Instructions:

- Record your answers to the multiple choice questions ("Problem 1") on the bubble sheet.
- To receive full credit on the worked problems, please <u>show all work</u> and <u>write neatly</u>. Draw a picture if possible. Be clear about what equations you are using, and why. Prove that you understand what is going on in the problem. It's generally a good idea to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Double-check your calculator work. Think about whether your answer makes sense; if not, go over your work again or try working the problem a different way to double-check things.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

(36 pts) **Problem 1**: Multiple choice questions, 1.5 pts each. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper for your own reference.

1.1. A power plant takes in steam at 300°C $\rightarrow 573k$ to power turbines and then exhausts the steam at 120°C $\rightarrow 393k$. Each second the turbines transform 100 megajoules of heat energy from the steam into usable work. The (theoretical maximum) possible power output of the power plant is:
  - a. 0 - 10 megawatts
  - b. 10 - 20
  - c. 20 - 30
  - **d. 30 - 40**
  - e. 40 - 50
  - f. 50 - 60
  - g. 60 - 70
  - h. 70 - 80
  - i. 80 - 90
  - j. 90 - 100 megawatts

$e = 1 - \frac{T_c}{T_h} \approx 1 - \frac{393}{573} = 31.4\%$ ← Carnot efficiency

$W_{net} = e \cdot Q_h = (.314)(100 \text{ MJ})$

$\approx \boxed{31.4 \text{ MJ per sec}}$

1.2. Heat is added to a monatomic ideal gas, causing its temperature to double. In which case does the most heat need to be added?
  - a. Constant volume change
  - **b. Constant pressure change**
  - c. (a) and (b) are the same

$Q = n C_v \Delta T$ vs $Q = n C_p \Delta T$

$C_p > C_v$, so Q is larger

1.3. Heat is added to a monatomic ideal gas, causing its temperature to double. In which case is the entropy change of the gas the largest?
  - a. Constant volume change
  - **b. Constant pressure change**
  - c. (a) and (b) are the same

$\Delta S = \int \frac{dQ}{T} = \int \frac{n C \, dT}{T} = n C \ln T_f/T_i$

$n C_v \ln T_f/T_i$ vs $n C_p \ln T_f/T_i$

$C_p > C_v$, so ΔS is large

1.4. As a wave travels into a medium in which its speed increases, its frequency ____.
  - a. decreases
  - b. increases
  - **c. remains the same** (as discussed in class)

1.5. As a wave travels into a medium in which its speed increases, its wavelength ____.
  - a. decreases
  - **b. increases**
  - c. remains the same

$v = f\lambda$ → if v increases and f stays the same, then λ increases

1.6. Consider the motion of transverse waves in a wire. Waves will travel fastest in a ____ wire.
  - a. tight and heavy
  - **b. tight and light**
 - c. loose and heavy
  - d. loose and light

$v = \sqrt{\frac{T}{\mu}}$ you want large T ("tight") and small μ ("light")

1.7. An airplane flies over your head, traveling faster than the speed of sound. When will you hear the sonic boom?
  - a. Before the airplane is directly over your head.
  - b. The instant the airplane is directly over your head.
  - **c. After the airplane is directly over your head.** (as discussed in class)

1.8. Consider the diagram shown with several circular waves created at various times and locations. The source was moving:
  (a) to the left *(like Dr Stokes' video in class)*
  b. to the right
  c. up
  d. down

A ⊙ B
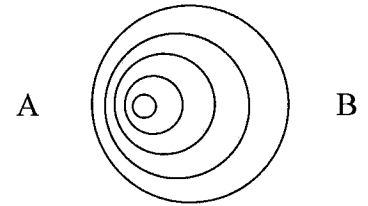
1.9. Same diagram. A person positioned at point A would perceive _____ frequency as the person positioned at point B.
  (a) a higher *More wavefronts in a given amount of time*
  b. a lower
  c. the same

1.10. An object is vibrating at its natural frequency. Repeated and periodic vibrations of the same frequency strike the object and the amplitude of its vibrations are observed to increase. This phenomenon is known as ____.
  a. beats
  b. interference
  c. overtones
  (d) resonance *like the bowling-ball video in class*
  e. transference

