unchanged
d. halved
e. quartered

$PV = nRT \rightarrow$ if T is constant, $P_1 V_1 = P_2 V_2$

if V is doubled, P must be halved

1.15. Two gases undergo an adiabatic compression where the volume decreases to 50% of the original amount. They each have one mole of molecules, but gas A is monatomic whereas gas B is diatomic. Which gas will end up at the higher temperature?

a. Gas A
b. Gas B
c. Same temperature

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$
$\left(\frac{nRT_1}{V_1}\right) V_1^{\gamma} = \left(\frac{nRT_2}{V_2}\right) V_2^{\gamma}$
$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$
$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

→ Which is bigger, $2^{5/3-1}$ or $2^{7/5-1}$?
↑ larger!

1.16. A gas in contact with a thermal reservoir undergoes an isothermal compression. The gas and the thermal reservoir are isolated from the rest of the universe. Which of the following is true?

heat flows out of gas

ΔS_{gas} = negative

and into reservoir

$\Delta S_{reservoir}$ = positive

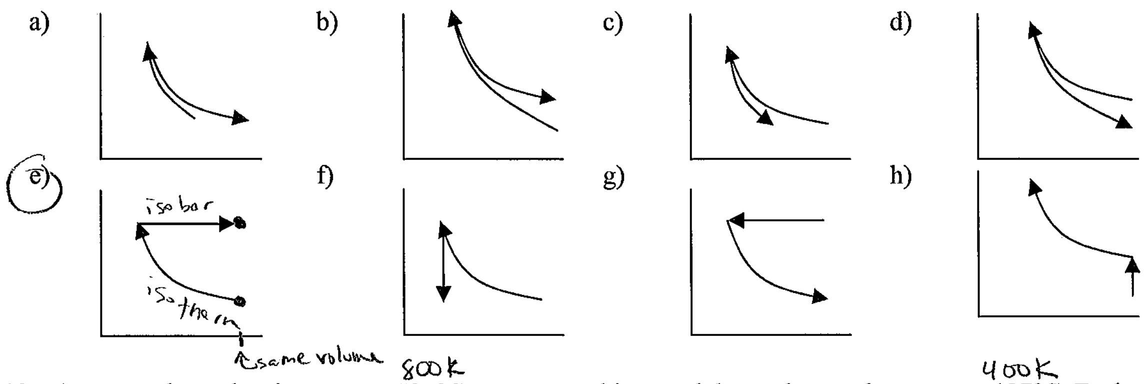
a. The entropy of the gas will increase. The entropy of the reservoir will increase.
b. The entropy of the gas will increase. The entropy of the reservoir will decrease.
c. The entropy of the gas will increase. The entropy of the reservoir will stay the same.
d. The entropy of the gas will decrease. The entropy of the reservoir will increase.
e. The entropy of the gas will decrease. The entropy of the reservoir will decrease.
f. The entropy of the gas will decrease. The entropy of the reservoir will stay the same.
g. The entropy of the gas will stay the same. The entropy of the reservoir will increase.
h. The entropy of the gas will stay the same. The entropy of the reservoir will decrease.
i. The entropy of the gas will stay the same. The entropy of the reservoir will stay the same.

1.17. If a gas undergoes a thermodynamic change whereby it somehow ends up in the same state it started in:

E_{int} is a state variable, so $\Delta E_{int} = 0$ for a cycle.

a. The internal energy of the gas will be less than when it started.
b. The internal energy of the gas will be greater than when it started.
c. The internal energy of the gas will be the same as when it started.
d. The change in internal energy will depend on the direction of the change (clockwise vs. counter-clockwise).

1.18. First, heat is removed from a gas while compressing it isothermally to 40% of its original volume. Next, heat is added to the gas while expanding it isobarically back to its original volume. Which of the following diagrams best represents the two processes on a standard P-V diagram?



