**Fall 2018** barcode here

**Physics 441**

**Exam 2**

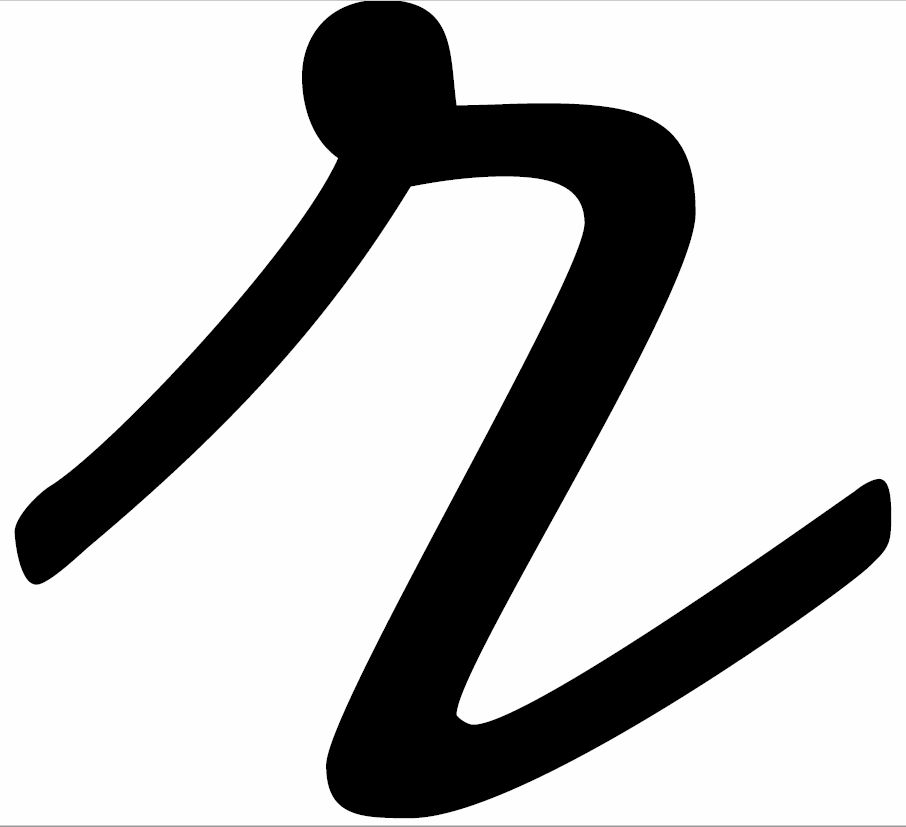
**Dr. Colton, cell: 801-358-1970**

**No time limit. Student calculators are allowed. One page of handwritten notes allowed (front & back). Books not allowed.**

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Instructions:* Please label & circle/box your answers. **Show your work**, where appropriate! And remember: **in any problems involving Gauss’s Law, you should explicitly show your Gaussian surface**. For all problems, unless otherwise specified you may assume that you are dealing with **electrostatics**, i.e. the charges are not moving and the fields have come to equilibrium.

*Griffiths front and back covers*

(similar things true for )

|  |  |
| --- | --- |
| Special case derivatives: |  |
|  |  |

Some miscellaneous mathematical stuff:

The first few Legendre polynomials:

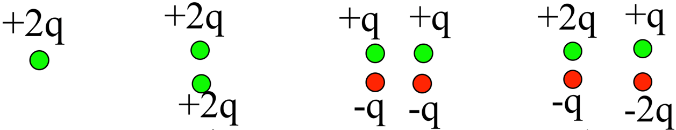
Some indefinite integrals:

Some definite integrals:

(20 pts) **Problem 1**. Multiple choice, 1.5 pts each. (Plus you get a free 0.5 point. ☺) Circle the correct answers for the multiple choice questions.

* 1. For an infinitesimal dA on the bottom surface of a cylinder, what would be the appropriate formula to use in cylindrical coordinates?
  2. Poisson’s equation tells us that . If the charge density throughout some volume is zero, what else must be true throughout that volume?