One wavelength, so
$$\lambda = \frac{2}{3} \cdot 6 = 4\,m$$

0.5m

6.0m
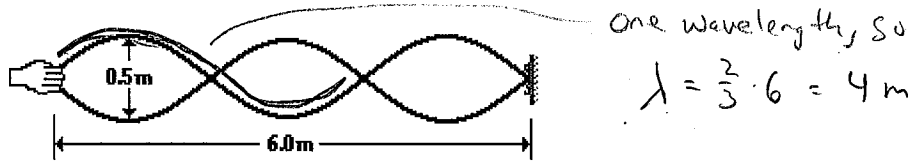
1.11. A standing wave experiment is performed to determine the speed of waves in a rope. The standing wave pattern shown above is established in the rope. The rope makes 90.0 complete vibrational cycles in exactly one minute. The speed of the waves is:
  a. 0.5 m/s
  b. 3
  (c) 6
  d. 180
  e. 360
  f. 540 m/s

$$V = f\lambda = \left(\frac{90\,cycles}{60\,sec}\right) \cdot 4\,m$$
$$= 6\,{}^m/_s$$

1.12. Suppose a rope has a measured speed of 10 m/s and a tension of 20 N. What is the rope's linear mass density?
  (a) 0.2 kg/m
  b. 0.5
  c. 2
  d. 5
  e. 20
  f. 50
  g. 200
  h. 500 kg/m

$$V = \sqrt{T/\mu} \rightarrow V^2 = T/\mu$$
$$\mu = T/V^2$$
$$\mu = \frac{20}{10^2} = .2\,{}^{kg}/_m$$

1.13. Consider a transverse traveling wave of the form: $y(x,t) = (3x - 12t)^4$. (You may assume that the numbers have the appropriate units associated with them to make x, y, and t be in standard SI units.) Is the wave moving in the +x or −x direction?
  (a) +x
  b. −x
  c. it cannot be determined

$$y = [3(x-4t)]^4$$

speed is 4, and it's traveling to the right. For example, when t=1, $[3(x-4)]^4$ has the same shape as $[3x]^4$, just shifted to the right by 4.

1.14. Same situation. What is the wave's speed?
  a. 1 m/s
  b. 3
  (c) 4
  d. 5
  e. 6
  f. 8
  g. 12
  h. 36
  i. 40 m/s

1.15. Two different ropes with different mass densities are attached to each other as in the above picture. A pulse is introduced into one end of the rope and approaches the boundary as shown. At the boundary, some of the energy is transmitted into the new medium and some is reflected. Which one of the diagrams below depicts the possible location and orientation of the pulse shortly after the incident pulse reaches the boundary? (Hint: you can judge the relative velocities of the two media by noting how close or far from the interface the pulses are at the instant shown.)

A $\times$ Same speed → would not get a reflection.

B $\times$ $v_2 < v_1$ → would get a $180°$ phase shift in reflected wave

C ✓ (circled)

D $\times$ } You never get a $180°$ phase shift in the transmitted wave.

E $\times$ }

1.16. A sound wave passes from medium A to medium B, at normal incidence (the wave travels perpendicular to the boundary). In which case will you get the most transmitted sound energy?

a. $v_A < v_B$
b. $v_A = v_B$ (circled) When $v_A = v_B$, then $r = 0$ and $t = 1$ → all energy is transmitted!
c. $v_A > v_B$

1.17. While attempting to tune a "concert A" string (A above middle C), a piano tuner hears 2 beats/s between a reference oscillator at 440 Hz and the string. When he tightens the string slightly, the frequency of the beats rises smoothly to 5 beats/s. What is the frequency of the string now?

a. 433 Hz → $f_{beat} = |f_1 - f_2|$, so
b. 435
c. 437 Could be 435 Hz or 445 Hz.
d. 438
e. 440

f. 442
g. 443
h. 445 (circled)
i. 447 Hz

string started too high, therefore it goes up to 445 Hz

1.18. The strings in an orchestra first tune themselves to a concert A (A above middle C) of 440 Hz. A trumpet playing in the orchestra tunes itself to a concert B-flat, however. That is a half-step higher. Assuming an equal temperament scale is used, what frequency is that?

a. Less than 452 Hz
b. 452 - 455
c. 455 - 458
d. 458 - 461

e. 461 - 464
f. 464 - 467 (circled)
g. 467 - 470
h. More than 470 Hz

$f_{Bflat} = f_A \times \sqrt[12]{2}$

$\approx 440 \cdot 2^{1/12} = \boxed{466.2 \text{ Hz}}$

Common musical intervals for use in the next question

Second = 2 half-steps
Minor third = 3 half-steps
Major third = 4 half-steps
Fourth = 5 half-steps
Fifth = 7 half-steps

