Phys 123 Section 2 Winter 2011 Final Exam Solns by Dr. Colton

(23 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. Which of the following is the best resolution of the barn paradox, as discussed in class and analyzed for homework? (Lee is the one running with the ladder; Cathy is the one at rest relative to the barn.)

- a. Cathy sees the ladder fit entirely within the barn, but Lee does not.
- b. Lee sees the ladder fit entirely within the barn, but Cathy does not.
- c. Each of them sees the ladder fit entirely within the barn.

Ladder is Lorentz-contracted to Cathy

1.2. In the relativity context, which of the following would be an example of an "event"?

- a. A light beam hits a sensor.
- b. A light beam travels through space.
- c. Bill travels on a very fast train.
- d. Ted observes Bill traveling on a fast train.

Something that happens at a particular place + time.

1.3. Frodo is eating breakfast on a train which moves at 2×10^8 m/s. Sam is sitting at a picnic table near the train tracks, also eating breakfast. Which pair of statements is correct (note, "slow motion" refers to the eating motions only, not to the overall train speed of 2×10^8 m/s, which is obviously anything but slow):

- a. To Frodo, it looks like Sam is eating in fast motion. To Sam, it looks like Frodo is eating in slow motion.
- b. To Frodo, it looks like Sam is eating in fast motion. To Sam, it looks like Frodo is eating in fast motion.
- c. To Frodo, it looks like Sam is eating in slow motion. To Sam, it looks like Frodo is eating in slow motion.
- d. To Frodo, it looks like Sam is eating in slow motion. To Sam, it looks like Frodo is eating in fast motion.

They each observe time-dilation effects on the other.

1.4. What is the maximum momentum that a particle with mass m and velocity v can have?

- a. mv
- b. mc
- c. mcv
- d. $2mv$
- e. $2mc$
- f. $2mcv$
- g. There is no maximum

$p = \gamma m v$
γ can be arbitrarily large.

1.5. A reference frame in which objects which do not experience forces do not accelerate is called a(n) __________ reference frame.

- a. accelerating
- b. depressed
- c. Einsteinian
- d. inertial
- e. Lorentz
- f. null
- g. proper
- h. relativisitic

1.6. Suppose Dr. Colton slams a book down on a desk, twice. The two book slams are (in Dr. Colton's frame) separated by 10 seconds. To an observer in a rocket moving at 0.7c relative to Dr. Colton, the two slams will be:

- a. separated by less than 10 seconds
- b. separated by more than 10 seconds
- c. separated by exactly 10 seconds

Observer will see Dr. Colton "in slow motion"

1.7. Emily is moving at 0.9c relative to Joshua. David is moving at 0.8c relative to Joshua. Emily's speed relative to David will be:

- a. less than 0.1c
- b. more than 0.1c
- c. exactly 0.1 c

Not $.9-.8$ but $\dfrac{.9-.8}{1+(.9)(.8)}$ ← denom < 1 so answer is $> .1$

1.8. A ballet dancer (mass m) stands on her toes during a performance with area A in contact with the floor. What is the pressure exerted by the floor over the area of contact if the dancer is jumping upwards with an acceleration of a?

- a. mg
- b. $m(g+a)$
- c. $m(g-a)$
- d. mg/A
- e. $m(g+a)/A$
- f. $m(g-a)/A$

$\Sigma F = ma$ ↑N ↓mg

$N - mg = ma$

$N = m(g+a)$

pressure $= \dfrac{\text{Force}}{\text{area}} = \dfrac{N}{A} = \dfrac{m(g+a)}{A}$

1.9. A boat is on a lake. If an anvil (that sinks) is pushed from the boat into the water, will the overall water level of the lake rise, fall or stay the same? (compared to when the anvil was in the boat)

a. rise

b. fall

c. stay the same

in boat: displaces weight ← bigger effect

in water: displaces volume

1.10. Water (no viscosity, incompressible) flows from a little pipe into a big pipe with no height change. The flow speed (m/s) in the little pipe will be ______ in the big pipe.

a. greater than

b. the same as

c. less than

$A_1 v_1 = A_2 v_2$ → small A = big velocity

1.11. Water (no viscosity, incompressible) flows from a little pipe into a big pipe while also increasing in height. (That is, the water is flowing uphill.) The volume flow rate (m^3/s) in the little pipe will be ______ in the big pipe.

