Fall 2016
Physics 441
Exam 3
Dr. Colton, cell: 801-358-1970

No time limit. Student calculators are allowed. One page of <u>handwritten</u> notes allowed (front & back). Books not allowed. A HANDOUT WITH FRONT AND BACK INSIDE COVERS OF GRIFFITHS TEXTBOOK SHOULD BE PROVIDED. If not, please ask the Testing Center for it and/or have them call me.

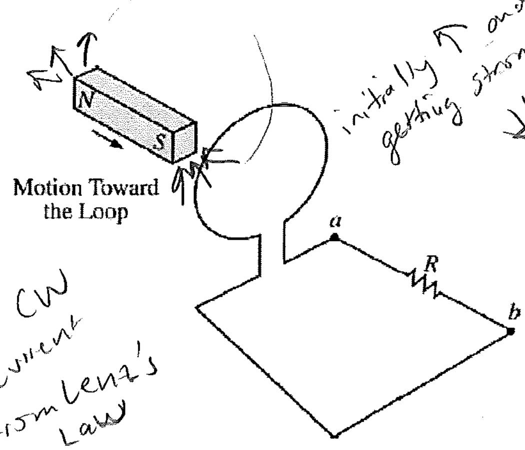
Name Solutions

Instructions: Please label & circle/box your answers. **Show your work**, where appropriate! And remember: **in any problems involving Gauss's Law, you should explicitly show your Gaussian surface**. For all problems, unless otherwise specified you may assume that you are dealing with **electrostatics**, i.e. the charges are not moving and the fields have come to equilibrium.

(No special math formulas needed for this exam.)

(20 pts) **Problem 1:** Multiple choice, 2 pts each. Circle the correct answer.

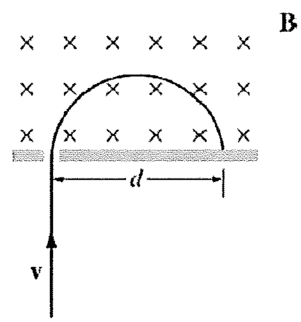
Later: ↑ is strong but getting weaker
⇓
causes CW current from Lenz's law



initially ↑ and getting stronger
⇓
causes CCW current from Lenz's law

1.1. The bar magnet shown in the figure is moved completely through the loop. Which of the following is a true statement about the direction of the current flow between the two point *a* and *b* in the circuit? (Hint: field lines from permanent magnets go from north pole to south pole.)
(a) No current flows between *a* and *b* as the magnet passes through the loop.
(b) Current flows from *a* to *b* as the magnet passes through the loop.
(c) Current flows from *b* to *a* as the magnet passes through the loop.
(d) Current flows from *a* to *b* as the magnet enters the loop and from *b* to *a* as the magnet leaves the loop.
(e) Current flows from *b* to *a* as the magnet enters the loop and from *a* to *b* as the magnet leaves the loop.

1.2. A particle with mass *m* and charge *q*, moving with a velocity *v*, enters a region of uniform magnetic field *B*, as shown in the figure. The particle strikes the wall at a distance *d* from the entrance slit. If the particle's velocity stays the same but its charge-to-mass ratio is doubled, at what distance from the entrance slit will the particle now strike the wall?



(a) $2d$
(b) $\sqrt{2}d$
(c) d
(d) $\frac{1}{\sqrt{2}}d$
(e) $\frac{1}{2}d$

$\sum F = ma_c = m\frac{v^2}{R}$

$q\cancel{v}B = \frac{mv^{\cancel{2}}}{R}$

$R = \frac{mv}{qB}$

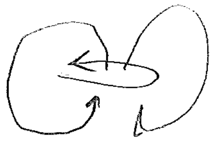
if q/m is doubled, R is halved

1.3. A current *I* in a circular loop of radius *b* produces a magnetic field. At a fixed point far from the loop, the strength of the magnetic field is proportional to which of the following combinations of *I* and *b*?
(a) Ib
(b) Ib^2
(c) I^2b
(d) $\frac{I}{b}$
(e) $\frac{I}{b^2}$