Sixth = 9 half-steps
Minor seventh = 10 half-steps
Major seventh = 11 half-steps
Octave = 12 half-steps

1.19. A piano is tuned to an equal temperament scale and two piano keys are played at random. The fundamental frequencies of the two notes are found to be 185 Hz and 311 Hz. What musical interval relates the two notes?

a. a second
b. a minor third
c. a major third
d. a fourth
e. a fifth

f. a sixth (circled)
g. a minor seventh
h. a major seventh
i. an octave

$\frac{311}{185} = 1.681$ How many half-steps?

$1.681 = 2^{x/12}$ → By trial and error,
$x \approx 9$
which is a sixth.

(or solve with logs:
$x = \frac{\log 1.681}{\log 2^{1/12}} = 8.99$)

1.20. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.
   a. A sound wave can travel through a vacuum. ✗ *Needs a medium*
   (b.) A sound wave is a pressure wave which can be thought of as fluctuations in pressure with respect to time. *True.*
   c. A sound wave is a transverse wave. ✗ *It's longitudinal*
   d. To hear the sound of a tuning fork, air molecules must travel from the tines of the fork to one's ear. ✗ *the energy is transferred via waves, not molecules.*

1.21. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.
   (a.) A machine produces a sound which is rated at 60 dB. If two of the machines were used at the same time, the decibel rating would be about 63 dB. *3 dB = ×2 in intensity*
   b. Intensity of a sound at a given location usually varies inversely with the distance from that location to the source of the sound: $I \sim 1/r$ ✗ *in general, intensity ~ $1/r^2$, not $1/r$. (Think of area of spherical wave front)*
   c. The ability of a human observer to hear a sound wave depends solely upon the intensity of the sound wave. ✗ *as in the class demo, it also depends on frequency.*
   (d.) The intensity of a sound wave has units of Watts/meter^2. ✓ *$I = P/Area$.*

1.22. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.
   (a.) A high pitched sound has a low wavelength. ✓ *$v = \lambda f \rightarrow$ if f = high, then λ = small*
   (b.) In general, sound waves travel faster in solids than they do in gases. *True (as discussed in class)*
   (c.) Sound waves travel faster on a warm day than a cool day. *True → the equation is $v = 343 \, m/s \sqrt{T/293K}$ from pg 1 of exam.*
   d. The speed of a sound wave moving through air is largely dependent upon the frequency of the wave. ✗ *there's not much dispersion, so mainly v doesn't depend on f.*

1.23. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.
   (a.) If a guitar string is touched directly in the middle, the second harmonic can be forced to sound. *True (as discussed in class)*
   b. The fundamental frequency of a guitar string corresponds to the standing wave pattern in which there is a complete wavelength within the length of the string. ✗ *or L = half a wavelength*
   c. The fundamental frequency of a guitar string is the highest frequency at which the string vibrates. ✗ *the lowest freq.*
   (d.) The frequency of the third harmonic of a string is 3/2 times the frequency of the second harmonic. *True → $f_3 = 3f_1$ and $f_2 = 2f_1$, so $f_3 = \frac{3}{2} f_2$*

1.24. Mark all items which are true. You must bubble in all true items, and no false items, in order to get credit.
   (a.) An "open-closed" air column that can play a fundamental frequency of 250 Hz will not have 500 Hz as a harmonic. ✗ *Just get odd harmonics so 2nd harmonic is not possible.*
   b. If an "open-closed" air column has a length of 20 cm, the wavelength of the first harmonic will be 5 cm. ✗ *will be 80cm*
   (c.) Long "open-closed" air columns will produce lower frequencies than short columns. *True → large L gives low freq.*
   (d.) "Panpipes" (also called a "pan flute") and narrow glass bottles can be modeled as open-closed systems. *True (as discussed in class)*

(10 pts) **Problem 2**. Give short answers/explanations to the following questions:

(a) What general kind of wave would result for each of these situations:

(1) A wave is created by adding together a large number of sine waves (or cosine) at the same frequency, ω_0, but with the sine waves all possibly having different amplitudes and phases.

