

What You Should Already Know About Electric Field and Potential by Dr. Colton (Fall 2019)

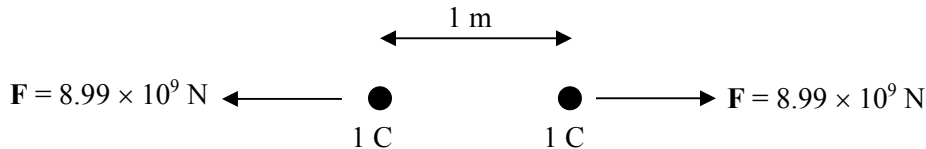
Electric Force

Coulomb's law describes the force between charged objects:

$$F_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d^2}$$

The constant ϵ_0 is called the “permittivity of free space”, and is equal to $8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$ in standard units. Sometimes $k_e = 1/(4\pi\epsilon_0)$, called the “Coulomb force constant,” is used instead of ϵ_0 , but we won't do that.

The direction of the force is given by the statement “Opposites attract. Like charges repel.” For example, the force between two 1 C positive charges* located 1 m apart would be $8.99 \times 10^9 \text{ N}$, and it can be drawn like this:



Electric Field

Electric field describes the electric force that *might* occur, if you were to bring an imaginary “test charge” close to another real charge that is present. It's the force per (test) charge, in the limit where the test charge becomes infinitely small so it doesn't impact the real charges which are present. The field of a point charge is therefore:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{d^2}$$

In other words force describes the **actual** effect on a charge; field describes the **potential** effect. Field is measured in N/C (newtons per coulomb), or alternatively V/m (volts per meter). $1 \text{ N/C} = 1 \text{ V/m}$.

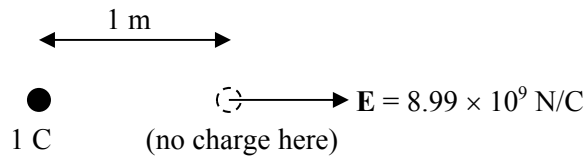
If you have worked out the field at a location, then you can easily figure out what the force will be on a charge Q put at that location. Assuming it doesn't disturb the other charges which are producing the field, you just multiply the field by the charge:

$$F = QE$$

The direction of the electric field is given by the direction a *positive* test charge would be pushed. This means that the field near a positive charge points away from it and the field near a negative charge points towards it.

* In practice, 1 C is a *huge* amount of charge, and you would never have that much extra charge in one place. But, to make the numbers work out easily, I'll use that for this example.

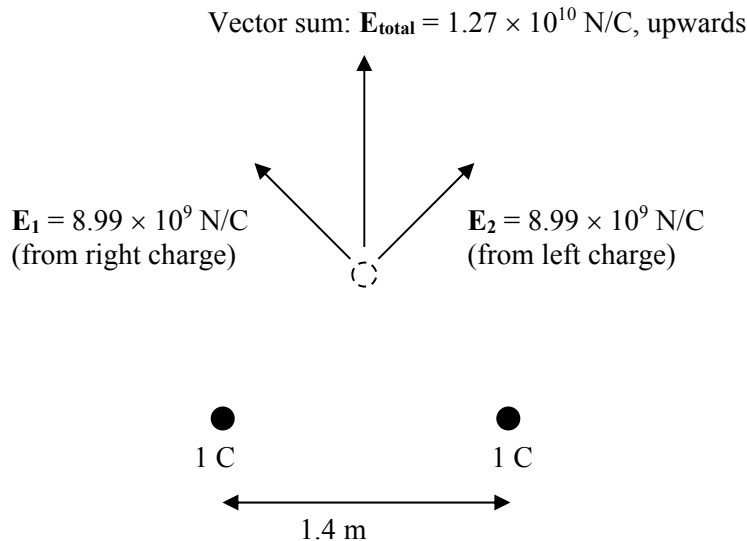
In the case shown above, the field from the left charge at a distance of 1 m away would be 1 N/C to the right:



Addition of Electric Field

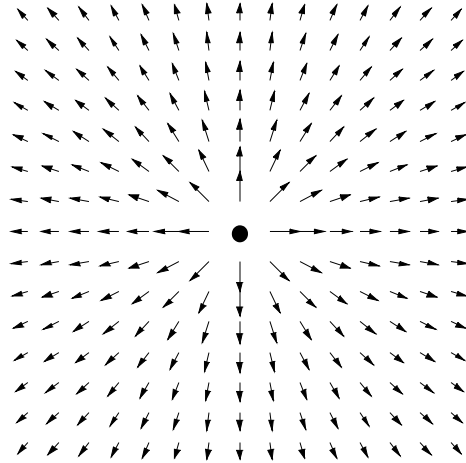
Suppose you have 2 charges and want to talk about the combined field from both of them. In this case, your “test charge” would be an imaginary 3rd charge. Since the forces on a 3rd charge from those two charges would just add, so do the electric fields. This is called the “superposition principle”. Note that since **the electric field has a magnitude and a direction, it is a vector quantity**. So make sure you add the electric fields as vectors; you can’t just add the magnitudes!