1.19. A power plant takes in steam at 527°C (800 K) to power turbines and then exhausts the steam at 127°C (400 K). Each second the turbines transform 100 megajoules of heat energy from the steam into usable work. The theoretical maximum possible power output of the power plant is:

a. 0 - 20 megawatts
b. 20 - 40
c. 40 - 60
d. 60 - 80
e. 80 - 100 megawatts

$e = \frac{W}{Q_h}$

$W = e \cdot Q_h = .5 \times 100 = \boxed{50\ MW}$

$e_c = 1 - T_c/T_h = 1 - \frac{400}{800} = 50\%$

1.20. A heat engine performs x joules of work in each cycle and has an efficiency of e. For each cycle of operation, how much energy is absorbed by heat?

a. x
b. x/e
c. xe
d. $(1-x)$
e. $(1-x)/e$
f. $(1-x)e$

$e = \frac{W}{Q_h}$

$Q_h = \frac{W}{e} = \frac{"x"}{e}$

1.21. A certain gasoline car engine operates with a compression ratio of 9:1. Assuming it does not lose a substantial amount of energy to items such as internal friction, what would you expect its efficiency to be?

a. 0 - 10%
b. 10 - 20
c. 20 - 30
d. 30 - 40
e. 40 - 50
f. 50 - 60
g. 60 - 70
h. 70 - 80
i. 80 - 90
j. 90 - 100%

$e_{otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{9^{1.4-1}}$

$= 58.5\%$

1.22. An engine, a refrigerator, and a heat pump all operate in ideal Carnot cycles with the same gas, between the same maximum and minimum temperatures. Which of the following properly expresses the relationships between efficiencies/coefficients of performance?

a. $e_{engine} < COP_{refrigerator} < COP_{heatpump}$
b. $e_{engine} < COP_{heatpump} < COP_{refrigerator}$
c. $COP_{refrigerator} < e_{engine} < COP_{heatpump}$
d. $COP_{refrigerator} < COP_{heatpump} < e_{engine}$
e. $COP_{heatpump} < e_{engine} < COP_{refrigerator}$
f. $COP_{heatpump} < COP_{refrigerator} < e_{engine}$
g. cannot be determined

$e = \frac{T_h - T_c}{T_h}$ is < 1

$COP_r = \frac{T_c}{T_h - T_c}$ is > 1

$COP_{hp} = \frac{T_h}{T_h - T_c}$ is > 1, and larger than

1.23. Heat is added to a monatomic ideal gas, causing its temperature to double. In which case is the entropy change of the gas the largest?

a. Constant volume change
b. Constant pressure change
c. (a) and (b) are the same

$\Delta S = \int \frac{\partial Q}{T} = \int \frac{nC\,dT}{T} = nC \ln T_2/T_1$

since $C_p > C_v$, this will be largest for CP

1.24. Suppose you flip 11 coins simultaneously. How many different ways can the coins land to give you 7 heads and 4 tails? (I.e., what is the number of microstates in the 7H 4T macrostate?)

a. 165
b. 330
c. 462
d. 495
e. 792
f. 2048
g. 7920
h. 665280
i. 1663200
j. 39916800

As per HW13.2, it will be $\binom{11}{4}$ or $\binom{11}{7}$, both of which give you

$$\frac{11!}{4!\,7!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2} = 330$$

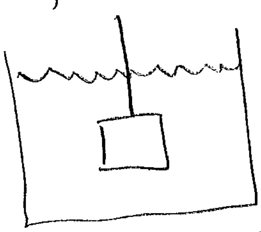
(12 pts) **Problem 2.** Give short answers/explanations to the following questions:

(a) Why is there a maximum length to how long a straw can be (and still function)?

There is no "sucking force". The liquid has to be pushed up the straw by atmospheric pressure, which is finite. Therefore there's a finite distance it can push the liquid.

(b) Give an example of a situation where the buoyant force on a object does not equal the object's weight.

Two easy ones:



$\uparrow T \uparrow B$

$\downarrow mg$

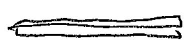
$T+B=mg$

$\uparrow N \uparrow B$

$\downarrow mg$

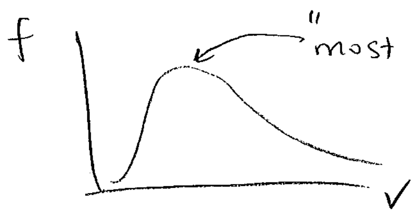
$N+B=mg$

(c) Why does a "bimetallic strip" curve as it is heated up?