$A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2) + \cdots.$ → can be added via complex numbers, $= A \cos(\omega t + \phi)$

will give you a single sine wave with a still different amplitude + phase.

(2) A wave is created by adding together an infinite number of sine waves at integer multiples of the same frequency, ω_0, with the sine waves all having the same phase but possibly different amplitudes.

$A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + \cdots$

This is a Fourier series! Will give you a wave with frequency ω, but with a non-sinusoidal shape.

 (or some other shape)
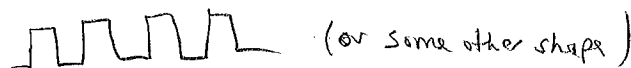
(3) A wave is created by adding together an infinite number of sine waves with frequencies spaced infinitely close to each other, centered around a particular frequency ω_0, extending up to frequencies $\omega_0 + \Delta\omega$ and down to frequencies $\omega_0 - \Delta\omega$, with the sine waves all having the same phase but possibly different amplitudes.
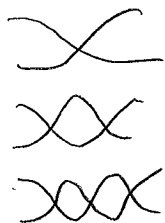
amplitude

$\omega_0 - \Delta\omega \quad \omega_0 \quad \omega_0 + \Delta\omega$
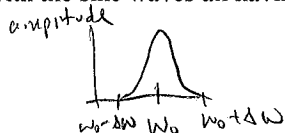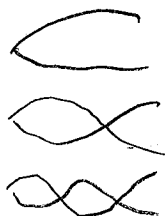
This will produce a wave packet (or pulse)

basic frequency will be ω_0

duration can be estimated by uncertainty principle
$\Delta t \, \delta\omega \approx \frac{1}{2} \rightarrow \Delta t \approx \frac{1}{2\Delta\omega}$

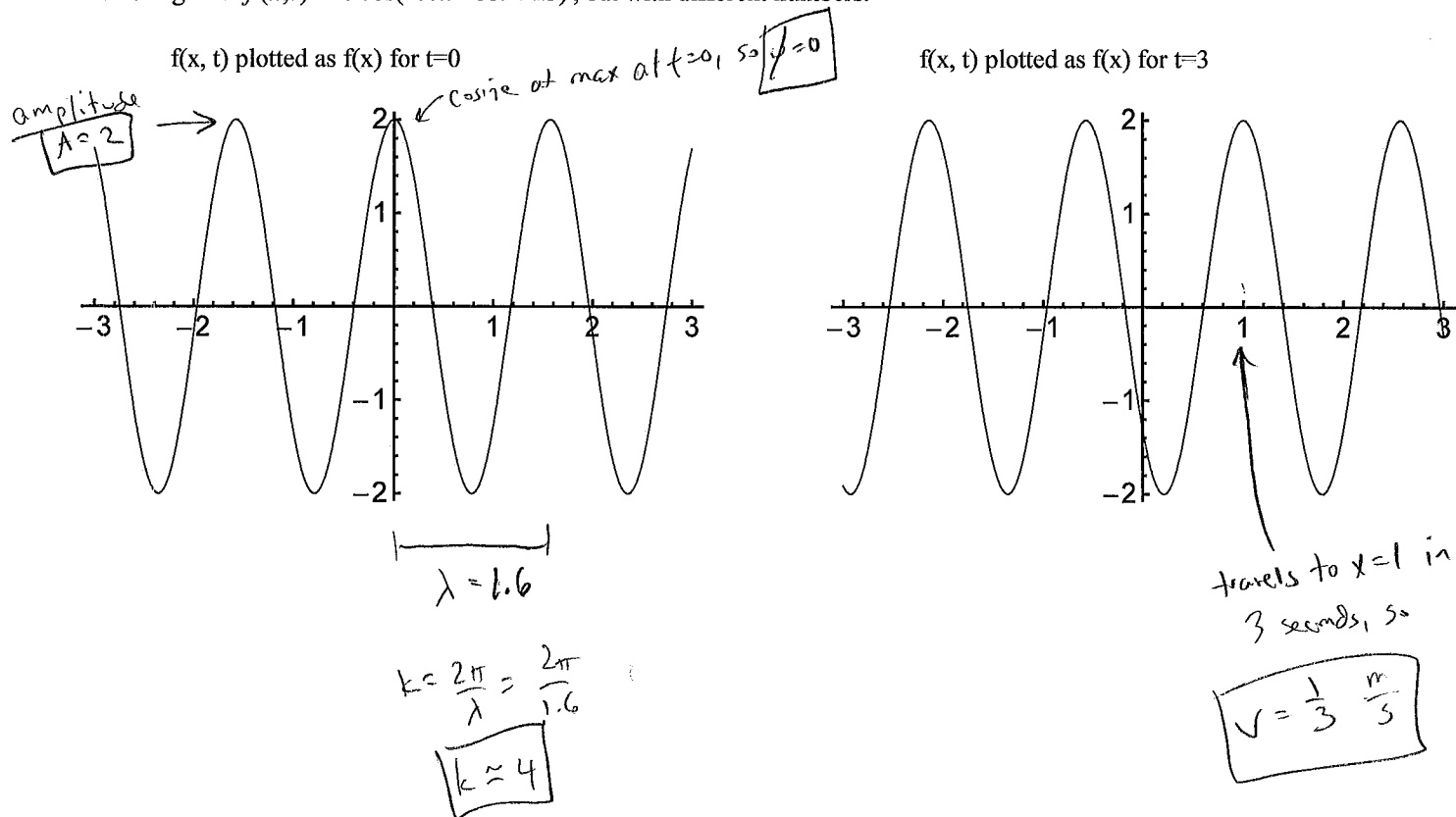(b) Sketch the first three standing wave patterns for these situations:

(1) an "open-open" pipe.

(2) an "open-closed" pipe.

(10 pts) **Problem 3.** The function f(x,t) is plotted at two different times. The wave moved smoothly to the right with no maxima crossing the origin between the two snapshots. Figure out the equation for the wave in terms of a cosine function. It will be something like $f(x,t) = 13\cos(100x - 33t + 23)$, but with different numbers.

f(x, t) plotted as f(x) for t=0

cosine at max at t=0, so $\phi = 0$

amplitude
$A = 2$

$\lambda = 1.6$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.6}$

$k \approx 4$

f(x, t) plotted as f(x) for t=3

travels to x=1 in 3 seconds, so

$v = \frac{1}{3} \frac{m}{s}$

Piecing together the info, using eqn. $f = A\cos\left(k(x-vt) + \phi\right)$,

$f(x,t) = 2\cos\left(4\left(x - \tfrac{1}{3}t\right)\right)$

or $2\cos\left(4x - \tfrac{4}{3}t\right)$

(10 pts) **Problem 4.** f_1 and f_2 are defined as follows:

$$f_1(t) = 4\cos(15t + 2)$$
$$f_2(t) = 5\cos(15t + 5)$$

f_3 is the sum of f_1 and f_2. Find f_3. It will be something like $f_3(t) = 13\cos(33t + 23)$, but with different numbers.

As described in class, this is like adding complex numbers in polar form

$$4 \angle 2 + 5 \angle 5$$

or in degrees, $4 \angle 114.6° + 5 \angle 286.5°$

Do it with components:

should give you this answer.

$$X = 4\cos 114.6° + 5\cos 286.5° = -0.246$$

$$Y = 4\sin(\ ) + 5\sin(\ ) = -1.157$$

$$r = \sqrt{x^2 + y^2} = \boxed{1.183}$$

this angle $= \tan^{-1}\left(\dfrac{1.157}{.246}\right)$

$= 77.99°$

$\phi = 180° + $ angle in picture

$\phi = 257.99°$

$= \boxed{4.503 \text{ radians}}$

Answer $= \boxed{1.183 \angle 4.503}$

or, more precisely, $\boxed{f_3 = 1.183\cos\left(15t + 4.503\right)}$

(11 pts) Problem 5. (This is like the demo done in class.) Two speakers are broadcasting an identical tone, frequency $f = 343$ Hz, in phase with each other. They are 2 m apart. This causes a pattern of maxima and minima to be distributed throughout the classroom. A student sits in a row a few meters away from the speakers; the student herself is exactly 5 m away from each of the speakers. Because the path length is the same for each of the speakers, the two sounds constructively interfere and the student hears one of the maxima. How far to the left (i.e., along her row) must the student move before she first hears a minimum in the sound level? Use 343 m/s as the speed of sound.