dipole approximation

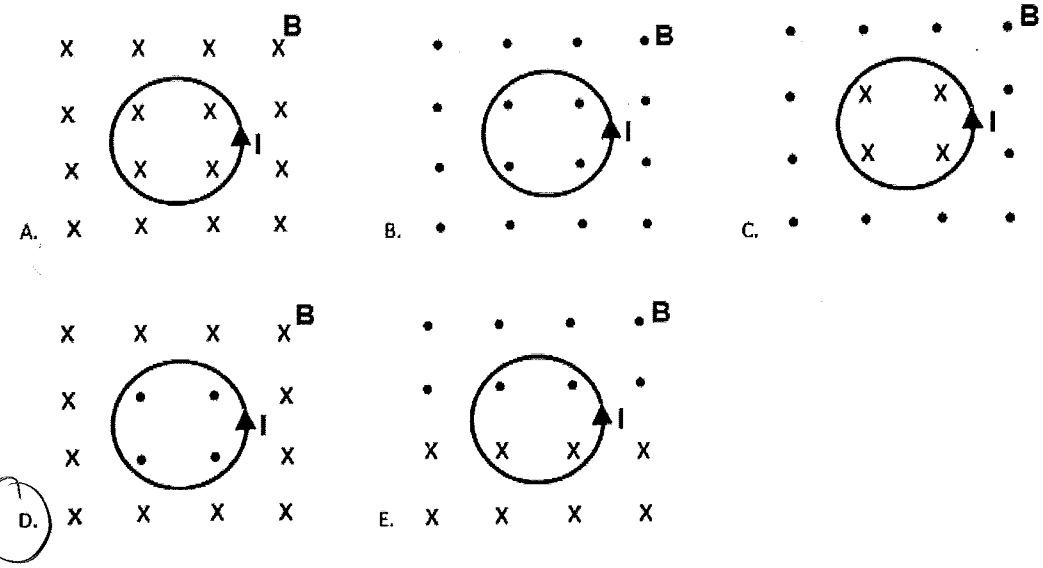
$B \sim m$, dipole moment

$m = I \times \text{area}$
$= I \pi b^2$

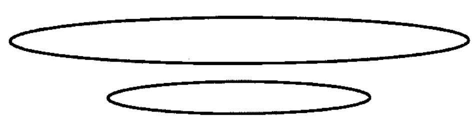


field of a loop, goes up (out of page) inside loop, goes down (into page) outside loop

1.4. Which of the following diagrams represents the magnetic field due to a circular current? (X's mean going into the page, dots mean coming out of the page. The symbols indicate only the direction of field; not necessarily its magnitude.)



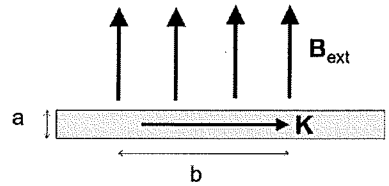
1.5. Which of these statements is not true of a static magnetic field?
(a) It can be generated by a DC current. True
(b) It can produce an electric field. False → needs to be changing magnetic field
(c) It doesn't have sinks or sources. True (no mono-poles)
(d) Magnetic flux lines are always closed. True (" ")
(e) None of the above (all are characteristic of static magnetic fields).



1.6. A large circular conducting loop is positioned directly over a smaller loop, as shown. What is true about the mutual inductance between the two loops?
(a) The effect of the larger loop on the smaller loop is greater than the effect of the smaller one on the larger one.
(b) The effect of the smaller loop on the larger loop is greater than the effect of the larger one on the smaller one.
(c) The effect of the larger loop on the smaller loop is equal to the effect of the smaller one on the larger one.
(d) Which effect is larger depends on other factors, such as loop positions and orientations.