The next diagram shows the electric field at a particular point (the dashed circle) due to two positive charges each 1 m away from the point. The charges are 1.4 m away from each other, and 1 m away from the point. Although the electric field from each charge is $8.99 \times 10^9 \text{ N/C}$, the total electric field for this case is only $1.27 \times 10^{10} \text{ N/C}$ because the two fields partially cancel each other out.



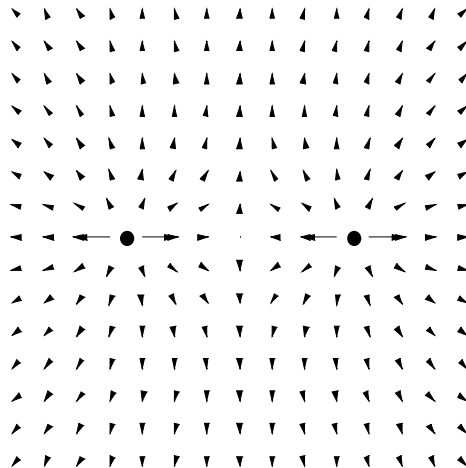
Electric Field Vectors

If you can envision the electric field at a given point as being a number combined with a direction, proportional to the force on a test charge placed at that point, then you’ve gone a long way to understanding electric field vectors. A number combined with a direction is a vector, and is often symbolized with an arrow. Each point in space thus has an arrow associated with it. Probably the best way to envision the electric field generated by a charge, is to think of it as the collection of all of the arrows for every point in space. Let me emphasize that: **the collection of arrows is the electric field**, in a very basic sense. For a single 1 C charge, as above, the collection of arrows would look something like this:



The length of an arrow should indicate the strength of the electric field at that point: the arrows close to the charge are longer than the arrows far away from the charge.* Of course, I can't draw every single arrow, since there's an infinite number of them—one at each point in space—but I think enough are plotted so that you can get the idea. Note that I only drew the arrows in 2 dimensions. Drawing them in 3D is generally too difficult to do by hand; you can use computer programs for that if you need to. All of the plots in this handout are sketches in the x - y plane.

For two positive charges, the field vectors would look something like this:



This distribution of arrows was generated by taking a set of arrows for one charge, as on the previous page, and adding them to a set of arrows for the second charge, shifted to the right. The arrows are added together as *vectors*, through the “tip to tail” method or through addition of components.

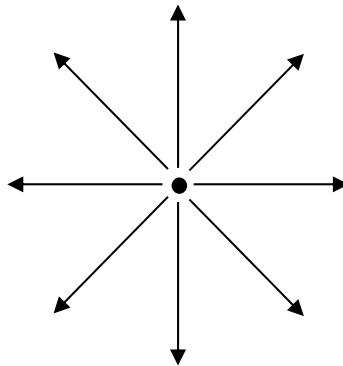
* I've actually fudged things here to make the distant arrows more easily seen. If the length of all the arrows were strictly proportional to the field strength in the plot, the distant arrows would be shorter than they are in the figure.

Electric Field Lines

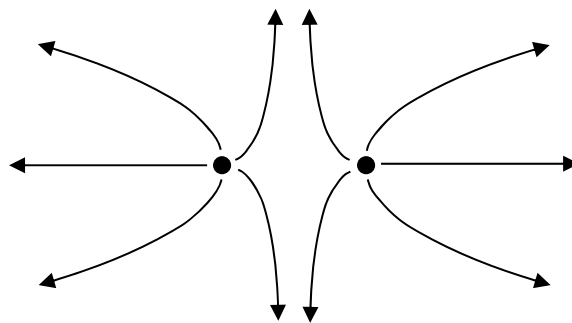
If you start with a collection of arrows (on paper, or in your head), you can obtain the electric field lines fairly easily:

1. Pick a point to start at.
2. Move very slightly in the direction the arrow points, drawing a line as you go.
3. You are now at a slightly different arrow. Move very slightly in the direction this *new* arrow points.
4. Repeat until you either run into another charge or get very far away from all charges.