You don't need to actually solve the equation. Call the distance that you are looking for "x", and set up an equation that you could potentially solve algebraically for x. That is, the equation must have no other unknowns but x in it.
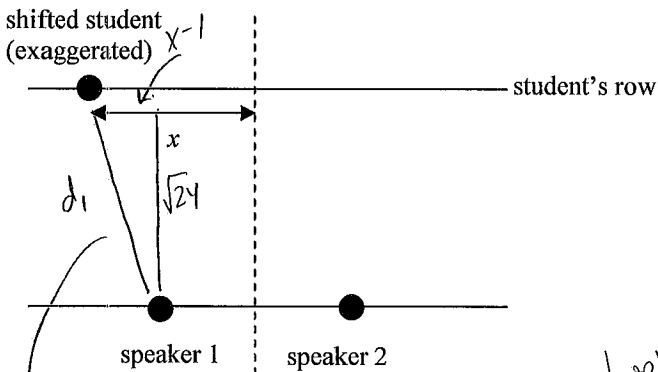
Hint: Among other things, you will need to figure out how far the students' row is from the line connecting the speakers.

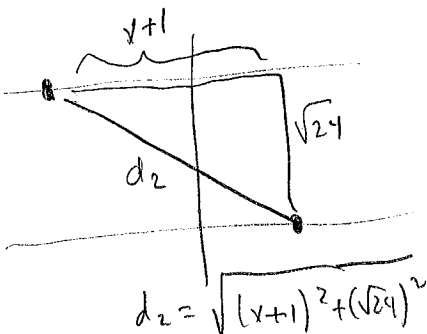$$v = \lambda \cdot f, \text{ so } \boxed{\lambda = 1\,m}$$

Before:

$$\sqrt{5^2 - 1^2} = \sqrt{24}$$
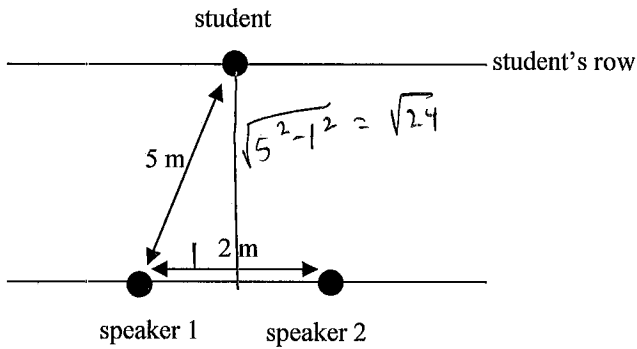
After:

$$d_1 = \sqrt{(x-1)^2 + (\sqrt{24})^2}$$

$$d_2 = \sqrt{(x+1)^2 + (\sqrt{24})^2}$$

Looking for a minimum, so $d_2 - d_1 = \frac{1}{2}\lambda$

Eqn is: $$\boxed{\sqrt{(x+1)^2 + 24} - \sqrt{(x-1)^2 + 24} = \frac{1}{2}\,m}$$

(10 pts) **Problem 6.** Trombones are usually tuned so that when the trombone slide is all the way in the fourth harmonic is 233.1 Hz. If the temperature of the air inside a certain trombone is 32°C, how long will the column of air be after it is tuned? Hint: do not use 343 m/s as the speed of sound.

$\hookrightarrow 305 \, K$

$4 \cdot f_1 = 233.1$

$f_1 = 58.275 \, Hz$

$v = 343 \frac{m}{s} \sqrt{\frac{T}{293 K}}$
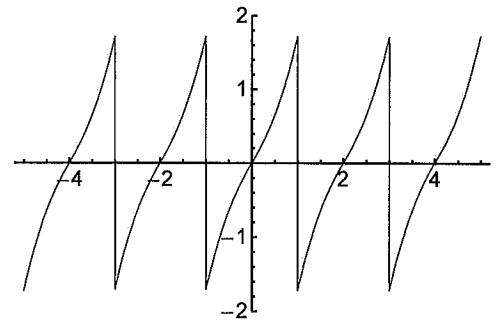
$= 343 \sqrt{\frac{305}{293}} = 349.95 \frac{m}{s}$

$f_1 = \frac{v}{2L} \longrightarrow L = \frac{v}{2f_1}$

$L = \frac{349.95}{2(58.275)} = \boxed{3.004 \, m}$

(10 pts) **Problem 7.** The function f(x), graphed on the right, is defined as follows:

$$f(x) = \begin{cases} e^x - 1, & \text{for } x \text{ between } 0 \text{ and } +1 \\ -e^{-x} + 1, & \text{for } x \text{ between } -1 \text{ and } 0 \end{cases}$$

(repeated with a period of $L = 2$)

I worked out the Fourier coefficients for this function, and found the following:

$$f(x) = 0 + 0.88\sin(\pi x) - 0.53\sin(2\pi x) + 0.35\sin(3\pi x) - 0.27\sin(4\pi x) + \ldots$$

I've rounded the numbers to two decimal places. Plots of f(x) for increasing numbers of terms in the summation are shown on the right.