As shown in class, $M_{12} = M_{21}$

1.7. A "ribbon" (width a, length b, and infinitely thin in the third dimension) with a uniform surface current density **K** to the right is in a uniform magnetic field $\mathbf{B}_{ext}$, oriented as shown. What is the magnitude of the force on the ribbon?



(a) KB_{ext}
(b) aKB_{ext}
(c) $abKB_{ext}$
(d) bKB_{ext}/a
(e) $KB_{ext}/(ab)$

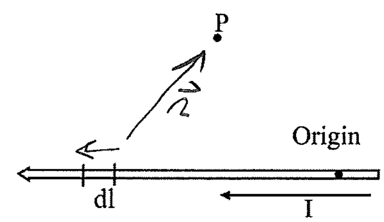
Force on a current $\vec F = I\vec l \times \vec B$

Here $I = Ka$ (because K is constant, otherwise $\int K\,dy$)

$l = b$

and $\vec l$ is $\perp$ to $\vec B$, so $F = KabB$

1.8. To find the magnetic field **B** at the point P due to a current-carrying wire we use the Biot-Savart law. What is the direction of the infinitesimal contribution d**B** created by current in the section of wire, d**l**, that is shown?



(a) Upwards
(b) Up and to the left
(c) Up and to the right
(d) Downwards
(e) Down and to the left
(f) Down and to the right
(g) Into the page
(h) Out of the page

$\vec F = \frac{\mu_0 I}{4\pi}\int \frac{d\vec l \times \vec r}{r^3}$

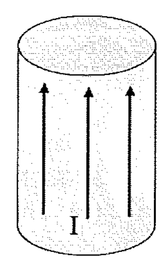
$d\vec l \times \vec r$ is into the page

1.9. With regards to the vector potential **A** (in general, i.e. not just in the Coulomb gauge), which of the following is continuous as you move past a boundary?

(a) **A**
(b) Not all of **A**, just the perpendicular component
(c) Not all of **A**, just the parallel component
(d) Nothing is guaranteed to be continuous regarding **A** for this situation

$\vec A$ is continuous, since $\vec B = \vec\nabla \times \vec A$ would go infinite if $\vec A$ were not continuous. (and then $\vec F$ would be infinite, $\vec F = q\vec v \times \vec B$)

1.10. A very long aluminum (paramagnetic!) rod carries a uniformly distributed current I along the +z direction. What is the direction of the bound <u>surface</u> current? Hint: the current will set up a B field inside the rod (as well as outside); that B field will induce a magnetization.



(a) It points parallel to I
(b) It points antiparallel to I
(c) It points clockwise (viewed from above)
(d) It points counter-clockwise (viewed from above)
(e) It is zero

The $\vec B$ field is (CCW viewed from above, $= \hat\phi$)

For paramag materials, $\vec B$, $\vec H$, and $\vec M$ all point in the same direction, so $\vec M$ is ($\hat\phi$)

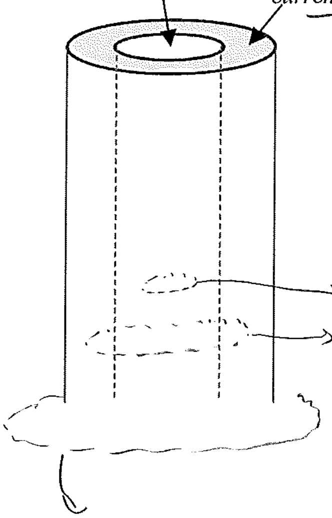
Bound Surface current $\vec K_b = \vec M \times \hat n$

$= \hat\phi \times \hat s = -\hat z$ direction

since (s, ϕ, z)

$\hat\phi \times \hat s$ is right to left

(16 pts) **Problem 2.** A long, straight, hollow cylindrical wire has an inner radius R and an outer radius $2R$. In between the inner and outer radii there is a uniform volume current density in the z-direction as shown. There is no surface current density. The total current is I. Determine the magnitude of the magnetic field produced by this current, as a function of the distance from the center of the wire, s, and make a reasonably accurate sketch of the result.



