Physics 441 Formula Sheet (18 Jun 2024 version)

ELECTRIC

Statics

1. $q = \int \lambda dl$
   $q = \int \sigma da$
   $q = \int \rho d\tau$
2. $\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0}\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}$
3. $\mathbf{F} = Q\mathbf{E}$
4. $\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i \boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3}$
   $\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} dl'$
   $\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} da'$
   $\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} d\tau'$
5. $\boxed{\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}}$
6. $\Phi_E = \int_S \mathbf{E}\cdot d\mathbf{a}$
7. $\oint_S \mathbf{E}\cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$
8. $\nabla\times\mathbf{E} = 0$ (this gets modified below)
9. $\mathbf{E} = -\nabla V$ (this gets modified in Phys 442)
10. $V(\mathbf{r}) = -\int_{\mathbf{ref}}^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l}$
11. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\mathbf{r}')}{\mathscr{r}} dl'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\mathbf{r}')}{\mathscr{r}} da'$
    $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}} d\tau'$
12. $U = \frac{1}{2}\sum_{i,j}\frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}$
    $U = \frac{1}{2}\sum_i q_i V(\mathbf{r}_i)$
    $U = \frac{1}{2}\int \rho(\mathbf{r}')V(\mathbf{r}')d\tau'$
    $U = \frac{\varepsilon_0}{2}\int E^2 d\tau$
13. $C = \frac{Q}{V}$
14. $U = \frac{1}{2}\frac{Q^2}{C}$
15. $E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}$
16. $\mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel}$
17. $V_1 = V_2$
18. $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$
19. $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\times \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\mathbf{r}')d\tau'$
20. $\mathbf{p} = \sum_i \mathbf{r}_i' q_i$
    $\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau'$
21. $V_{\text{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{\mathrm{r}^2}$
22. $\mathbf{E}_{\text{dip}}(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$
23. $\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$ (τ = torque here, not volume)
24. $\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E} = \nabla(\mathbf{p}\cdot\mathbf{E})$
25. $U = -\mathbf{p}\cdot\mathbf{E}$

Materials

26. $\mathbf{P}$ = electric dipole moment per unit volume
    $\mathbf{p} = \int \mathbf{P}(\mathbf{r}')d\tau'$
27. $V_{\text{pol.object}} = \frac{1}{4\pi\varepsilon_0}\int \frac{\mathbf{P}(\mathbf{r}')\cdot\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}d\tau'$
28. $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$
29. $\rho_b = -\nabla\cdot\mathbf{P}$
30. $\mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$
31. $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$
32. $\boxed{\nabla\cdot\mathbf{D} = \rho_f}$
33. $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E}$
34. $\oint \mathbf{D}\cdot d\mathbf{a} = q_{f,enc}$
35. $\varepsilon_r = 1 + \chi_e$
36. $U = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, d\tau$
37. $D_{1\perp} - D_{2\perp} = \sigma_{free}$

Dynamics

38. $\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$
39. $\boxed{\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}}$ $\quad \mathcal{E} = -\frac{d\Phi_{\mathrm{B}}}{dt}$
40. $\boxed{\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}}$ (unchanged for materials)

MAGNETIC

Statics

1. $I = \int K_{\perp} dl$
   $I = \int \mathbf{J}\cdot d\mathbf{a}$
2. No easy parallel for magnetic field
3. $\mathbf{F} = Q(\mathbf{v}\times\mathbf{B})$
4. $\mathbf{B(r)} = \frac{\mu_0}{4\pi}\int \frac{I\, d\mathbf{l}'\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}$
   $\mathbf{B(r)} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{K}(\mathbf{r}')\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} da'$
   $\mathbf{B(r)} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} d\tau'$
5. $\boxed{\nabla\cdot\mathbf{B} = 0}$
6. $\Phi_B = \int_S \mathbf{B}\cdot d\mathbf{a}$
7. $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{enc}$
8. $\nabla\times\mathbf{B} = \mu_0\mathbf{J}$ (this gets modified below)
9. $\mathbf{B} = \nabla\times\mathbf{A}$, $\nabla\cdot\mathbf{A} = 0$ (Coulomb gauge)
10. No direct parallel for the magnetic field
11. $\mathbf{A(r)} = \frac{\mu_0}{4\pi}\int \frac{I}{\mathscr{r}}\, d\mathbf{l}'$
    $\mathbf{A(r)} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{K}(\mathbf{r}')}{\mathscr{r}}\, da'$
    $\mathbf{A(r)} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}}\, d\tau'$
12. $U = \frac{1}{2}\int \mathbf{J}(\mathbf{r}')\cdot\mathbf{A}(\mathbf{r}')d\tau'$
    $U = \frac{1}{2\mu_0}\int B^2 d\tau$
13. $L = \frac{\Phi}{I}$
14. $U = \frac{1}{2}LI^2$
15. $B_1^{\perp} = B_2^{\perp}$
16. $\mathbf{B}_1^{\parallel} - \mathbf{B}_2^{\parallel} = \mu_0(\mathbf{K}\times\hat{\mathbf{n}})$
17. $\mathbf{A}_1 = \mathbf{A}_2$
18. $\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$
19. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\times \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\theta')d\mathbf{l}'$
20. $\mathbf{m} = I\int d\mathbf{a} = I\mathbf{a}$
21. $\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{\mathbf{r}^2}$
22. $\mathbf{B}_{\text{dip}}(r,\theta) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$
23. $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$
24. $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$
25. $U = -\mathbf{m}\cdot\mathbf{B}$