a. greater than

b. the same as

c. less than

VFR = constant (= Av)

(if density doesn't change)

1.12. A gas in contact with a thermal reservoir undergoes an isothermal expansion. The gas and the thermal reservoir are isolated from the rest of the universe. Which of the following is true?

a. The entropy of the gas will increase. The entropy of the reservoir will increase.

b. The entropy of the gas will increase. The entropy of the reservoir will decrease.

c. The entropy of the gas will increase. The entropy of the reservoir will stay the same.

d. The entropy of the gas will decrease. The entropy of the reservoir will increase.

e. The entropy of the gas will decrease. The entropy of the reservoir will decrease.

f. The entropy of the gas will decrease. The entropy of the reservoir will stay the same.

g. The entropy of the gas will stay the same. The entropy of the reservoir will increase.

h. The entropy of the gas will stay the same. The entropy of the reservoir will decrease.

i. The entropy of the gas will stay the same. The entropy of the reservoir will stay the same.

→ heat enters gas (otherwise temp. would decrease as it expands, instead of staying constant) Therefore heat leaves reservoir

$\Delta S = \int \frac{dQ}{T}$ → = positive for gas = negative for reservoir

1.13. When a gas expands:

a. The gas does positive work on its surroundings.

b. The surroundings do positive work on the gas.

c. Work is not necessarily done.

$W_{\text{by gas}} = \int P\,dV$ = positive if V increases

1.14. If a gas undergoes a thermodynamic change whereby it somehow ends up in the same state it started in:

a. The internal energy of the gas will be less than when it started.

b. The internal energy of the gas will be greater than when it started.

c. The internal energy of the gas will be the same as when it started.

d. The change in internal energy will depend on the direction of the change (clockwise vs. counter-clockwise).

internal energy is a state variable!

1.15. Suppose you flip 13 coins simultaneously. How many different ways could the coins land to give you 8 heads and 5 tails? (I.e., what is the number of microstates in the 8H 5T macrostate?)

a. 32

b. 256

c. 520

d. 688

e. 1287

f. 8192

g. 13980

h. 432432

i. 67108864

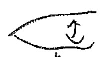
$\binom{13}{8} = \frac{13!}{8!\,5!} = 1287$

1.16. A "closed-closed" pipe and a "closed-open" pipe are the same length. Which will have the lower fundamental frequency?

a. closed-closed

b. closed-open

c. same fundamental frequency



longer λ, so smaller f

1.17. In which case will there be no reflection from a wave on a string hitting a boundary?

a. when the wave's frequency is the same on both sides of the boundary

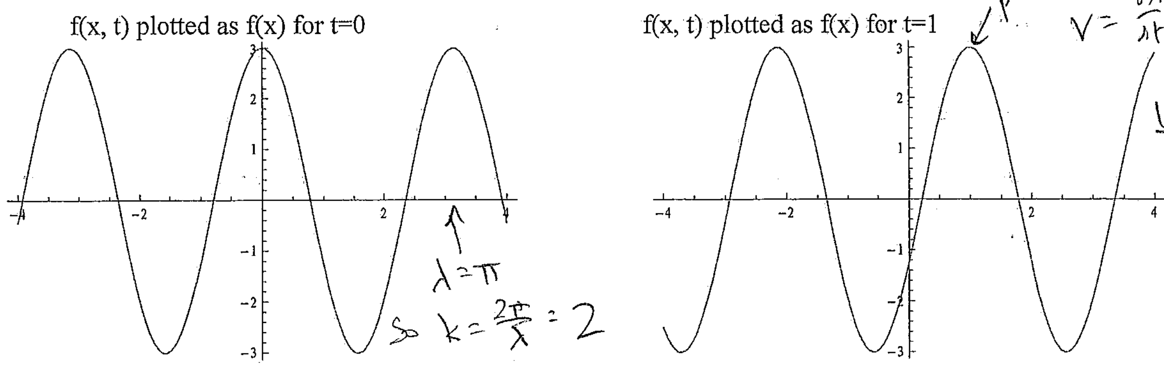
b. when the wave's speed is the same on both sides of the boundary

c. when the wave's wavelength is the same on both sides of the boundary

d. more than one of the above

(b) and (c) both true! freq will be the same, so if v is same, λ will also be same.