$J = \frac{I}{\text{area}}$ since J is constant

$= \frac{I}{\pi(2R)^2 - \pi R^2} = \frac{I}{3\pi R^2}$

$s<R$ Amperian loop $\quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = 0$

$B\cdot 2\pi s = 0 \rightarrow \boxed{B=0} \quad s<R$

$R<s<2R$ loop

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$I_{enc} = J \times \text{area}$ (where there is current) since J = constant

$= J\cdot(\pi s^2 - \pi R^2)$

$= \frac{I}{3\pi R^2}(\pi s^2 - \pi R^2)$

$B\cdot 2\pi s = \mu_0 \frac{I}{3\cancel{\pi} R^2}(\cancel{\pi} s^2 - \cancel{\pi} R^2)$

$$\boxed{\vec{B} = \frac{\mu_0 I}{6\pi R^2}\,\frac{1}{s}\,(s^2 - R^2)\,\hat{\phi}} \quad R<s<2R$$

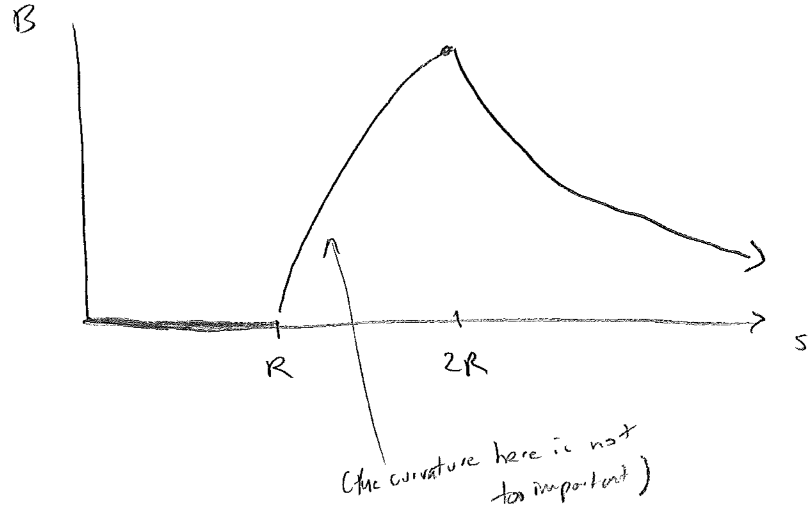
$s>2R$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$B\cdot 2\pi s = \mu_0 I$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}} \quad s>2R$$

Sketch



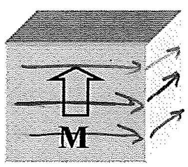
(12 pts) **Problem 3**. A cube of side a is centered on the origin. It is made out of a ferromagnetic material with a permanent magnetization, $\mathbf{M} = M_0\hat{\mathbf{z}}$. (a) What bound currents are implied by this magnetization? Make a sketch and calculate the magnitude(s). (b) Approximately what is the magnetic field produced by this cube at a point, (x, y, z), a large distance away from the cube? You can give your answer in terms of the usual spherical coordinates r, θ, and ϕ and the spherical unit vectors, if you like.

• (x, y, z)

(a) $\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$ since $\vec{M}$ is constant

