

Parallel Equations for the Electric and Magnetic Fields
Dr. Colton, Physics 441/442, Spring/Summer 2021

ELECTRIC

Statics

1. $q = \int \lambda dl$
 $q = \int \sigma da$
 $q = \int \rho d\tau$
2. $\mathbf{F} = \frac{qQ}{4\pi\epsilon_0} \frac{\mathbf{r}}{r^3}$
3. $\mathbf{F} = Q\mathbf{E}$
4. $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i \mathbf{r}_i}{r_i^3}$
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}') \mathbf{r}}{r^3} dl'$
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma(\mathbf{r}') \mathbf{r}}{r^3} da'$
 $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\mathbf{r}') \mathbf{r}}{r^3} d\tau'$
5. $\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}}$
6. $\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{a}$
7. $\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\epsilon_0}$
8. $\nabla \times \mathbf{E} = 0$ (this gets modified below)
9. $\mathbf{E} = -\nabla V$ (this gets modified in Phys 442)
10. $V(\mathbf{r}) = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$
11. $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}')}{r} dl'$
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\mathbf{r}')}{r} da'$
 $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r} d\tau'$
12. $U = \frac{1}{2} \sum_i q_i V(\mathbf{r}_i)$
 $U = \frac{1}{2} \int \rho(\mathbf{r}') V(\mathbf{r}') d\tau'$
 $U = \frac{\epsilon_0}{2} \int E^2 d\tau$
13. $C = \frac{Q}{V}$
14. $U = \frac{1}{2} \frac{Q^2}{C}$
15. $E_1^\perp - E_2^\perp = \frac{\sigma}{\epsilon_0}$
16. $\mathbf{E}_1^\parallel = \mathbf{E}_2^\parallel$

MAGNETIC

Statics

1. $I = \int K_\perp dl$
 $I = \int \mathbf{J} \cdot d\mathbf{a}$
2. No easy parallel for magnetic field
3. $\mathbf{F} = Q(\mathbf{v} \times \mathbf{B})$
4. $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l}' \times \mathbf{r}}{r^3}$
 $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_S \frac{\mathbf{K}(\mathbf{r}') \times \mathbf{r}}{r^3} da'$
 $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') \times \mathbf{r}}{r^3} d\tau'$
5. $\boxed{\nabla \cdot \mathbf{B} = 0}$
6. $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{a}$
7. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$
8. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (this gets modified below)
9. $\mathbf{B} = \nabla \times \mathbf{A}$, $\nabla \cdot \mathbf{A} = 0$ (Coulomb gauge)
10. No direct parallel for the magnetic field
11. $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{I}{r} d\mathbf{l}'$
 $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{r} da'$
 $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{r} d\tau'$
12. $U = \frac{1}{2} \int \mathbf{J}(\mathbf{r}') \cdot \mathbf{A}(\mathbf{r}') d\tau'$
 $U = \frac{1}{2\mu_0} \int B^2 d\tau$
13. $L = \frac{\Phi}{I}$
14. $U = \frac{1}{2} LI^2$
15. $B_1^\perp = B_2^\perp$
16. $\mathbf{B}_1^\parallel - \mathbf{B}_2^\parallel = \mu_0 \mathbf{K}$

$$17. V_1 = V_2$$

$$18. \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$19. V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta') \rho(\mathbf{r}') d\tau'$$

Materials

$$20. \mathbf{p} = \sum_i \mathbf{r}'_i q_i$$

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d\tau'$$

$$21. V_{\text{dip}} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

$$22. \mathbf{E}_{\text{dip}}(r, \theta) = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} (2\cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}})$$

$$23. \boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (\tau = \text{torque here, not volume})$$

$$24. \mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} = \nabla(\mathbf{p} \cdot \mathbf{E})$$

$$25. U = -\mathbf{p} \cdot \mathbf{E}$$

$$26. \mathbf{P} = \text{electric dipole moment per unit volume}$$

$$\mathbf{p} = \int \mathbf{P}(\mathbf{r}') d\tau'$$

$$27. V_{\text{pol.object}} = \frac{1}{4\pi\epsilon_0} \int \frac{\mathbf{P}(\mathbf{r}') \cdot \mathbf{z}}{r'^3} d\tau'$$

$$28. \sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$$

$$29. \rho_b = -\nabla \cdot \mathbf{P}$$

$$30. \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$31. \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$32. \boxed{\nabla \cdot \mathbf{D} = \rho_f}$$

$$33. \mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E}$$

$$34. \oint \mathbf{D} \cdot d\mathbf{a} = q_{f, \text{enc}}$$

$$35. \epsilon_r = 1 + \chi_e$$

$$36. U = \frac{1}{2} \int \mathbf{D} \cdot \mathbf{E} d\tau$$

$$37. D_{1\perp} - D_{2\perp} = \sigma_{\text{free}}$$

Dynamics

$$38. \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

$$39. \boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$40. \boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} \quad (\text{unchanged for materials})$$

$$17. \mathbf{A}_1 = \mathbf{A}_2$$

$$18. \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$19. \mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint (r')^n P_n(\cos\theta') d\mathbf{l}'$$

Materials

$$20. \mathbf{m} = I \int d\mathbf{a} = I \mathbf{a}$$

$$21. \mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}$$

$$22. \mathbf{B}_{\text{dip}}(r, \theta) = \frac{\mu_0 m}{4\pi r^3} (2\cos\theta \hat{\mathbf{r}} + \sin\theta \hat{\boldsymbol{\theta}})$$

$$23. \boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

$$24. \mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$

$$25. U = -\mathbf{m} \cdot \mathbf{B}$$

$$26. \mathbf{M} = \text{magnetic dipole moment per unit vol.}$$

$$\mathbf{m} = \int \mathbf{M}(\mathbf{r}') d\tau'$$

$$27. \mathbf{A}_{\text{magn.object}} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{M}(\mathbf{r}') \times \mathbf{z}}{r'^3} d\tau'$$

$$28. \mathbf{J}_b = \nabla \times \mathbf{M}$$

$$29. \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

$$30. \mathbf{M} = \chi_m \mathbf{H}$$

$$31. \mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$$

$$32. \boxed{\nabla \cdot \mathbf{B} = 0} \quad (\text{still})$$

$$33. \mathbf{H} = \mathbf{B}/\mu = \mathbf{B}/(\mu_0 \mu_r)$$

$$34. \oint \mathbf{H} \cdot d\mathbf{l} = I_{f, \text{enc}}$$

$$35. \mu_r = 1 + \chi_m$$

$$36. U = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} d\tau$$

$$37. \mathbf{H}_{1\parallel} - \mathbf{H}_{2\parallel} = \mathbf{K}_{\text{free}} \times \hat{\mathbf{n}}$$

Dynamics

$$38. \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (\text{same equation as in left hand column; connects charge to current})$$

$$39. \boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}}$$

$$40. \boxed{\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}}$$