$A = 3$



$\lambda = \pi$

So $k = \frac{2\pi}{\lambda} = 2$

peak moved 1 to right

$v = \frac{\Delta x}{\Delta t} = 1$ m/s

$\frac{\omega}{k} = v$

$\rightarrow \omega = vk$

$\rightarrow \omega = 2$

1.18. Which wave function f(x,t) is represented by the two graphs displayed? The left-hand graph is the wave function plotted for t=0; the right hand graph is the wave function plotted for t=1.

a. f(x,t) = 3 cos(2x - 2t) ← only choice with $k = 2$, $\omega = 2$, $A = 3$
b. f(x,t) = 3 cos(2x - 4t)
c. f(x,t) = 3 cos(3x - 3t)
d. f(x,t) = 3 cos(4x - 2t)
e. f(x,t) = 3 cos(4x - 4t)
f. f(x,t) = 6 cos(2x - 2t)
g. f(x,t) = 6 cos(2x - 4t)
h. f(x,t) = 6 cos(3x - 3t)
i. f(x,t) = 6 cos(4x - 2t)
j. f(x,t) = 6 cos(4x - 4t)

1.19. Light going from a low index of refraction to a high index of refraction will always experience a 180° phase shift, regardless of the angle of the light ray relative to the boundary. At normal incidence that always happens, but not at grazing incidence. Brewster's angle marks the boundary (for p-polarized light.... s-polarized always has phase shift)

a. true
b. false

1.20. The critical angle for total internal reflection exists on both sides of a material interface.

a. true
b. false   Just from high n to low n

1.21. When you are designing a coating for a piece of glass that needs to minimize reflections for a given wavelength, there is only one coating thickness *d* that is allowed.

a. true
b. false   You get equations like $2d + \text{phaseshift} = (m + \frac{1}{2})\lambda$ ← lots of possible m's, so lots of possible d's.

1.22. When light diffracts through two wide slits, the resulting diffraction pattern will be the same as the pattern from a single wide slit, times the pattern of two infinitely narrow slits.

a. true   See extra credit problem from exam 3.
b. false

1.23. In transparent glass, which travels faster: red light (λ=630 nm) or green light (λ=500 nm)?

a. red light
b. green light
c. both travel at the same speed

green has higher n, so slower speed. $(v = \frac{c}{n})$

$\gamma = 2.294$

(10 pts) **Problem 2.** Disgusted with their final exam, the Physics 123 students gang up and ship Dr. Colton out from Earth on a fast rocket traveling at 0.90c, to live the remainder of his days in isolation from the rest of humanity. He lives for 60 more years (in his frame of reference), all the while wishing he had been kinder to the students. How far is he from the Earth when he dies (in the Earth's frame of reference)?

From Colton's point of view:

He travels 60 yrs, traveling at $\beta = .9$, so goes 54 ly

But distance is contracted to him, so in the Earth's frame, he traveled $54 \times \gamma = \boxed{123.9 \text{ ly}}$

From Earth point of view:

Colton travels more than 60 yrs, because he is time-dilated.

He travels for $\gamma \cdot 60$ yrs, going .9c the whole time. Therefore he went $\gamma \cdot 60 \cdot (.9c) = \boxed{123.9 \text{ ly}}$

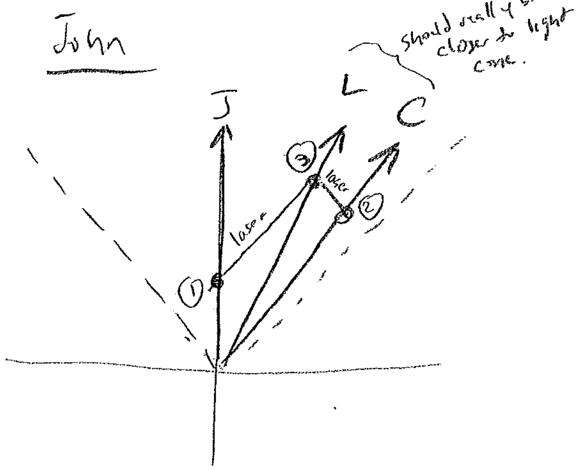
(13 pts) **Problem 3.** Lee is traveling at +0.7c relative to John. Cathy is traveling at +0.9c relative to Lee.

