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Summer 2024
Physics 442 Exam 2
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No time limit. Student calculators are allowed. Notes not allowed. Books not allowed.

Name Solutions

Instructions: Please label & circle/box your answers. **Show your work**, where appropriate.

Griffiths front and back covers

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl:
$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem: $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

Special case derivatives: $\nabla \cdot \frac{\hat{\mathbf{r}}}{r^2} = 4\pi\delta(\mathbf{r})$ $\quad \nabla^2 \frac{1}{r} = -4\pi\delta(\mathbf{r})$
(similar things true for $\mathscr{r}$)

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Larmor formula: $P = \frac{\mu_0}{6\pi c}q^2 a^2$

FUNDAMENTAL CONSTANTS

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$ (permittivity of free space)

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$ (permeability of free space)

$c = 3.00 \times 10^8\ \mathrm{m/s}$ (speed of light)

$e = 1.60 \times 10^{-19}\ \mathrm{C}$ (charge of the electron)

$m = 9.11 \times 10^{-31}\ \mathrm{kg}$ (mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Potentially useful mathematical information

quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\int_{-L/2}^{L/2} \sin\left(\frac{\pi z}{L}\right) z\,dz = \frac{2L^2}{\pi^2}$$

$$\int_{-L/2}^{L/2} \cos\left(\frac{\pi z}{L}\right) z\,dz = 0$$

$$\int_{-L/2}^{L/2} \sin\left(\frac{\pi z}{L}\right) z^2\,dz = 0$$

$$\int_{-L/2}^{L/2} \cos\left(\frac{\pi z}{L}\right) z^2\,dz = \frac{L^3(\pi^2-8)}{2\pi^3}$$

$$\int_0^1 J_0(u_{0m}r)\,J_0(u_{0n}r)\,r\,dr = \begin{cases} 0 & \text{if } n \neq m \\ \frac{1}{2}\left(J_1(u_{0m})\right)^2 & \text{if } n = m \end{cases}$$

$$\int_0^1 J_\alpha(u_{\alpha m}r)\,J_\alpha(u_{\alpha n}r)\,r\,dr = \begin{cases} 0 & \text{if } n \neq m \\ \frac{1}{2}\left(J_{\alpha+1}(u_{\alpha m})\right)^2 & \text{if } n = m \end{cases}$$

Some zeroes of the first few Bessel functions and the Bessel function derivatives, to five sig figs.

m	J_0	J_1	J_2	J_0'	J_1'	J_2'
1	2.4048	3.8317	5.1357	3.8317	1.8412	3.0542
2	5.5201	7.0156	8.4172	7.0156	5.3314	6.7061
3	8.6537	10.173	11.620	10.173	8.5363	9.9695
4	11.792	13.324	14.796	13.324	11.706	13.170

(14 pts) **Problem 1**. The Physics 441 scalar and vector potentials of an electric dipole are $V = \frac{p}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$ and $\mathbf{A} = 0$.

(a) Find a gauge transformation to a new set of potentials V' and $\mathbf{A}'$ such that $V' = 0$. What is $\mathbf{A}'$?

$$V' = V - \frac{\partial \lambda}{\partial t} \qquad \vec{A}' = \vec{A} + \nabla\lambda$$

$\rightarrow$ force $V' = 0$

$$\frac{\partial \lambda}{\partial t} = V$$

$$\frac{\partial \lambda}{\partial t} = \frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^2}$$

$$\underline{\lambda = \frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^2}\, t}$$

then $\vec{A}' = \cancel{\vec{A}} + \nabla\lambda$

$$= \frac{\partial\lambda}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\lambda}{\partial\theta}\hat{\theta} + \cancel{\frac{1}{r\sin\theta}\frac{\partial\lambda}{\partial\phi}\hat{\phi}}^{\,0}$$

$$\boxed{\vec{A}' = -2\frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^3}\, t\,\hat{r} - \frac{p}{4\pi\epsilon_0}\frac{\sin\theta}{r^3}\, t\,\hat{\theta}}$$