The strip has two different metals on it, 

, which expand differently when heated because α_A and α_B will in general not be equal. This causes the strip to bend.

(d) Explain with words how you would use calculus to calculate the "most probable" speed of a collection of molecules, given the Maxwell-Boltzmann velocity distribution: $f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$. You don't have to actually do the calculation. You will receive no credit by simply saying $v_{most\ probable} = \sqrt{\frac{2k_BT}{m}}$.

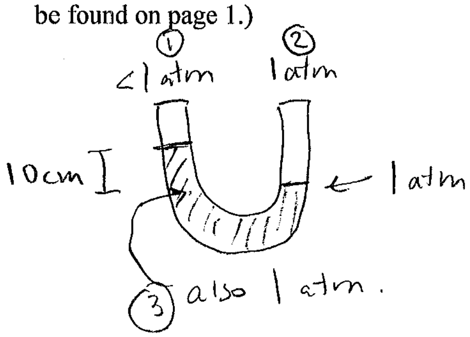


"most probable" is the velocity of the peak.

To find the peak, take $\frac{df}{dv}$, set equal to zero, and solve for v.

(14 pts) **Problem 3.**

(a) A U-tube, open on both ends, is filled with water. It is 0.6 cm in diameter. The right end is then shielded while air is blown across the left end. This creates a decrease in pressure by the left end, which "sucks" the water up. How fast must the air be blown in order for the water in the left-hand section to end up 10 cm higher than the water in the right-hand section. (Densities of air and water can be found on page 1.)



Bernoulli: $P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g h_2} + \cancel{\frac{1}{2}\rho v_2^2}$

$$P_1 = P_2 - \frac{1}{2}\rho_{air} v_1^2$$

Also $P_3 = P_1 + \rho_w g h$

$$P_3 = \left(P_2 - \frac{1}{2}\rho_a v^2\right) + \rho_w g h$$

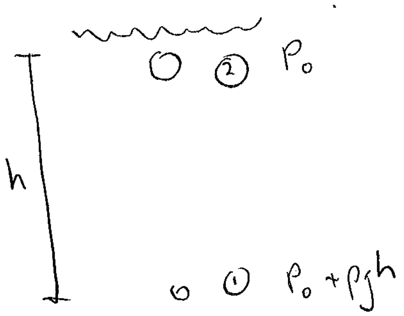
$\uparrow = P_2$, so it cancels with P_2

$$\frac{1}{2}\rho_a v^2 = \rho_w g h$$

$$v = \sqrt{2\frac{\rho_w}{\rho_a} g h} = \sqrt{2 \cdot \frac{1000}{1.29} \cdot 9.8 \cdot 0.1}$$

$$= \boxed{38{,}98 \text{ m/s}}$$

(b) An air bubble has a volume of 1 cm^3 when it is released by a diver 50 m below the surface of a lake. What is the volume of the bubble when it reaches the surface? Assume that the temperature and the number of air molecules in the bubble remain constant during the ascent.



$PV = nRT$

$\rightarrow$ if T = constant,

$$P_1 V_1 = P_2 V_2$$

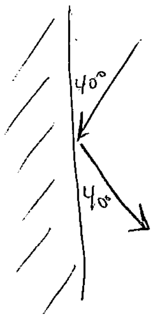
$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)$$

$$= V_1 \left(\frac{P_o + \rho g h}{P_o}\right)$$

$$= 1\text{cm}^3 \left(\frac{101000 \text{ Pa} + 1000 \cdot 9.8 \cdot 50 \text{ Pa}}{101000 \text{ Pa}}\right)$$

$$= \boxed{5.851 \text{ cm}^3}$$

(10 pts) **Problem 4.** Hailstones strike a glass window at an angle of 40° to the window surface, and at a rate of 200 hailstones per minute. Each collision lasts 0.03 seconds on average. The window is rectangular, with dimensions of 0.8 m by 0.7 m. Each hailstone has a mass of 5 g and a speed of 8 m/s. The collisions are elastic. What is the average pressure exerted by the hailstones on the window?