$\vec{K}_b = \vec{M} \times \hat{n} = 0$ for top + bottom

$=$ as shown in figure for sides, magnitude M_0



surface currents flow around surface like

(b) Large distance : dipole approx

$m = \int M \, d\tau = M \times \text{volume}$

$m = M_0 a^3$

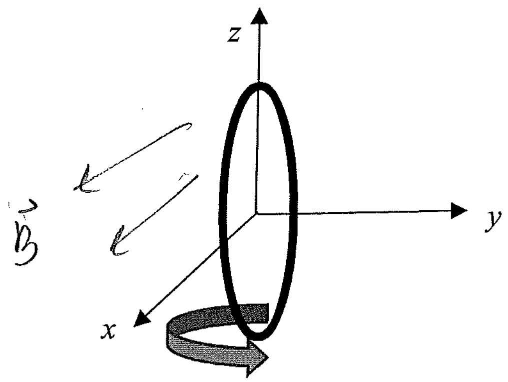
$$\vec{B}_{dipole} = \frac{\mu_0}{4\pi} \frac{m}{r^3} \left(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right)$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{M_0 a^3}{r^3} \left(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta}\right)}$$

where $r, \theta, \hat{r}$ and $\hat{\theta}$ could be determined from (x, y, z) if we really needed to (but the problem says not needed)

(12 pts) **Problem 4.** A conducting circular loop (radius R) is centered at the origin, and originally positioned such that it lies in the x-z plane; see the figure. There is a constant magnetic field present in the area, $\mathbf{B} = B_0\hat{\mathbf{x}}$ (not shown). The loop is then rotated about the z-axis counter-clockwise as viewed from above, at an angular frequency ω.

(a) Find the induced EMF in the loop as a function of time.



$$\Phi = \int \vec{B} \cdot d\vec{a}$$

$$= B_0 \pi R^2 \sin \omega t$$

from the dot product, combined with a rotation in the angle

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

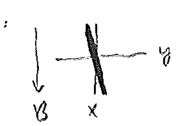
$$\boxed{\mathcal{E} = -B_0 \omega \pi R^2 \cos \omega t}$$

(b) After one full rotation, the loop returns to its original position (and continues to rotate). What direction is the current in the loop—clockwise or counter clockwise as viewed from the positive y axis on the right?

The flux is a minimum (zero) but increasing.

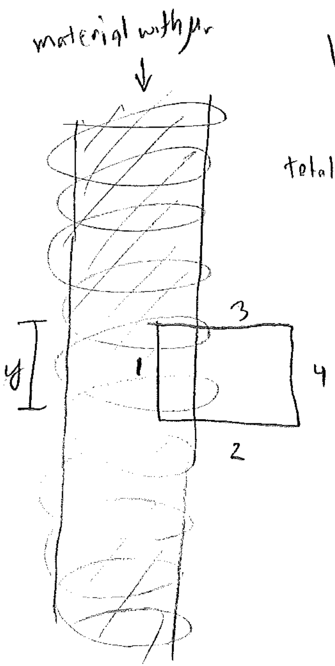
Lenz's Law: current will try to reduce the flux

Top view:



To reduce flux, current goes out of page / into page } this is **CCW as viewed from the right**

(16 pts) **Problem 5**. Derive the equation for inductance of an almost infinite solenoid filled with a material of relative permeability μ_r: $L = \mu_r\mu_0 n^2 lA$, where n is the number of turns per length, l is the length, and A is the cross sectional area. Be clear (and accurate!) about each step.



$L = \frac{\Phi}{I}$ by definition

total Φ = $B \cdot A \cdot$ # loops

B found through H : $B = \mu_0\mu_r H$

H found through Ampere's Law

$\oint \vec{H} \cdot d\vec{l} = I_{\text{free enclosed}}$

$\hookrightarrow = I \times$ # loops
$\hookrightarrow = ny$ } $I_{\text{free enc}} = Iny$