(b) Explicitly determine the electric field predicted by your new set of potentials, as a function of r and θ.

$$\vec{E} = -\cancel{\nabla V'} - \frac{\partial \vec{A}'}{\partial t}$$

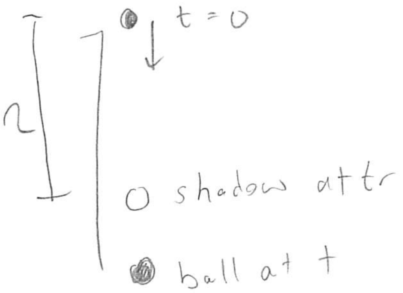
$$\vec{E} = +2\frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^3}\hat{r} + \frac{p}{4\pi\epsilon_0}\frac{\sin\theta}{r^3}\hat{\theta}$$

$$\boxed{\vec{E} = \frac{p}{4\pi\epsilon_0}\frac{1}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$

familiar dipole field from 441

(20 pts) **Problem 2**. Suppose you drop a charged ball (charge q) off a cliff; neglecting air resistance the ball falls at a constant acceleration g. From Physics 121 we know that the ball's position vs. time is given by $y = -\frac{1}{2}gt^2$ (the origin is at the top of the cliff); and the ball's velocity vs. time is given by $v = -gt$.

(a) Determine r as a function of time for yourself as the observer (i.e. the distance between yourself and the ball's "shadow"). Hint: the quadratic formula is given on page 2 of the exam.



$$t_r = t - \frac{r}{c}$$

$$\Downarrow$$

Since $|y| = \frac{1}{2} g t^2$, $\quad r = \frac{1}{2} g t_r^2$

$$t_r = \sqrt{\frac{2r}{g}}$$

Therefore

$$\sqrt{\frac{2r}{g}} = t - \frac{r}{c}$$

$$\frac{2r}{g} = \left(t - \frac{r}{c}\right)^2$$

$$\frac{2r}{g} = t^2 - \frac{2tr}{c} + \frac{r^2}{c^2}$$

$$r^2 - \left(\frac{2c^2}{g} + 2ct\right) r + c^2 t^2 = 0$$

Quad. formula

$$r = \frac{\cancel{2}\frac{c^2}{g} + \cancel{2}tc \pm \sqrt{\left(\cancel{2}\frac{c^2}{g} + \cancel{2}ct\right)^2 - \cancel{4}c^2t^2}}{\cancel{2}}$$

$$r = \frac{c^2}{g} + ct \pm \sqrt{\frac{c^4}{g^2} + \frac{2c^3t}{g} + \cancel{c^2t^2} - \cancel{c^2t^2}}$$

r must equal 0 at time $t = 0$ therefore the negative sign must be correct

$$\boxed{r = \frac{c^2}{g} + ct - \sqrt{\frac{c^4}{g^2} + \frac{2c^3t}{g}}}$$

(continued on next page)

(b) The situation of 1-dimensional particle motion with observer on the axis of motion was treated in one of your homework problems, for which you should have found that (replacing all x's with y's) for field points with smaller y than the observer:

√ at retarded time

$$\mathbf{E} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{\mathscr{r}^2}\frac{c-v}{c+v}\hat{\mathbf{y}}$$

Here v is the velocity of the shadow, and it is positive if the particle is traveling in the $+\hat{\mathbf{y}}$ direction and negative if traveling in the $-\hat{\mathbf{y}}$ direction. Use that equation along with your answer from (a) to deduce $\mathbf{E}(t)$ at your location.

$$v = -g t_r = -g\left(t - \frac{\mathscr{r}}{c}\right)$$

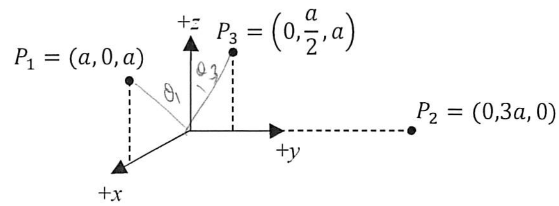
$$\text{so} \quad \vec{E} = -\frac{q}{4\pi\epsilon_0}\frac{1}{\mathscr{r}^2}\frac{c+g\left(t-\frac{\mathscr{r}}{c}\right)}{c-g\left(t-\frac{\mathscr{r}}{c}\right)}\hat{y}$$

where $\mathscr{r}$ = answer from a.