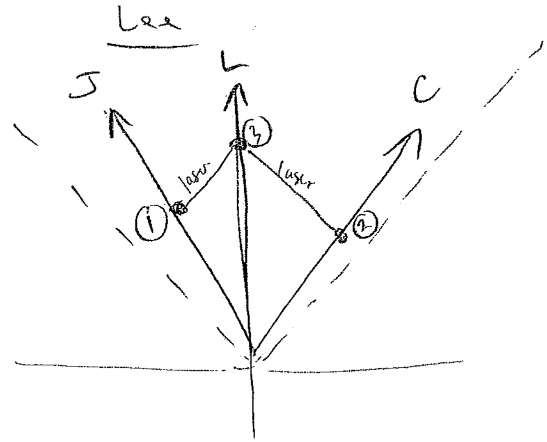
(a) How fast is Cathy traveling relative to John?

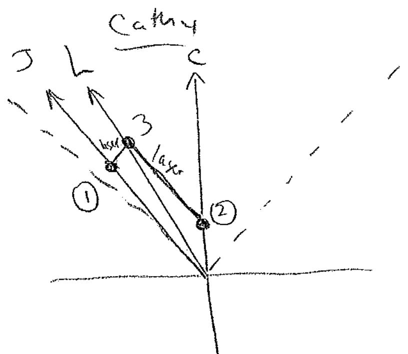
$$\beta_{CJ} = \frac{\beta_{CL} + \beta_{LJ}}{1 + \beta_{CL}\beta_{LJ}} = \frac{.9 + .7}{1 + (.9)(.7)} = \boxed{.982\,c}$$

(b) John and Cathy both fire laser guns at Lee, and coincidentally both of their shots (traveling at the speed of light) arrive at Lee at the same time. Draw the situation on three different space-time diagrams: from John's perspective, from Lee's perspective, and from Cathy's perspective. On each diagram label the world-lines from each of the three people, the two laser beams, and label these three events:

1 = John fires laser gun
2 = Cathy fires laser gun
3 = Lee gets hit by both guns







(c) In John's frame of reference, who fires first? In Lee's frame? In Cathy's frame? (You should be able to use your pictures from part (b) to answer this; no equations needed.)

John's frame: John fires first ($t_1 < t_2$)
Lee's frame: Cathy fires first ($t_2 < t_1$)
Cathy's frame: Cathy fires first ($t_2 < t_1$)

(12 pts) **Problem 4.**

(a) How fast would a 1 kg block need to be traveling in order to have as much momentum as a 0.999c electron?

almost certainly will be nonrelativistic

$\gamma = 22.366$

$$(mv)_{block} = (\gamma m v)_{electron}$$

$$(1\,kg)\, v_{block} = (22.366)(9.11\cdot 10^{-31}\,kg)(.999\cdot 3\cdot 10^{8}\,\tfrac{m}{s})$$

$$\boxed{v_{block} = 6.107\cdot 10^{-21}\ m/s}$$

(yep, non relativistic!)

(b) When you burn a match, about 1000 J of energy are released. If all of that energy goes into accelerating an electron (that is, it all gets turned into kinetic energy), how fast would that electron be traveling? Note: because your answer will be very close to the speed of light, please write your answer as, for example, $(1 - 7.7\times10^{-11})c$, instead of 0.999999999923c (or whatever the correct answer turns out to be). Hint: remember that $(1+x)^n \approx 1+nx$ when x = small.

$$KE = (\gamma - 1) mc^2$$

$$\gamma = 1 + \frac{KE}{mc^2} = 1 + \frac{1000}{(9.11\cdot 10^{-31})(3\cdot 10^{8})^2} = 1.220\cdot 10^{16}$$

Also, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$\frac{1}{\gamma^2} = 1 - \beta^2$$

$$\beta = \Big(1 - \underbrace{\tfrac{1}{\gamma^2}}_{\text{very small!}}\Big)^{1/2} = 1 - \frac{1}{2}\frac{1}{\gamma^2}$$

