Parallel Equations for the Electric and Magnetic Fields

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ELECTRIC

Statics

1. $q = \int \lambda dl$
   $q = \int \sigma da$
   $q = \int \rho d\tau$
2. $\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0} \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}$
3. $\mathbf{F} = Q\mathbf{E}$
4. $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i \boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3}$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\mathbf{r}') \boldsymbol{\mathscr{r}}}{\mathscr{r}^3} dl'$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(\mathbf{r}') \boldsymbol{\mathscr{r}}}{\mathscr{r}^3} da'$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\mathbf{r}') \boldsymbol{\mathscr{r}}}{\mathscr{r}^3} d\tau'$
5. $\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}}$
6. $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a}$
7. $\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$
8. $\nabla \times \mathbf{E} = 0$ (this gets modified below)
9. $\mathbf{E} = -\nabla V$ (this gets modified in Phys 442)
10. $V(\mathbf{r}) = -\int_{\mathbf{ref}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$
11. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{\mathscr{r}_i}$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\mathbf{r}')}{\mathscr{r}} dl'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\mathbf{r}')}{\mathscr{r}} da'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{\mathscr{r}} d\tau'$
12. $U = \frac{1}{2} \sum_i q_i V(\mathbf{r}_i)$
    $U = \frac{1}{2} \int \rho(\mathbf{r}') V(\mathbf{r}') d\tau'$
    $U = \frac{\varepsilon_0}{2} \int E^2 d\tau$
13. $C = \frac{Q}{V}$
14. $U = \frac{1}{2} \frac{Q^2}{C}$
15. $E_1^\perp - E_2^\perp = \frac{\sigma}{\varepsilon_0}$
16. $\mathbf{E}_1^\parallel = \mathbf{E}_2^\parallel$

MAGNETIC

Statics

1. $I = \int K_\perp dl$
   $I = \int \mathbf{J} \cdot d\mathbf{a}$
2. No easy parallel for magnetic field
3. $\mathbf{F} = Q(\mathbf{v} \times \mathbf{B})$
4. $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I\, d\mathbf{l}' \times \boldsymbol{\mathscr{r}}}{\mathscr{r}^3}$
   $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_S \frac{\mathbf{K}(\mathbf{r}') \times \boldsymbol{\mathscr{r}}}{\mathscr{r}^3} da'$
   $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') \times \boldsymbol{\mathscr{r}}}{\mathscr{r}^3} d\tau'$
5. $\boxed{\nabla \cdot \mathbf{B} = 0}$
6. $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{a}$
7. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$
8. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (this gets modified below)
9. $\mathbf{B} = \nabla \times \mathbf{A}$, $\nabla \cdot \mathbf{A} = 0$ (Coulomb gauge)
10. No direct parallel for the magnetic field
11. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{\mathscr{r}} d\mathbf{l}'$
    $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{\mathscr{r}} da'$
    $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} d\tau'$
12. $U = \frac{1}{2} \int \mathbf{J}(\mathbf{r}') \cdot \mathbf{A}(\mathbf{r}') d\tau'$
    $U = \frac{1}{2\mu_0} \int B^2 d\tau$
13. $L = \frac{\Phi}{I}$
14. $U = \frac{1}{2} L I^2$
15. $B_1^\perp = B_2^\perp$
16. $\mathbf{B}_1^\parallel - \mathbf{B}_2^\parallel = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$

17. $V_1 = V_2$
18. $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$
19. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\mathbf{r}')d\tau'$

17. $\mathbf{A}_1 = \mathbf{A}_2$
18. $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$
19. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\theta')d\mathbf{l}'$

Materials

20. $\mathbf{p} = \sum_i \mathbf{r}_i' q_i$
$\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau'$
21. $V_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{\mathrm{r}^2}$
22. $\mathbf{E}_{\text{dip}}(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$
23. $\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$ (τ = torque here, not volume)
24. $\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E} = \nabla(\mathbf{p}\cdot\mathbf{E})$
25. $U = -\mathbf{p}\cdot\mathbf{E}$
26. $\mathbf{P}$ = electric dipole moment per unit volume
$\mathbf{p} = \int \mathbf{P}(\mathbf{r}')d\tau'$
27. $V_{\text{pol.object}} = \frac{1}{4\pi\varepsilon_0}\int \frac{\mathbf{P}(\mathbf{r}')\cdot\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}d\tau'$
28. $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$
29. $\rho_b = -\nabla\cdot\mathbf{P}$
30. $\mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$ (if linear, isotropic)
31. $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$
32. $\boxed{\nabla\cdot\mathbf{D} = \rho_f}$
33. $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E}$ (if linear, isotropic)
34. $\oint \mathbf{D}\cdot d\mathbf{a} = Q_{f,enc}$
35. $\varepsilon_r = 1 + \chi_e$ (if linear, isotropic)
36. $U = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\,d\tau$
37. $D_1^{\perp} - D_2^{\perp} = \sigma_{free}$

Materials

20.
$\mathbf{m} = I\int d\mathbf{a} = I\mathbf{a}$
21. $\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{\boldsymbol{r}^2}$
22. $\mathbf{B}_{\text{dip}}(r,\theta) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$
23. $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$
24. $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$
25. $U = -\mathbf{m}\cdot\mathbf{B}$
26. $\mathbf{M}$ = magnetic dipole moment per unit vol.
$\mathbf{m} = \int \mathbf{M}(\mathbf{r}')d\tau'$
27. $\mathbf{A}_{\text{magn.object}} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{M}(\mathbf{r}')\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}d\tau'$
28. $\mathbf{J_b} = \nabla\times\mathbf{M}$
29. $\mathbf{K_b} = \mathbf{M}\times\hat{\mathbf{n}}$
30. $\mathbf{M} = \chi_m\mathbf{H}$ (if linear, isotropic)
31. $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$
32. $\boxed{\nabla\cdot\mathbf{B} = 0}$ (unchanged for materials)
33. $\mathbf{H} = \mathbf{B}/\mu = \mathbf{B}/(\mu_0\mu_r)$ (if linear, isotropic)
34. $\oint \mathbf{H}\cdot d\mathbf{l} = I_{f,enc}$
35. $\mu_r = 1 + \chi_m$ (if linear, isotropic)
36. $U = \frac{1}{2}\int \mathbf{H}\cdot\mathbf{B}\,d\tau$
37. $\mathbf{H}_1^{\parallel} - \mathbf{H}_2^{\parallel} = \mathbf{K}_{\boldsymbol{free}}\times\hat{\mathbf{n}}$

Dynamics

38. $\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$
39. $\boxed{\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}}$
40. $\boxed{\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}}$ (unchanged for materials)

Dynamics

38. [same equation as in left hand column; connects charge to current]
39. $\boxed{\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}}$
40. $\boxed{\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}}$ (for materials)