Piecing together,

$$\vec{E} = -\frac{q}{4\pi\epsilon_0}\frac{1}{\left(\frac{c^2}{g}+ct-\sqrt{\frac{c^4}{g^2}+\frac{2c^3t}{g}}\right)^2}\left(\frac{c+gt-\frac{g}{c}\left[\frac{c^2}{g}+ct-\sqrt{\frac{c^4}{g^2}+\frac{2c^3t}{g}}\right]}{c-gt+\frac{g}{c}\left[\frac{c^2}{g}+ct-\sqrt{\frac{c^4}{g^2}+\frac{2c^3t}{g}}\right]}\right)\hat{y}$$

(16 pts) **Problem 3.** An oscillating electric dipole is at the origin, with dipole moment in the z-direction. It produces radiation which is detected at points P_1, P_2, and P_3, whose locations are indicated in the diagram below. The length a is just a reference distance, with $a \gg \frac{c}{\omega} \gg (size\ of\ dipole)$.



(a) Which of the three points will detect the LEAST amount of radiation?

(b) If we call the amount of radiation detected at that point, "X", in terms of X how much radiation is detected at the other two points?

Power radiated $\sim \frac{\sin^2\theta}{r^2}$

For P_1: $\left.\begin{array}{l}\theta = 45^\circ \\ r = a\sqrt{2}\end{array}\right\} \rightarrow P_1 \sim \frac{\sin^2 45^\circ}{(a\sqrt{2})^2} = \frac{1}{4a^2} = \underline{0.25 \frac{1}{a^2}}$

For P_2: $\left.\begin{array}{l}\theta = 90^\circ \\ r = 3a\end{array}\right\} \rightarrow P_2 \sim \frac{\sin^2 90^\circ}{(3a)^2} = \frac{1}{9a^2} = \underline{0.111 \frac{1}{a^2}}$

For P_3: $\left.\begin{array}{l}\theta = \tan^{-1}\left(\frac{a/2}{a}\right) = 26.57^\circ \\ r = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} = \frac{a\sqrt{5}}{2}\end{array}\right\} \rightarrow P_3 \sim \frac{\sin^2(26.57^\circ)}{\left(\frac{a\sqrt{5}}{2}\right)^2} = \underline{0.16 \frac{1}{a^2}}$

$\boxed{P_2 \text{ has the least radiation}}$

If $P_2 = X$ then $P_1 = \frac{0.25}{0.111} X = \boxed{2.25 X}$

$P_3 = \frac{0.16}{0.111} X = \boxed{1.44 X}$

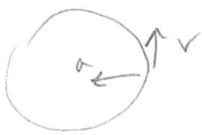
(20 pts) **Problem 4.** (a) Apply Liénard's generalization of the Larmor formula to the situation of a particle going in a circle, to deduce the power radiated by the particle as a function of charge q, relativistic gamma factor γ, acceleration a, and fundamental constants μ_0 and c.

Liénard: $P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v}\times\vec{a}}{c}\right|^2\right)$

circular motion, $\vec{v} \perp \vec{a}$ so $\vec{v}\times\vec{a} = va$ so parentheses $= \left(a^2 - \frac{va}{c}\right)^2$

$= a^2(1-\beta)^2$

$= \frac{a^2}{\gamma^2}$



$$\boxed{P = \frac{\mu_0 q^2 \gamma^4 a^2}{6\pi c}}$$

(b) The Advanced Light Source synchrotron at LBNL sends electrons around in a circle with a relativistic energy of 1.9 GeV ($= \gamma mc^2$). During one section of the synchrotron the electrons are bent around a part of a circle with radius of 5 m. How much power does an electron radiate in that section? How much power would the regular Larmor formula predict? Hints: 1 eV $= 1.602 \times 10^{-19}$ J. Also, centripetal acceleration is still $\frac{v^2}{R}$ even for relativistic situations.