(first 2 terms)

(a) Why is the constant term in my expression equal to zero?

You can tell by looking at the function that the average value is zero.

(Alternatively the function is odd ... which forces the average value to be zero.)

(b) Why are there no cos(nπx/L) terms in the series?

The function is odd

(first 5 terms)

(c) Why are all of the sine function arguments multiples of πx?

the ~~fundamental~~ period L is 2

Therefore the fundamental k is $\frac{2\pi}{L} = \pi$

All frequencies present will be multiples of the fundamental, so the k-values are $\pi, 2\pi, 3\pi$, etc.

(first 100 terms)

(d) Set up an integral that you could presumably solve to determine the coefficient of an arbitrary term of the series (the "n^{th}" term, if you like). Don't just copy down the formula from page one of the exam, actually put the integral into a form that you could (for example) type into Mathematica to get the coefficient for a given n. That is, your equation must have no other variables in it besides x (the variable of integration) and n.

$$b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \sin \frac{2\pi n x}{L} dx$$

$$b_n = \frac{2}{2} \int_{-1}^{1} f(x) \sin \frac{2\pi n x}{2} dx$$

$$\boxed{b_n = \int_{-1}^{0} \left(-e^{-x} + 1\right) \sin \pi n x \, dx + \int_{0}^{1} \left(e^x - 1\right) \sin \pi n x \, dx}$$
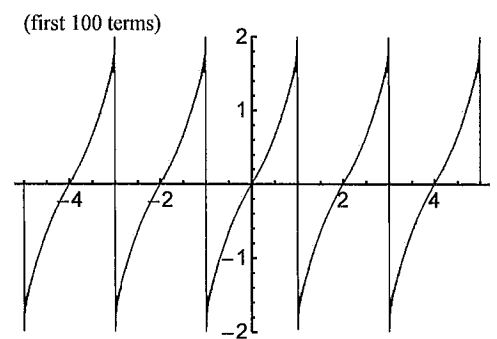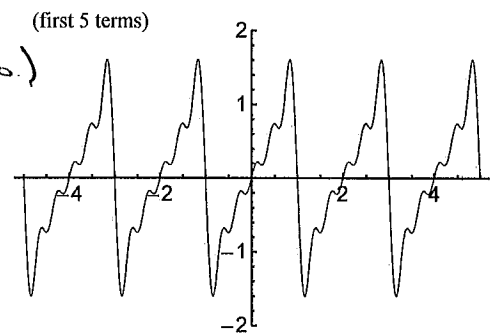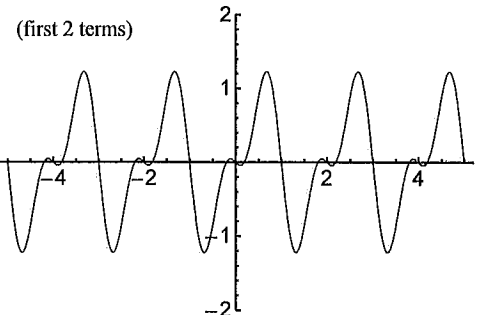
which also equals

$$2\int_{0}^{1} \left(e^x - 1\right) \sin \pi n x \, dx$$

(3 pts, no partial credit) **Problem 8.** Explain with words how you would use calculus to calculate the "most probable" speed of a collection of molecules, given the Maxwell-Boltzmann velocity distribution: $f(v) = 4\pi \left(\dfrac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-mv^2/2k_B T}$. You don't have to actually do the calculation. You will receive no credit by simply saying $v_{most\,probable} = \sqrt{\dfrac{2k_B T}{m}}$.