$\underbrace{\int_1}_{H \cdot y} + \underbrace{\int_2 + \int_3}_{\text{zero, from the dot product (B will be in z direction)}} + \underbrace{\int_4}_{\text{zero, because infinite solenoid will have neglible field outside}}$

piecing together Amp's Law

$H \cancel{y} = In\cancel{y}$

$H = nI$

then $B = \mu_0\mu_r H$

$B = \mu_0\mu_r nI$

then $\Phi = BA \cdot (nl)$

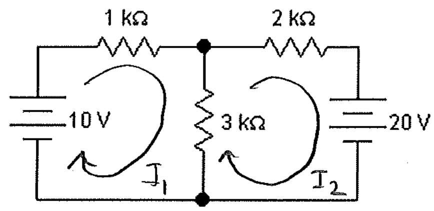
$= (\mu_0\mu_r nI) A (nl)$

$= \mu_0\mu_r n^2 IAl$

then $L = \frac{\Phi}{I}$

$\boxed{L = \mu_0\mu_r n^2 I l}$

(10 pts) **Problem 6**. (a) Write down the two KVL mesh-current equations for this circuit, in terms of the left hand loop's virtual current I_1 and the right hand loop's virtual current I_2. Make each loop go counter-clockwise, as we did in class.



left loop: $+10 - 1000 I_1 - 3000 I_1 + 3000 I_2 = 0$

right loop: $-3000 I_2 + 3000 I_1 - 2000 I_2 - 20 = 0$

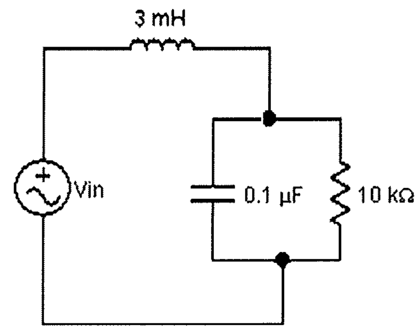
(b) Supposing you solve for I_1 and I_2 (you don't have to do this), how would you determine the voltage of the 3 kΩ resistor? Assume the top of the resistor is at a higher voltage than the bottom.

Ohm's Law $V = I R$

$I = I_1 - I_2$ (assuming actual current flow is top to bottom)

$$\boxed{V_{3k} = (I_1 - I_2) \cdot 3000}$$

(14 pts) **Problem 7**. For $V_{in} = 5\cos(1300t)$, assuming steady-state conditions, find (a) the amplitude and phase of the *current* through the inductor, and (b) the amplitude and phase of the *voltage* of the inductor. Give your answers in terms of real quantities, not complex. (Hint for part b: apply the complex version of Ohm's law to the inductor, and recall that $i = e^{i\pi/2}$.) If you don't want to go through the complex number algebra by hand to put the complex impedance into polar form, that's fine—just be clear about which steps you'd turn to Mathematica for to get which quantities, and what commands you would use. Put the rest of your work in terms of those quantities.



$$Z_{tot} = Z_L + Z_C \parallel Z_R$$

$$= i\omega L + \left(\frac{1}{\frac{1}{i\omega C}} + \frac{1}{R}\right)^{-1}$$

$$Z_{tot} = i\omega L + \left(i\omega C + \frac{1}{R}\right)^{-1}$$

type this into Mathematica with $\omega = 1300$, $L = 3\cdot 10^{-3}$, $C = .1\cdot 10^{-6}$, $R = 10\cdot 10^{3}$

then use $\left.\begin{aligned} Z_{abs} &= \text{Abs}[Z_{tot}] \\ \phi &= \text{Arg}[Z_{tot}] \end{aligned}\right\}$ to get in polar form

(a) then $\tilde{I}_{tot} = \dfrac{\tilde{V}_0}{\tilde{Z}} \rightarrow \boxed{I_{magn.} = \dfrac{5}{Z_{abs}}}$

$\boxed{\text{phase of } I = -\phi}$

(b) Ohm's law for inductor

$$\tilde{V}_L = \tilde{I} Z_L$$

$$= \left(\frac{5}{Z_{abs}} e^{-i\phi}\right)(\underbrace{i}_{e^{i\pi/2}} \omega L)$$

$$= \frac{5\omega L}{Z_{abs}} e^{i(\pi/2 - \phi)}$$

Amplitude is $\boxed{\dfrac{5\omega L}{Z_{abs}}}$ phase is $\boxed{\pi/2 - \phi}$