$\gamma mc^2 = 1.9\text{ GeV}$

$\gamma = \frac{1.9\text{ GeV}}{mc^2}$

$= \frac{1.9\cdot 10^9 \cdot 1.602\cdot 10^{-19}}{9.11\cdot 10^{-31}\cdot (3\cdot 10^8)^2}$

$= 3712.4$

$a = \frac{v^2}{R} = \frac{(3\cdot 10^8)^2}{5}$

$= 1.8\cdot 10^{16}\ m/s^2$

v is so close to c as to make the small difference here irrelevant

$P = \frac{\mu_0 q^2 \gamma^4 a^2}{6\pi c}$

$= \frac{4\cancel{\pi}\cdot 10^{-7}\cdot (1.602\cdot 10^{-19})^2 (3712)^4 (1.8\cdot 10^{16})^2}{6\cancel{\pi}\cdot 3\cdot 10^8}$

$$\boxed{P = 3.51\cdot 10^{-7}\ W}$$

Regular Larmor is lower than that by factor of γ^4

$P_{regular} = \frac{3.51\cdot 10^{-7}}{(3712.4)^4} = \boxed{1.85\cdot 10^{-21}\ W}$

(16 pts) **Problem 5**. At $(x,t)_{Andrew} = (0,0)$ Andrew offers a prayer for his sister who is located at $x_{Andrew} = 10^6$ m. At $(x,ct)_{Andrew} = (10^6, 10^5)$ m, a truck nearly misses his sister. Event A is "Andrew offers prayer" and event B is the near miss. Meanwhile, Josh is speeding along (moving to the right relative to Andrew) and passes Andrew just as Andrew offers the prayer. How fast does Josh need to be moving relative to Andrew for event B to come at $ct_{Josh} = -10^5$ m? In other words, in Josh's frame event the near miss occurs before the prayer that was offered. Draw space-time diagrams from both Andrew's and Josh's point of view. Include worldlines for Andrew, Josh, Andrew's sister, and mark the two events on each diagram. Remember to include the space contraction of the distance between Andrew & his sister, in Josh's frame.

$$\begin{pmatrix} ct \\ x \end{pmatrix}_J = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}_A \rightarrow \text{top row } ct_J = \gamma ct_A - \gamma\beta x_A$$