f

This is what $f(v)$ looks like

$v_{most\,probable}$ is the peak.
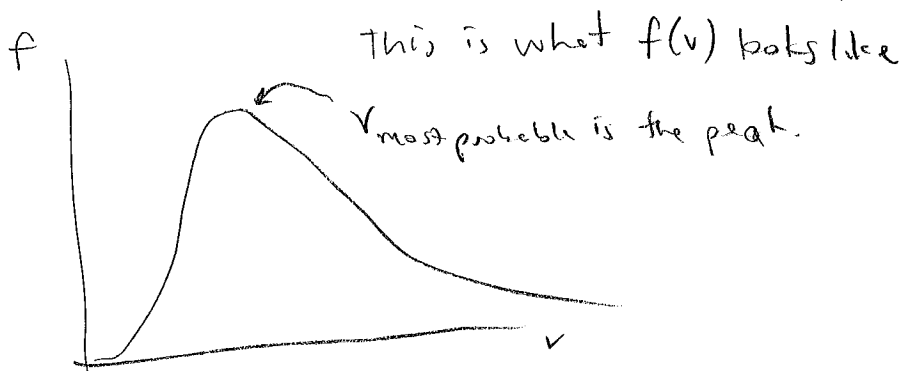
To find the peak, you just take $\dfrac{df}{dv}$, set equal to zero, and solve for v.

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)

(a) Solve the equation in Problem 5 for x.

$$\sqrt{(x+1)^2 + 24} - \sqrt{(x-1)^2 + 24} = \frac{1}{2}$$

$$\sqrt{(x+1)^2 + 24} = \frac{1}{2} + \sqrt{(x-1)^2 + 24}$$

$$(x+1)^2 + 24 = \frac{1}{4} + 2 \cdot \frac{1}{2} \cdot \sqrt{(x-1)^2 + 24} + ((x-1)^2 + 24)$$

$$\cancel{x^2} + 2x + 25 = \frac{1}{4} + \sqrt{(x-1)^2 + 24} + \cancel{x^2} - 2x + 25$$

$$4x - \frac{1}{4} = \sqrt{(x-1)^2 + 24}$$

$$16x^2 - 2 \cdot 4x \cdot \frac{1}{4} + \frac{1}{16} = (x-1)^2 + 24$$

$$16x^2 - \cancel{2x} + \frac{1}{16} = x^2 - \cancel{2x} + 25$$

$$15x^2 = 24\frac{15}{16}$$

$$x = \sqrt{\frac{24\frac{15}{16}}{15}}$$

$$\boxed{x = 1.289 \text{ m}}$$

(b) Find the numerical value of the coefficient for the $n = 5$ term of the Fourier series in Problem 7.

Hint: this integral is useful: $\int e^x \sin(Cx)\,dx = \dfrac{e^x(-C\cos(Cx)+\sin(Cx))}{1+C^2}$

$$b_5 = 2\int_0^1 (e^x - 1)\sin 5\pi x\,dx$$

$$= 2\int_0^1 e^x \sin 5\pi x\,dx - 2\int_0^1 \sin 5\pi x\,dx$$

$$= 2e^x\left(\frac{-5\pi\cos 5\pi x + \sin 5\pi x}{1+(5\pi)^2}\right)\Big|_{x=0}^{1} + 2\frac{\cos 5\pi x}{5\pi}\Big|_{x=0}^{1}$$

$$= \frac{2}{25\pi^2 + 1}\left[e(-5\pi\cos 5\pi + \sin 5\pi) - 1(-5\pi)\right] + \frac{2}{5\pi}(\cos 5\pi - 1)$$

$$= \frac{2}{25\pi^2 + 1}\left[e(5\pi) + 5\pi\right] - \frac{4}{5\pi} = \boxed{0.2169}$$

(c) Do the calculus-related calculation in problem 8 to derive the equation, $v_{most\,probable} = \sqrt{\dfrac{2k_B T}{m}}$.

$$f = stuff \cdot v^2\, e^{-mv^2/2kT}$$

$$\frac{df}{dv} = stuff\left[2v\,e^{-mv^2/2kT} + v^2\,e^{-mv^2/2kT}\left(-\frac{2mv}{2kT}\right)\right]$$

$$0 = \cancel{stuff\,e^{-mv^2/2kT}}\left[2v + v^2\left(-\frac{mv}{kT}\right)\right]$$

$$2v = v^3\frac{m}{kT}$$

$$v^2 = \frac{2kT}{m}$$

$$\boxed{v = \sqrt{\frac{2kT}{m}}}$$

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