As with the arrows, there are an infinite number of field lines. When you sketch them, you don't have to worry about all of the lines, but do sketch enough to make it clear what the pattern is—either 4 or 8 lines per charge should be about right for most situations. For the 1 C charge shown above, the field lines look like this:



For two 1 C charges, the field lines look something like this:

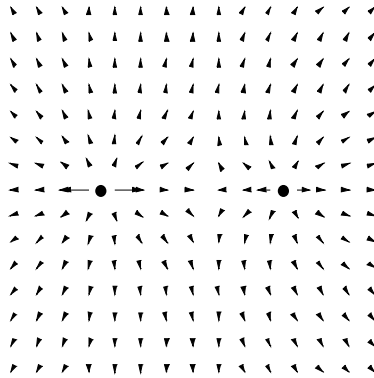


Facts about field lines:

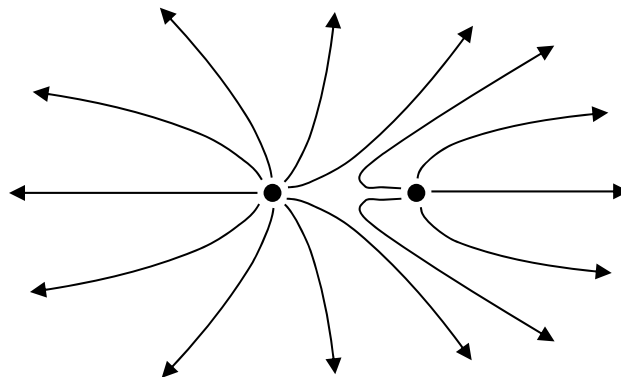
1. Field lines never cross. If they did, that would mean that the field had two different directions at the point of crossing, which is impossible by definition.
2. Field lines can only originate and end at charges. If you compare electric field to water flow, you'd say positive charges are the faucets and negative charges are the drains.
3. The density of field lines is the highest where the field is strongest (look at how the lines are closest to each other near the charges in the pictures above).

4. As a corollary to the previous item, if you have charges with different magnitudes you should draw more field lines entering/leaving the larger ones.
5. When you get close enough to a charge, the field lines should look like the field of single charge.
6. When you get far enough away from all charges, the field lines should look like the field of a single charge (if there is a net charge).

Look at how the last two facts are put into place in this drawing of field lines from two positive charges: +2 C (on the left) and +1 C (on the right). First the field arrows, done with a computer program:



Now the field lines, sketched by myself using MS Word:



Notice that near each charge, the lines just go straight out from the circles; far away from both charges, the lines just go straight out from the center of the charges. And notice that the lines are most closely spaced near the charges, where the field is strongest—except for one point in between the two charges where the fields cancel each other out.

Electric Work & Potential Energy

As you learned in Physics 121, the term “work” has a precise meaning in physics, related to energy: it’s the amount of energy expended or recovered when a force pushes an object over a certain distance. For cases where the force is constant and in the same direction as the distance, work is just force \times distance. For more situations where the force is changing (but still in the same direction as the distance), the work is given by this integral:

$$W = \int_{x_1}^{x_2} F dx$$

Work is measured in units of energy (joules). If you want to bring a positive charge close to another positive charge, you have to overcome the electric repulsive force, so it takes work.

For two point charges, integrating the force given by Coulomb's Law yields this expression for the work needed to bring one charge to a distance d away from another charge (assuming it starts infinitely far away):

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}$$

This is also the Coulomb potential energy for two charges:

$$PE = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}$$

Electric Potential

The key here is an analogy: **potential is to potential energy, as field is to force**. Remember that! That means that electric potential describes the potential energy that *would* exist (or equivalently, the work that would have to be done), if you were to bring a test charge close to another charge or charges. Just like potential energy, electric potential requires a reference point. It uses the symbol V , and has units of V (volts). $1 V$ is $1 J/C$. That's analogous to units of field being force per coulomb. The potential a distance d from a point charge, referenced to zero potential being at infinity, is this:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{d}$$

Unlike field, potential is not a vector. **Electric potential is a scalar quantity**. You can just add together the potential from multiple charges without worrying about directions.