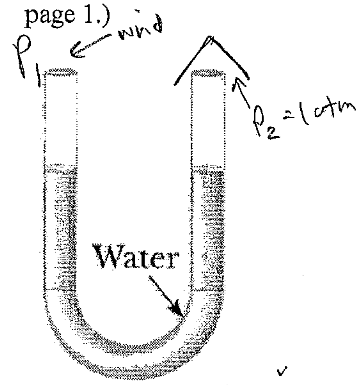
$$\beta = 1 - \frac{1}{2}\left(\frac{1}{1.22\cdot 10^{16}}\right)^2$$

$$\beta = 1 - 3.361\cdot 10^{-33}$$

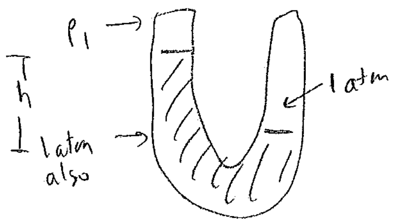
$$\boxed{v = (1 - 3.361\cdot 10^{-33})c}$$

(a) 9 pts
(b) 5 pts

(14 pts) **Problem 5.**
(a) A U-tube, open on both ends, is filled with water. The right end is then shielded while air is blown across the left end. This creates a decrease in pressure by the left end, which "sucks" the water up. How fast must the air be blown in order for the water in the left-hand section to end up 10 cm higher than the water in the right-hand section. (Densities of air and water can be found on page 1.)



after "sucked up"



Bernoulli: $P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g h_2} + \cancel{\frac{1}{2}\rho v_2^2}$

$$P_1 = 1\,atm - \tfrac{1}{2}\rho_{air} v_1^2$$

$$1\,atm - P_1 = \tfrac{1}{2}\rho_{air} v_1^2$$

Pressure change w/ depth:

$$1\,atm = P_1 + \rho_{water} g h$$

$$1\,atm - P_1 = \rho_{water} g h$$

Combine two eqns:

$$\tfrac{1}{2}\rho_{air} v_1^2 = \rho_{water} g h$$

$$v_1 = \sqrt{2gh\frac{\rho_{water}}{\rho_{air}}}$$

$$= \sqrt{\frac{2(9.8)(.10)(1000)}{1.29}} = \boxed{38.98\ \frac{m}{s}}$$

(b) An air bubble has a volume of 1 cm^3 when it is released by a diver 50 m below the surface of a lake. What is the volume of the bubble when it reaches the surface? Assume that the temperature and the number of air molecules in the bubble remain constant during the ascent.

$$PV = nRT = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)$$

pressure w/ depth: $P_1 = P_2 + \rho g h$

$$V_2 = (1\,cm^3)\left(\frac{1.01 \cdot 10^5 + 1000 \cdot 9.8 \cdot 50}{1.01 \cdot 10^5}\right) = 1\,cm^3 \left(\frac{5.91 \cdot 10^5}{1.01 \cdot 10^5}\right)$$

$$V_2 = \boxed{5.85\ cm^3}$$

(12 pts) **Problem 6.** An insulated chest has a surface area of 1.2 m^2 and a wall thickness of 3 cm. The temperature of the inner surface is 4°C, and the temperature of the outside surface is 28°C. The chest is made out of a material that has a thermal conductivity of 0.06 in standard SI units.

(a) What *are* the standard SI units for thermal conductivity?

Eqn: $P = \frac{kA}{\ell}\Delta T \rightarrow k = \frac{P\ell}{A\Delta T}$

units: $[k] = \frac{[W][m]}{[m^2][K]} \Rightarrow \boxed{\frac{W}{m \cdot K}}$ or $\boxed{\frac{W}{m \cdot {}^\circ C}}$ since ΔT in °C is same as ΔT in kelvin

(or just look on pg 1 of exam, where some values are given w/ units...)

(b) If 5 kg of ice at 0°C are put in the chest, how long (in hours) will it take for the ice to melt?

$$\frac{Q}{t} = \frac{kA\Delta T}{\ell}$$

for melting ice at 0°C, $Q = mL$

$$\frac{mL}{t} = \frac{kA\Delta T}{\ell}$$

$$t = \frac{mL\ell}{kA\Delta T}$$

$$= \frac{(5)(3.33 \cdot 10^5)(.03)}{(.06)(1.2)(24)}$$

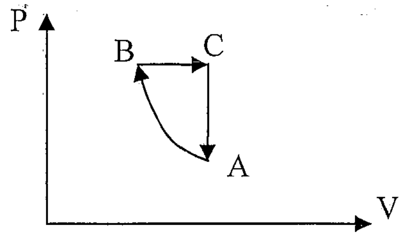
$$= 28906 \text{ seconds}$$

$$= \boxed{8.03 \text{ hrs}}$$

$\gamma = 7/5,\ C_p = \frac{7}{2}R,\ C_v = \frac{5}{2}R$

(16 pts) **Problem 7.** An engine using 0.12 moles of a diatomic ideal gas is driven by this cycle: starting from state A, the gas is compressed adiabatically until it reaches state B. Then, the gas is heated at constant pressure until it reaches state C. Finally, the gas is cooled at constant volume back to the original state. Various pressures, volumes, and temperature are given in the table.