$\vec{F}\Delta t = m\Delta\vec{v}$ (impulse equation)

→ only $\vec{v}$ perpendicular to surface will "count"

$v\sin 40°$

$v\cos 40°$

That's the sine component

$$F = \frac{m \cdot \Delta v}{\Delta t} = \frac{m \cdot (2v\sin 40°)}{\Delta t}$$

If you want average force (not instantaneous) then use the time between collisions, not the time of a single collision (as discussed in class)

$$P = \frac{F_{ave}}{area}$$

$$= \frac{m\ 2v\sin 40°}{A\ \Delta t}$$

$$= \frac{(.005\,kg)\ 2 \cdot 8\,m/s\ \sin 40°}{(.8m \times .7m)\left(\frac{60\ sec}{200\ stones}\right)}$$

$$= \boxed{0.3061\ Pa}$$

(12 pts) **Problem 5.** An insulated chest has a surface area of 1.2 m^2 and a wall thickness of 3 cm. The temperature of the inner surface is 0°C, and the temperature of the outside surface is 28°C. The chest is made out of a material that has a thermal conductivity of 0.06 in standard SI units.

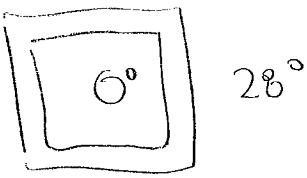
(a) What *are* the standard SI units for thermal conductivity?

Eqn is $P = \frac{k A \Delta T}{\ell}$, so $k = \frac{P \cdot \ell}{A \Delta T}$

units must be $\frac{W \cdot \cancel{m}}{m^{\cancel{2}} \cdot {}^\circ C} = \boxed{\frac{W}{m\,{}^\circ C}}$

or $\frac{J}{s \cdot m \cdot {}^\circ C}$ if you like.

(b) If 5 kg of ice at 0°C are put in the chest, how long (in hours) will it take for the ice to melt? Water has a specific heat of 4186 J/kg·°C, a latent heat of fusion of 333,000 J/kg, and a latent heat of vaporization of 2,260,000 J/kg.



$$P = \frac{Q}{t} = \frac{mL}{t}$$

$$\frac{mL}{t} = \frac{k A \Delta T}{\ell}$$

$$t = \frac{mL\ell}{kA\Delta T}$$

$$= \frac{(5\,kg)(333000\,\frac{J}{kg})(.03\,m)}{(.06\,\frac{W}{m\,{}^\circ C})(1.2\,m^2)(28\,{}^\circ C)}$$

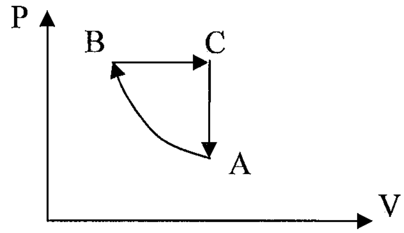
$$= 24777 \text{ seconds}$$

$$= \boxed{6.882 \text{ hrs}}$$

$\to C_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R$

(16 pts) **Problem 6.** An engine using 0.12 moles of a diatomic ideal gas is driven by this cycle: starting from state A, the gas is compressed isothermally until it reaches state B. Then, the gas is heated at constant pressure until it reaches state C. Finally, the gas is cooled at constant volume back to the original state. Various pressures, volumes, and temperatures are given in the table.

	P (kPa)	V (m^3)	T (K)
A	100	0.0030	300
B	800	?	300
C	800	0.0030	2400



(a) What is the volume of state B?

$T_A = T_B \to P_A V_A = P_B V_B$

$V_B = V_A \left(\frac{P_A}{P_B}\right) = (.003)\left(\frac{100}{800}\right) = \boxed{.000375\ m^3}$

(b) Find the heat added to the gas during each of the three legs.