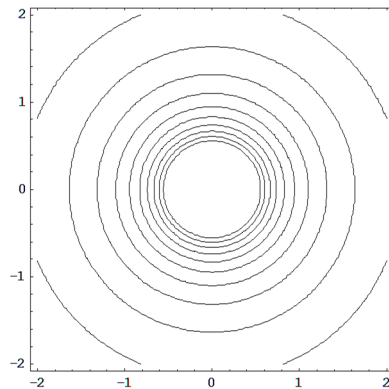
Just as force on a charge can be calculated by multiplying the field at the charge's location by the charge itself, the work needed to be done in moving a charge can be calculated by multiplying the potential difference between the charge's locations, by the charge itself:

$$W = q \Delta V$$

Equipotential Lines

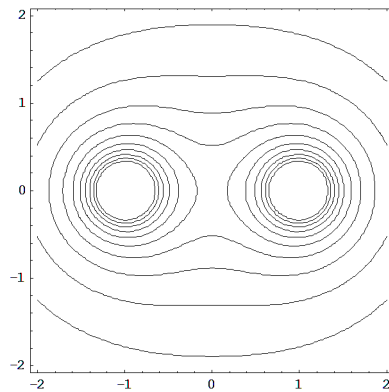
Just as field lines can be used to help you visualize the electric field, "equipotential lines" (or equipotential surfaces, in 3D) can be used to help you visualize the electric potential. Those are regions of space where the potential is constant. For a single charge, the equipotential lines are just circles; for multiple charges, the equipotential lines become more complex. The equipotential lines are sketched below for the same three cases the field lines were sketched above: a single charge, and a pair of equal positive charges, and a pair of unequal positive charges.

Single charge:

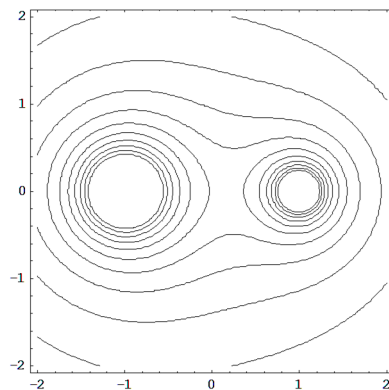


(The circles get very close to each other in the middle at the location of the charge, so beyond a certain point they are not plotted.)

Two equal positive charges:



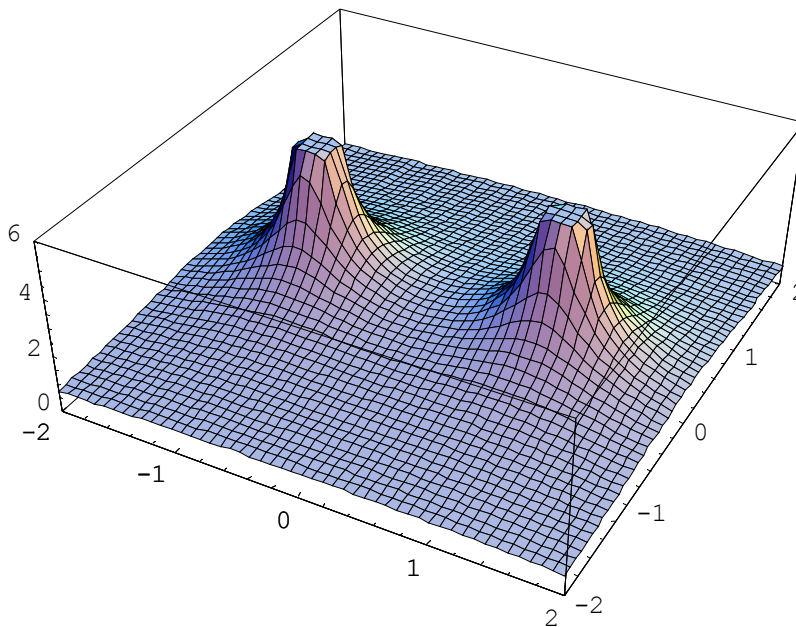
Two unequal positive charges:



When the charges lie in a plane, a mental image of a topological map works very well to envision the relationship between field and potential lines:

- **The field is the slope, how steep the terrain is.**
- **The direction of the field is downhill, i.e. the direction a ball would roll.**
- **The equipotential lines are the contour lines (constant elevation).**
- **Positive charges are sharp mountains; negative charges are sharp crevices.**

Notice how the contours lines for this 3D plot of a “mountain range” with 2 equal positive charges would look like the equipotential lines on the previous page:



(The peaks get too large at the location of the charges, so beyond a certain point the plot cuts them off.)

Facts about equipotential lines:

1. Equipotential lines are always perpendicular to electric field lines (field lines point “down the hill”; equipotential lines point “around the hill”).
2. No work is needed to move a charge along an equipotential line (same potential energy).
3. Equipotential lines are most closely spaced with the field is strongest (just like contours on a map are most closely spaced where the ground is the steepest).
4. When you get close enough to a charge, the equipotential lines should look like those of a single charge.
5. When you get far enough away from all charges, the equipotential lines should look like those of a single charge (if there is a net charge).

Fact #1 can be shown by superimposing an equipotential plot on a field lines (or equivalently, field arrows) plot; notice how the arrows always point perpendicularly to the contours!