1. *V* = 0
2. E = 0
3. Both *V* and E must be zero
4. None of the above is necessarily true.
   1. Which of the following is true of Laplace’s equation?
5. It can have more than one solution for a given set of boundary conditions.
6. The solutions in one dimension must be sines and cosines (or a linear combination).
7. It is only valid in regions of space that contain no charges.
8. It requires that nowhere within the region of interest can the potential be zero
9. More than one of the above.
   1. In class we solved Laplace’s equation in 2D, obtaining *V*(*x*, *y*). You have also worked one or more 2D problems for homework. Which of the following is true about situations like that where we can write *V* is a function of *x* and *y* only?
10. The third dimension must be infinitely thin, or close enough that it’s a good approximation.
11. The third dimension must be finite.
12. The third dimension must be infinitely long, or close enough that it’s a good approximation.
    1. A cube of side length *a* is somehow maintained at the following potential: *V* = 0 for all sides except for one, and is a constant on that side. The formula for the potential inside the surface will involve:
13. An infinite sum of sines/cosines and/or exponentials.
14. A finite sum of sines/cosines and/or exponentials.
15. An infinite sum of Legendre polynomials in .
16. A finite sum of Legendre polynomials in .
17. An infinite sum of Bessel functions.
18. A finite sum of Bessel functions.
    1. A localized charge distribution has no net charge, zero electric dipole moment, but a nonzero electric quadrupole moment. At a large distance *r* from the distribution, the electric field will fall off like:
19. No fall off; it’s constant
20. 1/*r*
21. 1/*r*2
22. 1/*r*3
23. 1/*r*4
24. 1/*r*5
    1. Which of these charge distributions produce an electric potential which varies as 1/*r*2 when you get far away?



(i) (ii) (iii) (iv)

1. (i) only
2. (ii) only
3. (iii) only
4. (iv) only
5. (i) and (ii)
6. (i) and (iii)
7. (i) and (iv)
8. (ii) and (iii)
9. (ii) and (iv)
10. (iii) and (iv)
    1. Points A and B are the same large distance from an electric dipole, but in different directions as per the figure. The dipole is depicted by the arrow (dipole moment in the direction of the arrow). What is true of the magnitude of the E field at point A compared to point B?



A

B

1. are both
2. ,
3. ,
4. (and both )
5. (and both )
6. (and both )
   1. A dielectric is placed in an external electric field whose magnitude was *E0* before the dielectric was put in the region. (Initially the region contained only a vacuum.) Which is true about the total electric field inside the dielectric, Etot?
7. *Etot* < *E0*, and they are in the same direction
8. *Etot* = *E0*, and they are in the same direction
9. *Etot* > *E0*, and they are in the same direction
10. *Etot* < *E0*, and they are in the opposite direction
11. *Etot* = *E0*, and they are in the opposite direction
12. *Etot* > *E0*, and they are in the opposite direction
    1. Same situation. *D*0 is the magnitude of the D field before the dielectric was put in the region. What is true about the total D field inside the dielectric, Dtot? (Assume no edge effects, if it matters.)
13. *Dtot* < *D0*, and they are in the same direction
14. *Dtot* = *D0*, and they are in the same direction
15. *Dtot* > *D0*, and they are in the same direction
16. *Dtot* < *D0*, and they are in the opposite direction
17. *Dtot* = *D0*, and they are in the opposite direction
18. *Dtot* > *D0*, and they are in the opposite direction
    1. Same situation. What is true about how the D field compares to the E field inside the dielectric?
19. *Dtot* < *ϵ*0*Etot*, and they are in the same direction
20. *Dtot* = *ϵ*0*Etot*, and they are in the same direction
21. *Dtot* > *ϵ*0*Etot*, and they are in the same direction
22. *Dtot* < *ϵ*0*Etot*, and they are in the opposite direction
23. *Dtot* = *ϵ*0*Etot*, and they are in the opposite direction
24. *Dtot* > *ϵ*0*Etot*, and they are in the opposite direction
    1. A boundary exists between two linear dielectric materials, i.e. *ϵr* has two different values above and below the boundary. There are no free charges in the region considered. What must be continuous across the boundary?

