Physics 441 Formula Sheet (18 Jun 2024 version)

ELECTRIC

<u>Statics</u>

1. $q = \int \lambda dl$
   $q = \int \sigma da$
   $q = \int \rho d\tau$
2. $\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0}\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}$

3. $\mathbf{F} = Q\mathbf{E}$
4. $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i \boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3}$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}dl'$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}da'$
   $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}d\tau'$

5. $\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}}$
6. $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a}$
7. $\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$
8. $\nabla \times \mathbf{E} = 0$ (this gets modified below)
9. $\mathbf{E} = -\nabla V$ (this gets modified in Phys 442)
10. $V(\mathbf{r}) = -\int_{\mathbf{ref}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$
11. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\mathbf{r}')}{\mathscr{r}}dl'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\mathbf{r}')}{\mathscr{r}}da'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}}d\tau'$

12. $U = \frac{1}{2}\sum_{i,j} \frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$
    $U = \frac{1}{2}\sum_i q_i V(\mathbf{r}_i)$
    $U = \frac{1}{2}\int \rho(\mathbf{r}')V(\mathbf{r}')d\tau'$
    $U = \frac{\varepsilon_0}{2}\int E^2 d\tau$
13. $C = \frac{Q}{V}$
14. $U = \frac{1}{2}\frac{Q^2}{C}$
15. $E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}$
16. $\mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel}$
17. $V_1 = V_2$
18. $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$
19. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \times$
    $$\sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta')\rho(\mathbf{r}')d\tau'$$
20. $\mathbf{p} = \sum_i \mathbf{r}_i' q_i$
    $\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau'$
21. $V_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{\mathrm{r}^2}$
22. $\mathbf{E}_{\text{dip}}(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$

<u>**Equations that don't fit anywhere else**</u>

Stress = force/area on a conductor: $\textbf{stress} = \frac{\sigma^2}{2\varepsilon_0}\hat{\mathbf{n}}$

$Q_{ij} = \int \left(\frac{3}{2}r_i' r_j' - \frac{1}{2}r'^2\delta_{ij}\right)\rho(\mathbf{r}')d\tau'$

$V_{quad}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r^3}\sum_{i,j} \hat{r}_i \hat{r}_j \, Q_{ij}$