A-B $\Delta E = Q + W_{on} \to Q = \Delta E - W_{on}$ or $= \Delta E + W_{by}$

isothermal $\to \Delta E = 0$

$Q = W_{by} = \int P\,dV = nRT \ln V_B/V_A = (.12)(8.31)(300)\ln\left(\frac{.000375}{.003}\right) = \boxed{-622\ J}$

B-C const pressure $\to Q = nC_P\Delta T = .12\left(\frac{7}{2}\cdot 8.31\right)(2400-300)$

$= \boxed{7329\ J}$

C-A const volume $\to Q = nC_V\Delta T = .12\left(\frac{5}{2}\cdot 8.31\right)(300-2400)$

$= \boxed{-5235\ J}$

(c) How much net work is done by the gas each cycle?

$Q_h = 7329\ J$

$Q_c = 622 + 5235 = 5857$

$W_{net} = Q_h - Q_c = \boxed{1472\ J}$

(d) What is the efficiency of the engine?

$e = \frac{W}{Q_h} = \frac{1472}{7329} = \boxed{20.1\%}$

(e) What is the maximum theoretical efficiency for an engine operating between the same minimum and maximum temperatures?

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{2400} = \boxed{87.5\%}$

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)

(a) In my research I study electrons in the material "gallium arsenide" (GaAs). When the material is put into a magnetic field, the regular electron state splits into two; the energy of the lower state is given by $E_{lower} = -\frac{1}{2} g \mu_B B$ and the energy of the upper state is given by $E_{upper} = +\frac{1}{2} g \mu_B B$. In these equations B represents the magnetic field,* g is a constant which is 0.44 for GaAs (that's called the "g-factor", and is different for each material), and μ_B is a constant which is equal to 9.274×10^{-24} J/T (called the "Bohr magneton" constant). These two states relate to the "spin" of the electrons, a quantum mechanical property that makes the electrons act in some ways as if they were spinning. The "spin polarization" is defined to be $(N_{upper} - N_{lower})/N_{total}$. What is the spin polarization of electrons in GaAs at a field of 1 tesla and a temperature of 1.5 K?

	E	BF	Probability = BF/sum of BFs
upper	$+\frac{1}{2} g \mu_B B = \frac{1}{2}(.44)(9.274 \cdot 10^{-24} \frac{J}{T})(1T) = 2.04 \cdot 10^{-24} J$	$e^{-E/k_BT} = e^{\frac{-2.04 \cdot 10^{-24}}{1.381 \cdot 10^{-23} \cdot 1.5}} = .9062$	$\frac{.9062}{2.00971} = 45.091\%$
lower	$-\frac{1}{2} g \mu_B B = -2.04 \cdot 10^{-24} J$	$e^{-E/k_BT} = e^{\frac{+2.04 \cdot 10^{-24}}{1.381 \cdot 10^{-23} \cdot 1.5}} = 1.1035$	$\frac{1.1035}{2.00971} = 54.91\%$

$$\text{Polarization} = \frac{54.91 - 45.09}{100} = \boxed{9.81\%}$$

(b) For the situation described in Problem 5 (a chunk of ice is put into an insulated chest, and then melts), calculate the entropy change of the ice as well as the entropy change of the surrounding air. Then add them together to find the total entropy change of the universe, which must come out to be positive. Hint: the ice stays at 0°C and the air stays at 28°C during the process.

$0°C \to 273.15\,K$ $\quad$ $28°C \to 301.15\,K$

Both are isothermal processes.

For isothermal, $\Delta S = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{Q}{T}$

ice $\quad \Delta S = \frac{mL}{T_{ice}} = \frac{5(333000)\,J}{273.15\,K} = 6095.6\ J/K$

air $\quad \Delta S = \frac{-mL}{T_{air}} = \frac{-5(333000)\,J}{301.15\,K} = -5528.8\ J/K$

negative, since heat flows out of the air

$$\Delta S_{universe} = \Delta S_{ice} + \Delta S_{air} = 6095.6 - 5528.8\ J/K$$
$$= \boxed{+566.7\ J/K}$$

* If you haven't run across this before, the S.I. unit of magnetic field is called the "tesla", and given the symbol T.