i) *E*// ii) *E*⊥ iii) *D*// iv) *D*⊥

1. (i) only
2. (ii) only
3. (iii) only
4. (iv) only
5. (i) and (ii)
6. (i) and (iii)
7. (i) and (iv)
8. (ii) and (iii)
9. (ii) and (iv)
10. (iii) and (iv)
    1. Which of the following is true of bound charges?
11. Bound charges mark the start and end of the displacement field.
12. Bound charges are free to move within an object but cannot be transferred from one object to another object.
13. Bound charges are normally found in conductors.
14. Bound charges are the result of dipole moments related to a material.
15. Bound charges can exist in the absence of a polarization field.
16. More than one of the above.

(11 pts) Problem 2. Short answers.

(a) A positive charge +*q* is close to two grounded conducting semi-infinite planes, as shown (the arrows are conductors which extend infinitely into and out of the page, as well as infinitely in the direction of the arrows).

*+q*

Draw the appropriate image charge configuration that could be used to determine the electric potential near the charge *q*. How do you know your answer is the appropriate configuration? Be specific. *Hint*: think about the drawing to the right.

*+q*

X

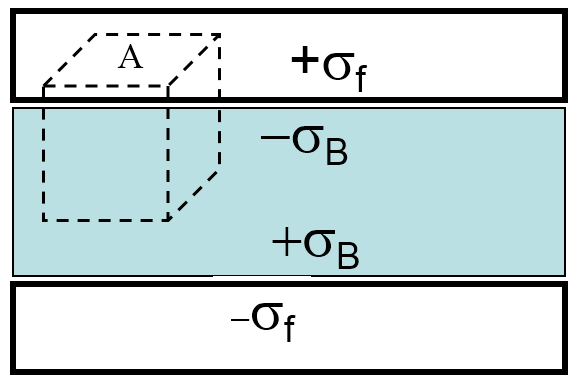
**p**

(b) Given a dipole moment p and point X as shown in the figure, what is the direction of that must be used in the dipole electric field equation, namely ? Draw the direction on point X in the figure just above.

X

**p**

(c) Same situation… what is the direction of ? Draw the direction on the figure just above.

(8 pts) **Problem 3**.A parallel plate capacitor (separation *d*) is charged up (total charge *Q*, charge per area *σf*) with a dielectric material (dielectric constant *r*) present between the plates. The dielectric polarizes, and bound charge densities +*σB* and –*σB* are produced as shown. What is the magnitude of the **D** field inside the dielectric? (Ignore fringing fields as usual.) *Hint*: use the Gaussian surface shown to analyze the situation.

(12 pts) **Problem 4**. A ferroelectric infinite slab with thickness 2*d* is centered on the *x*-*y* plane. A section is shown. It has a polarization . Determine the electric field *inside* the slab, both magnitude and direction, as a function of *z*, for 0 < z < *d*.

(19 pts) **Problem 5**. A sphere has the following potential on its surface at : . Find the potential outside the sphere, as a function of *R* and *θ*.

(extra page for work in case you need it)

(18 pts) **Problem 6**. An infinite line of charge (charge density λ) goes along the *z*-axis.

(a) What is the electric field from the line of charge?

(b) An infinitely long thick cylindrical shell (inner radius *a*, outer radius *b*, dielectric constant *ϵr*) is centered on the line charge. What is the electric field in the three regions, (i) , (ii) , and (iii) ?

(c) Find the bound volume charge density within the dielectric and also the bound surface charge densities at the inner and outer surfaces of the dielectric shell.

(12 pts) **Problem 7**. Derive the specific condition you could use in a computational relaxation problem with a square dielectric to set the potential of the (*i*, *j*) cell which is located at the bottom boundary of the dielectric *(*dielectric constant *ϵr*) as shown. Write your answer as “*Vi*,*j* = …” where the right hand side is given in terms of the potential of neighboring cells. Yes, this is just like part of the computational homework problem.

This cell is (*i*, *j*). Cell just below is (*i*+1, *j*).