	P (kPa)	V (m^3)	T (K)
A	100	0.0030	300
B	800	0.00068	545
C	800	0.0030	?



(a) What is the temperature of state C?

$PV = nRT \rightarrow T = \frac{PV}{nR} = \frac{(800\cdot 10^3)(.003)}{(.12)(8.31)} = \boxed{2407\text{ K}}$

Note: Alternate methods may give $T = 2400$K. That's because I have rounded the values given in the table.

(b) Find the heat added to the gas during each of the three legs.

A-B Adiabatic : $\boxed{Q = 0}$

B-C const P : $Q = nC_p\Delta T = (.12)(\frac{7}{2}\cdot 8.31)(2407-545) = \boxed{6499\text{ J}}$

$= 6474$ J if $T_c = 2400$K is used

C-A const V : $Q = nC_v\Delta T = (.12)(\frac{5}{2}\cdot 8.31)(300-2407) = \boxed{-5253\text{J}}$

$= -5235$ J if $T_c = 2400$K is used

(c) ~~(b)~~ How much net work is done by the gas each cycle?

$\Delta U_{cycle} = 0 \rightarrow W_{by} = Q_{net}$ from 1st Law

$W_{by} = 6499 - 5253 = \boxed{1246\text{ J}}$

$= 1239$ if $T_c = 2400$K is used

(d) ~~(c)~~ What is the efficiency of the engine?

$\text{eff} = \frac{W_{net}}{Q_h} = \frac{1246}{6499} = \boxed{19.2\%}$

$= 19.1\%$

(e) ~~(d)~~ What is the maximum theoretical efficiency for an engine operating between the same minimum and maximum temperatures?

$e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{2407} = \boxed{87.5\%}$

(12 pts) **Problem 8.** While attempting to tune the note C at 523.3 Hz, a piano tuner hears 2 beats/s between a reference oscillator (at 523.3 Hz) and the string.

$\hookrightarrow f = 523.3 \pm 2$

(a) When she tightens the string slightly, the beat frequency she hears rises smoothly to 3.5 beats/s. What is the frequency of the string now?

$\hookrightarrow$ f must have started higher than 523.3

Therefore f rose to 523.3 + 3.5 Hz

$= \boxed{526.8 \text{ Hz}}$

(b) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

$v = \sqrt{T/\mu}$

also $v = \lambda f$

$\lambda f = \sqrt{T/\mu}$

Easiest way is to take ratio:

$$\frac{\lambda_1 f_1 = \sqrt{\frac{T_1}{\mu_1}}}{\lambda_2 f_2 = \sqrt{\frac{T_2}{\mu_2}}} \Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

since λ and μ are constant

$$\frac{T_2}{T_1} = \left(\frac{f_2}{f_1}\right)^2$$

$$T_2 = \left(\frac{523.3}{526.8}\right)^2 \times T_1$$

$$T_2 = .9868\, T_1$$

Tension needs to be lowered by $\boxed{1.32\ \%}$

(14 pts) **Problem 9.** As a car drives past Steve, the driver sounds the horn. Steve has perfect pitch, and notices that as the car approached, the horn sounded the A above middle C (440.0 Hz). After the car passed, the pitch dropped down three half steps to an F-sharp. How fast was the car going?