$$-10^5 = \gamma(10^5) - \gamma\beta(10^6)$$

$$-1 = \gamma - \gamma\beta \cdot 10$$

$$-1 = \gamma(1 - 10\beta)$$

$$-1 = \frac{1}{\sqrt{1-\beta^2}}(1 - 10\beta)$$

$$\cancel{1} - \beta^2 = (\cancel{1} - 20\beta + 100\beta^2)$$

$$0 = -20\beta + 101\beta^2$$

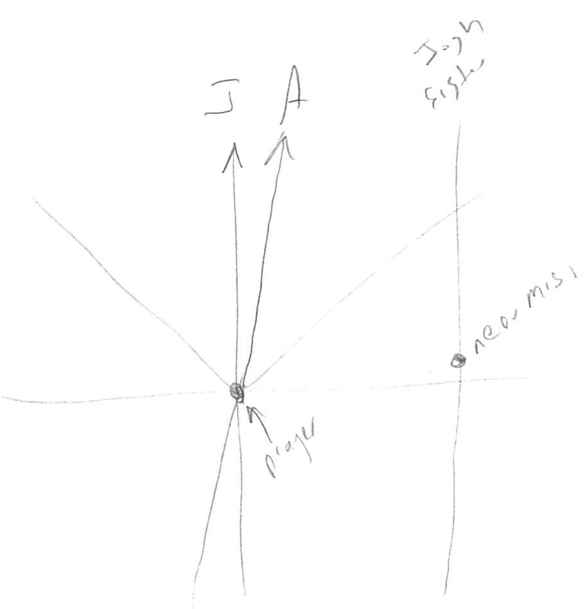
$$0 = \beta(-20 + 101\beta)$$

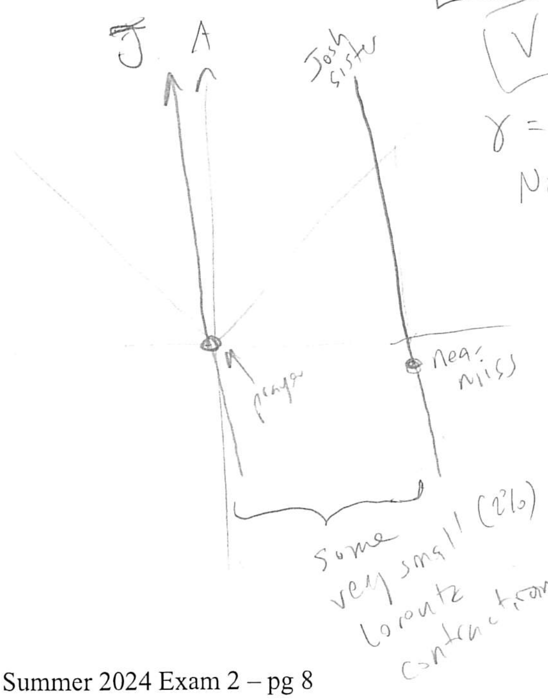
$$\boxed{\beta = \frac{20}{101}}$$

$$\boxed{V = .198c}$$

$\gamma = 1.0202$

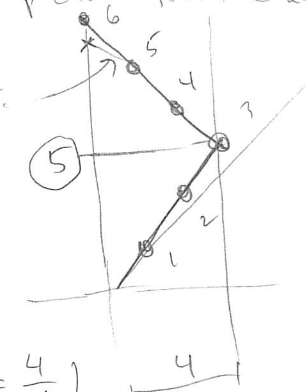
Not much length contraction!





(14 pts) **Problem 6**. In class we discussed the twin paradox in this context: Josh and Andrew are twins. Josh leaves the Earth on their birthday in a rocket ship traveling at 0.8c (which has $\gamma=5/3$), going to a planet which is 4 light years away from the Earth. Josh arrives at the planet exactly 3 years (to him) later, having celebrated two birthdays along the way and the third right as he arrives. Then Josh immediately turns around and returns home at that same speed, celebrating another two birthdays on the trip and the final one (the sixth) just as he arrives back home. Therefore, to Josh himself, he has aged 6 years.

picture from class, Andrew frame



signal sent on 5th birthday

(a) To Andrew, how many years has Josh aged?

$\boxed{6 \text{ years}}$ they both must agree on that!

(b) To Andrew, how many years has Andrew aged?

$\boxed{10 \text{ years}}$

In Andrew's frame it take Josh 5 years to get there and 5 years to get back (since $t = d/v = \frac{4}{.8}$)

(b) On his fifth birthday away (to Josh), Josh sends a "Happy Birthday!" message to Andrew at the speed of light. When exactly does that message reach Andrew? Hint: the coordinates of a point on a line which is a fraction f along the way between two endpoints is $(1-f) \times (coords. of\ starting\ point) + f \times (coords. of\ ending\ point)$.

Pt 5 is $2/3$ of the way from 3 to 6

So coords of pt 5 are $\frac{1}{3}(4,5) + \frac{2}{3}(10,0)$

$= \left(\frac{4}{3}, \frac{25}{3}\right) = (1.333, 8.333)$

The message travels at a 45° angle line, so it goes left $\frac{4}{3}$ and up $\frac{4}{3}$, putting its final coords at $\left(0, \frac{29}{3}\right)$

Andrew gets the message at $\boxed{9.667 \text{ years}}$

so just .333 years before Josh arrives home for 6^{th} birthday.