Materials

26. $\mathbf{M}$ = magnetic dipole moment per unit vol.
    $\mathbf{m} = \int \mathbf{M}(\mathbf{r}')d\tau'$
27. $\mathbf{A}_{\text{magn.object}} = \frac{\mu_0}{4\pi}\times \int \frac{\mathbf{M}(\mathbf{r}')\times\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}d\tau'$
28. $\mathbf{J_b} = \nabla\times\mathbf{M}$
29. $\mathbf{K_b} = \mathbf{M}\times\hat{\mathbf{n}}$
30. $\mathbf{M} = \chi_m\mathbf{H}$
31. $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$
32. $\boxed{\nabla\cdot\mathbf{B} = 0}$ (still)
33. $\mathbf{H} = \mathbf{B}/\mu = \mathbf{B}/(\mu_0\mu_r)$
34. $\oint \mathbf{H}\cdot d\mathbf{l} = I_{f,enc}$
35. $\mu_r = 1 + \chi_m$
36. $U = \frac{1}{2}\int \mathbf{H}\cdot\mathbf{B}\, d\tau$
37. $\mathbf{H}_{1\parallel} - \mathbf{H}_{2\parallel} = \mathbf{K}_{free}\times\hat{\mathbf{n}}$

Dynamics

38. $\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$ (same equation as in left hand column; connects charge to current)
39. $\boxed{\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}}$
40. $\boxed{\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}}$

Equations that don't fit anywhere else

Stress = force/area on a conductor: **stress** $= \frac{\sigma^2}{2\varepsilon_0}\hat{\mathbf{n}}$

$Q_{ij} = \int\left(\frac{3}{2}r_i' r_j' - \frac{1}{2}r'^2\delta_{ij}\right)\rho(\mathbf{r}')d\tau'$

$V_{quad}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r^3}\sum_{i,j}\hat{r}_i\hat{r}_j\, Q_{ij}$

SOV results:

2d Cartesian, no z dependence (X and Y can be reversed)

$$X = \begin{cases}\sin kx\\ \cos kx\end{cases},\quad Y = \begin{cases}e^{ky}\\ e^{-ky}\end{cases} \text{ or } \begin{cases}\sinh ky\\ \cosh ky\end{cases}$$

3d Cartesian (X, Y, and Z can be permuted)

$$X = \begin{cases}\sin k_x x\\ \cos k_x x\end{cases},\ Y = \begin{cases}\sin k_y y\\ \cos k_y y\end{cases}\ Z = \begin{cases}e^{\sqrt{k_x^2+k_y^2}\,z}\\ e^{-\sqrt{k_x^2+k_y^2}\,z}\end{cases} \text{ or } \begin{cases}\sinh \ldots\\ \cosh \ldots\end{cases}$$

Spherical, no ϕ dependence

$$R = \begin{cases}r^{\ell}\\ \frac{1}{r^{\ell+1}}\end{cases},\quad \Theta = \begin{cases}P_\ell(\cos\theta)\\ Q_\ell(\cos\theta)\end{cases}$$

Cylindrical, no z dependence

$$k = 0:\quad S\Phi = \begin{cases}C\\ \ln s\end{cases}$$

$$k = 1,2,\ldots,\infty:\quad S = \begin{cases}s^k\\ s^{-k}\end{cases},\quad \Phi = \begin{cases}\sin k\phi\\ \cos k\phi\end{cases}$$

Clausius-Mossotti: $\alpha = \frac{3\varepsilon_0}{N}\frac{\chi_e}{\chi_e+3}$

$\mathbf{J}_D = \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$ (or sometimes $= \frac{\partial\mathbf{D}}{\partial t}$) $\quad \mathbf{J}_{Faraday} = -\frac{\partial\mathbf{B}}{\partial t}$