3 half steps: going away freq was $\frac{440}{2^{3/12}} = \underline{369.99 \text{ Hz}}$

Doppler: $f' = f \frac{v \pm v_o}{v \mp v_s}$, $v_o = 0$

when car going towards: $f_{towards} = f \frac{v}{v - v_s}$

away $f_{away} = f \frac{v}{v + v_s}$

We don't know f, so easiest way to solve is to divide eqns

$$\frac{f_{towards}}{f_{away}} = \frac{\cancel{f} \frac{\cancel{v}}{v - v_s}}{\cancel{f} \frac{\cancel{v}}{v + v_s}} = \frac{v + v_s}{v - v_s}$$

Now solve for v_s

$$\frac{f_t}{f_a} v - \frac{f_t}{f_a} v_s = v + v_s$$

$$\left(\frac{f_t}{f_a} - 1\right) v = \left(\frac{f_t}{f_a} + 1\right) v_s$$

$$v_s = \frac{\frac{f_t}{f_a} - 1}{\frac{f_t}{f_a} + 1} \times v$$

$$= \frac{\frac{440}{369.99} - 1}{\frac{440}{369.99} + 1} \times 343 \frac{m}{s}$$

$$= \boxed{29.64 \frac{m}{s}}$$

(10 pts) **Problem 10**. You book lists this as the "lens-maker's equation":

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

However, when I consulted Wikipedia, http://en.wikipedia.org/wiki/Lens_(optics), I found a slightly different version of that equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n-1)d}{nR_1R_2}\right)$$

In that equation, d is the thickness of the lens. Assuming the Wikipedia equation is correct (which I believe it is), apparently the textbook's equation is making an approximation that the lens is infinitely thin. But, of course, lenses are not infinitely thin.

If I have a bi-convex lens (the normal lens shape, curved outward on both sides), with both radii of curvature equal to 10 cm in magnitude, and my lens is 0.6 cm thick, what is the difference in the focal lengths predicted by the two equations? (The index of refraction of the material is 1.55.)

Book eqn: $f=\left[(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]^{-1}$ I'll use cm for all distance units

$=\left[.55\left(\frac{2}{10\text{ cm}}\right)\right]^{-1} = \underline{9.091\text{ cm}}$

R_2 is negative, so both terms add

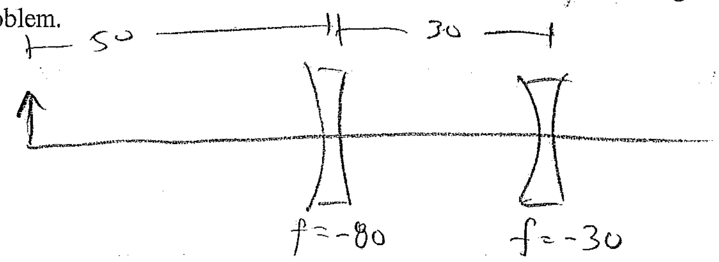
Wikipedia eqn: $f=\left[.55\left(\frac{2}{10}-\frac{.55\times.6}{1.55(10)(10)}\right)\right]^{-1} = 9.189$

minus sign because R_2 = negative

The difference in focal lengths is $\boxed{0.098\text{ cm}}$

Seems like the book eqn will work fine for all normal-sized lenses

(14 pts) **Problem 11.** An object is placed 50 cm to the left of lens 1 (diverging, f = –80 cm). Lens 1 is placed 30 cm to the left of lens 2 (diverging, f = –40 cm). How far (in magnitude) from lens 2 will the final image be formed? Will the image be to the left or the right of lens 2? Will it be real or virtual? What will be the total magnification? You do not have to provide any ray diagrams for this problem.



lens 1: $q_1 = \left(\frac{1}{f} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{-80} - \frac{1}{50}\right)^{-1} = \underline{-30.769}$

30.769 to <u>left</u> of lens 1

$M_1 = \frac{-q_1}{p_1} = -\frac{(-30.769)}{50} = \underline{+.615}$

lens 2: image of lens 1 = object of lens 2

= 60.769 to left of lens 2

$p_2 = +60.769$

$q_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-40} - \frac{1}{60.769}\right)^{-1} = \boxed{-24.12}$

24.12 to <u>left</u> of lens 2

$M_2 = \frac{-q_2}{p_2} = -\frac{(-24.12)}{60.769} = +.397$

$M_{tot} = M_1 \times M_2$
$= (.615)(.397) = \boxed{24.4\%}$

|q| = 24.12 cm (away from lens 2)

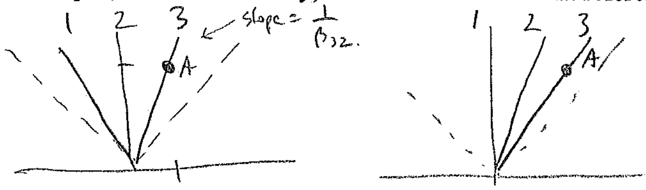
Final image will be to the left (left/right) of lens 2

real vs. virtual: virtual

total magnification: 24.4%

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)

(a) Use the Lorentz transformations to prove the relativistic velocity addition formula. *Hint*: Pick an arbitrary velocity in reference frame 1, find a point on the world-line corresponding to that velocity, do the Lorentz transformation to reference frame 2, then find the slope (and hence velocity) of the world-line in that reference frame.



Step 1: pick a pt on worldline 3, I'll choose $(1, \frac{1}{\beta_{32}})$

Step 2: transform to frame 1, "β" = β_{21}

$$\begin{pmatrix} x \\ ct \end{pmatrix}_A = \begin{pmatrix} \gamma & +\gamma\beta \\ +\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} 1 \\ 1/\beta_{32} \end{pmatrix} = \begin{pmatrix} \gamma + \gamma\beta/\beta_{32} \\ \gamma\beta + \gamma/\beta_{32} \end{pmatrix}$$

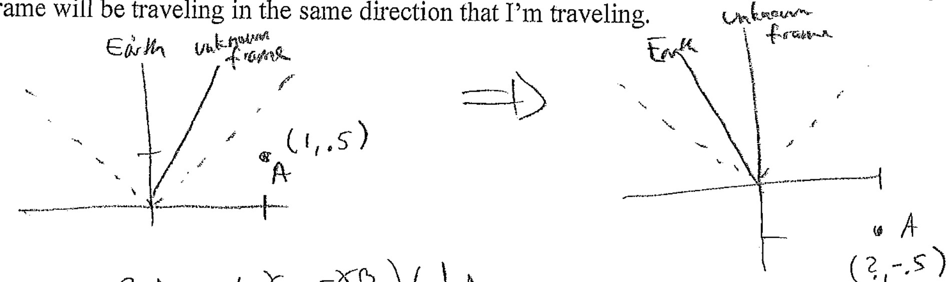
Step 3. That gave us coords of pt A on worldline 3, in frame 1

Speed of that worldline $= \frac{1}{\text{slope}} = \frac{x \text{ coord}}{ct \text{ coord}}$

$$\beta_{31} = \frac{\gamma + \gamma\beta/\beta_{32}}{\gamma\beta + \gamma/\beta_{32}} \times \frac{\beta_{32}}{\beta_{32}}.$$ The γ's cancel!

$$\boxed{\beta_{31} = \frac{\beta_{32} + \beta_{21}}{\beta_{32}\beta_{21} + 1}}$$ Yay! The formula I gave in class.

(b) Suppose I developed the ability to travel faster than the speed of light. That means (in the Earth frame of reference) I could travel one light year in 0.5 years. This would cause a tremendous problem for causality! Namely, the endpoint of my trip and the start of my trip will have a "spacelike" relationship to each other instead of a "timelike" relationship. And, as I claimed in class, if two points on a space-time diagram have a spacelike relationship, then one can always find a reference frame where one point happened before the other, and another reference frame where it's the other way around. Therefore, there is some reference frame where my trip's endpoint happened before my trip's start! In that frame, cause and effect are reversed! In fact, there are an infinite number of such reference frames. Find the speed of the specific reference frame where I arrived 0.5 years before I left. *Hint*: that reference frame will be traveling in the same direction that I'm traveling.



$$\begin{pmatrix} ? \\ -.5 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} 1 \\ .5 \end{pmatrix}$$

Just look at 2nd row:

$-.5 = -\gamma\beta + .5\gamma$

Need to solve for β, so plug in for $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$-.5 = -\frac{\beta}{\sqrt{1-\beta^2}} + \frac{.5}{\sqrt{1-\beta^2}}$

$-.5\sqrt{1-\beta^2} = -\beta + .5$

$.25(1-\beta^2) = \beta^2 - \beta + .25$

$\cancel{.25} - .25\beta^2 = \beta^2 - \beta + \cancel{.25}$

$1.25\beta^2 - \beta = 0$

$\beta(1.25\beta - 1) = 0$

$\beta = 0$ not correct or $1.25\beta - 1 = 0$

$\beta = \frac{1}{1.25}$

$$\boxed{\beta = .8}$$